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Aspects of Tournament Models: A Survey*

A recent development in the literature on contracting is the tournament model. The tournament, by offering a "carrot" to the winner or a "stick" to the loser, can elicit effort to mitigate the moral hazard problem present if the monitoring of input is costly. Tournament models have obvious applications in sports, and also apply to executive compensation, "up-or-out" policies in partnerships and academia, and of course to grading on a pure curve. Under a broader interpretation, the principal features of tournaments apply to any compensation scheme which bases pay on relative performance.

The purpose of this survey is to collect the results from the small but growing literature on tournaments. Since the several papers to be surveyed use disparate notations, several specifications of preferences and production, and a variety of extensions on the basic model, it is difficult to catalogue the results across papers. I develop a consistent notation to compare the many results across model specifications. Quadratic approximations to preferences and the cost of effort are employed, and symmetry of a particular density function is assumed. I do this to focus on the main results, as the details are available in the cited papers. The simplifications are quite powerful in highlighting the common structure of the solutions across models and specifications.

Although the paper is primarily a survey, several original contributions and extensions are also provided. For instance, I establish general

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conditions under which tournaments with risk neutral contestants elicit the first-best level of effort; and I develop structural implications in a regression setting which are testable, if not on naturally generated data, on experimental data.

The analysis is divided into five main sections and a conclusion. In the first section, I set up the contracting framework, and analyze the two-contestant tournament with comparisons to simple individualistic contracts, and variations on the basic tournament such as tournaments with "gaps" and endogenous "precision," payment by relative performance, and unfair tournaments. Comparisons are made across models and specifications, and for risk neutral versus risk averse agents. In section II, the tournament model is extended to the case of n contestants with up to n prizes. Allowing for heterogeneous contestants, implications for self-sorting, entry credentials and up-front money are generated in section III. I evaluate the advantages and disadvantages of tournaments relative to some simple contracts in section IV. In section V, I use these evaluations to indicate some testable implications regarding the conditions under which tournaments are likely to be observed. Also in section V, I develop structural implications of the tournament model which are testable in the regression setting. The empirical evidence from the literature is also discussed.

I. CONTRACTS AND TWO-CONTESTANT TOURNAMENTS

I begin by examining the general contract model which nests the tournament as a special case. Assume firms are risk neutral elements of a competitive industry, so expected profit is zero in equilibrium. To be competitive the firm offers an employment contract which satisfies the zero expected profit condition, is consistent with each employee's incentives given incomplete monitoring, and maximizes each employee's expected utility. That is, the firm's problem for some employee i is to choose the contract parameters R and effort level μ_i to

$$(1.0) \qquad \underset{\{R;\mu_i\}}{\text{Max}} \; EU_i = \int U(y_i, \mu_i) dJ(y_i)$$

subject to

$$(1.1) \qquad P \cdot E[q_i] = E[y_i]$$

$$(1.2)) \qquad \mu_i = \mu^*(\eta) \in \underset{\{\mu_i\}}{\text{argmax}} \left[\int U[\Phi(q_1, \ldots, q_n; R), \mu_i] dF(\epsilon) \right]$$

$$(1.3) \qquad y_i = \Phi(q_1, \ldots, q_n; R)$$

$$(1.4) \qquad q_i = q(\mu_i, \epsilon_i, \eta).$$

The random variable y_i is the payment, and its cumulative distribution function is denoted J. Φ defines the class of admissible contractual payment schemes with parameters R; e.g., tournament, linear piece rate, quota, etc.. Output for employee or agent i, i = 1, ..., n, is denoted q_i. Constraint (1.4) is the output equation which determines how effort and the stochastic elements interact. The stochastic environment is divided into the n individual-specific components ϵ_i and the common component η. The n+1 disturbances are distributed independently with cumulative distribution functions $F(\epsilon_i)$ and $H(\eta)$. The zero-profit condition (1.1) with product price P incorporates the assumption of independence in the production process across agents. Equation (1.2) is the incentive compatibility constraint with equation (1.4) implicit in the arguments of its integrand. Note that in equation (1.2), the expectation is not taken over η because η is assumed to be observed by agent i prior to his choice of effort μ_i.

The core sections of this paper consider two alternative forms of the utility function and output equation. In introducing the concept of tournaments, Lazear and Rosen (1981) employ a utility function in which the effect of income on effort is zero, and an output equation which is additive in effort and the disturbances.[1]

$$(2.0) \qquad U(y_i, \mu_i) = U[y_i - C(\mu_i)], \qquad U', C', C'' > 0, \qquad U'' \leq 0;$$

$$(2.1) \qquad q_i = \mu_i + \epsilon_i + \eta, \qquad \mu_i \geq 0, \; E[\epsilon_i] = E[\eta] = 0.$$

Nalebuff and Stiglitz (1983) employ a preference structure separable in income and effort, and an output equation with a multiplicative common disturbance.[2]

$$(3.0) \qquad U(y_i, \mu_i) = U(y_i) - V(\mu_i), \qquad U', V', V'' > 0, \qquad U'' \leq 0;$$

$$(3.1) \qquad q_i = \eta\mu_i + \epsilon_i, \qquad \mu_i \geq 0, \quad E[\epsilon_i] = 0, \text{ and } E[\eta] = 1.$$

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[1]The presence of η makes this output equation more general than the one utilized in most of Lazear and Rosen's (1981) paper.

[2]Under risk neutrality with the normalization that $U' = 1$, and constant $\eta = E[\eta]$, both specifications reduce to: $U(\cdot) = y_i - C(\mu_i)$; $q_i = \mu_i + \epsilon_i$.

In analyzing the solution to the basic two-contestant tournament, variations on its theme, and simple individualistic contracts, I refer to the first-best level of effort. This is the solution to the problem of maximizing the value of output net of the cost of effort, and has a familiar form: the value of the marginal product of effort equals the marginal cost of effort. For the Lazear-Rosen specification, the first-best effort level satisfies $P = C'(\mu_i)$. For Nalebuff and Stiglitz, $P\eta = V'(\mu_i)/U'(\tilde{y}) \equiv C'(\mu_i)$.[3] The substantive difference is that with a multiplicative disturbance, the marginal product of effort, and hence the first-best level of effort, depends on η.

Two-Contestant Tournaments

Letting j denote the "other" contestant, the payment scheme for a two-contestant tournament is defined by the two parameter class

$$
\begin{aligned}
(4) \qquad y_i &= w_1 \text{ if } q_i > q_j, \\
&= w_2 \text{ otherwise,}
\end{aligned}
$$

with $w_1 > w_2$. With $EU_i = \int EU_i(\eta)h(\eta)d\eta$, the expected utility conditional on η for the tournament problem is

$$
(5) \qquad EU_i(\eta) = \{1 - \Pr[q_i < q_j]\}\cdot U(w_1, \mu_i) + \Pr[q_i < q_j]\cdot U(w_2, \mu_i).
$$

Substituting Lazear and Rosen's utility function (2.0) and output equation

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[3] I normalize V' to units comparable with the C' of Lazear and Rosen. This facilitates the comparisons below.

(2.1) into equation (5) gives

$$(6) \qquad EU_i(\eta) = [1 - G(\mu_j-\mu_i)]\cdot U[w_1 - C(\mu_i)] + G(\mu_j-\mu_i)\cdot U[w_2 - C(\mu_i)].$$

G denotes the distribution function for the random variable $\xi \equiv \epsilon_i - \epsilon_j$. Its density g is assumed to be symmetric, and concave at a single mode. By the additive property of the disturbances, η falls out of the expression.

Next consider the incentive compatibility condition in the Lazear-Rosen tournament. μ^* maximizes (6) with respect to μ_i with $\mu_j = \mu^*$, that is at the Nash solution. Therefore, the symmetric Nash solution $\mu^*(w_1, w_2, g(0))$ is the root of the following equation:

$$(7) \qquad C'(\mu_i) = 2g(0)\cdot\left[\frac{U(y_i^1) - U(y_i^2)}{U'(y_i^1) + U'(y_i^2)}\right],$$

with y_i^1 and y_i^2 denoting $w_1 - C(\mu_i)$ and $w_2 - C(\mu_i)$ respectively. Notice that μ^* is not a function of η.

The zero expected-profit condition for the Lazear-Rosen tournament is

$$(8) \qquad P\mu_i = [1 - G(\mu_j-\mu_i)]\cdot w_1 + G(\mu_j-\mu_i)\cdot w_2 = (w_1 + w_2)/2.$$

The simplification follows at the Nash solution since $G(0) = 1/2$ by the symmetry assumption. Invoking the Nash solution throughout and collecting these three results generates Lazear and Rosen's tournament problem:

$$\text{(9.0)} \qquad \underset{\{w_1, w_2; \mu_i\}}{\text{Max}} \; EU_i = ½\{U[w_1 - C(\mu_i)] + U[w_2 - C(\mu_i)]\}$$

subject to

$$\text{(9.1)} \qquad P\mu_i = (w_1 + w_2)/2$$

$$\text{(9.2)} \qquad C'(\mu_i) = 2g(0)\cdot\left[\frac{U[w_1-C(\mu_i)] - U[w_2-C(\mu_i)]}{U'[w_1-C(\mu_i)] + U'[w_2-C(\mu_i)]}\right].$$

Next consider Nalebuff and Stiglitz's variety of the tournament problem. Expected utility conditional on η is

$$\text{(10)} \qquad EU_i(\eta) = \{1 - G[\eta\cdot(\mu_j-\mu_i)]\}\cdot U(w_1) + G[\eta\cdot(\mu_j-\mu_i)]\cdot U(w_2) - V(\mu_i).$$

The incentive compatibility condition is

$$\text{(11)} \qquad V'(\mu_i) = \eta g(0)\cdot[U(w_1) - U(w_2)].$$

Notice that for this specification μ^* is a function of η as well as of w_1, w_2, and g(0). The zero expected profit condition is

$$\text{(12)} \qquad P\cdot E[\eta\mu_i] = (w_1 + w_2)/2,$$

recognizing that μ_i will depend on η. Collecting these results yields the problem for the Nalebuff-Stiglitz tournament:

$$\max_{\{w_1, w_2; \mu_i\}} EU_i = ½\{U(w_1) + U(w_2)\} - E[V(\mu_i)] \tag{13.0}$$

subject to

$$PE[\eta\mu_i] = (w_1 + w_2)/2 \tag{13.1}$$

$$V'(\mu_i) = \eta g(0)\cdot[U(w_1) - U(w_2)]. \tag{13.2}$$

In analyzing the properties of the solutions to these two tournament problems, interest focuses on the prize spread $\Delta w \equiv w_1 - w_2$ and the level of effort μ^*. Lazear and Rosen (1981, 844-46) show that under risk neutrality a tournament is capable of generating the first-best effort level, and hence first-best utility. Similarly, Nalebuff and Stiglitz (1983, 27-28) indicate that their tournament achieves "the full information first-best outcome" if (and only if) contestants are risk neutral. The result on the optimality of the tournament under risk neutrality is more general than either of these two specifications.

<u>Proposition</u>: For any set of preferences that reduce under risk neutrality to a form additively separable in income and effort, the optimal tournament with risk neutral agents attains the first-best effort level if and only if output takes the form

$$q_i = q^1(\mu_i, \eta) + q^2(\epsilon_i, \eta). \tag{14}$$

A sketch of the proof is as follows. If more generally $q_i = q(\mu_i, \epsilon_i, \eta)$, then the first-best level of effort solves $C'(\mu_i) = P\cdot\partial q/\partial\mu_i(\mu_i, \epsilon_i, \eta)$. For this most general form, first-best effort depends on ϵ_i. Since ϵ_i is not observable to the agent prior to his choice of effort, the optimal tournament cannot in general induce first-best effort. Necessity is immediate since equation (14) is the least restrictive specification of output which is separable in effort and the individual-specific disturbance. Establishing sufficiency is a straightforward exercise which is included in an appendix.

The output equations employed in the Lazear-Rosen and Nalebuff-Stiglitz specifications satisfy restriction (14), so both tournament models are first-best under risk neutrality. For both parameterizations, the optimal prize spread is $\Delta w^{*} = P/g(0)$. The invariance of the prize spread to the specification of the common disturbance in the output equation does not survive the generalization to risk aversion.

Under risk aversion, the analysis becomes complicated. Since the employee is interested in higher moments of his income distribution, the solution admits suboptimal effort in exchange for a degree of income insurance. I examine the properties of the second-best solution in three ways: by comparing it with the first-best solution, by making comparisons across the two model specifications, and through comparative static results with respect to product price, the degree of risk aversion, the curvature of the cost of effort, the riskiness of the distribution G, and the variance of the common disturbance.

Analysis of the properties of the two specifications, equations (9.0)-(9.2) and (13.0)-(13.2), is formidable. I follow Lazear and Rosen in solving the quadratic approximation to these problems, and in doing so, limit the higher moments to variances. For the Lazear-Rosen specification, a second-order Taylor series expansion of the utility function around $\bar{y}_i = (w_1 +$

$w_2)/2 - C(\mu_i)$ reduces the objective function given by equation (9.0) to

$$\text{(15)} \qquad EU_i \approx \tfrac{1}{2}\{U(\bar{y}_i) + U'(\bar{y}_i)(y_i^1 - \bar{y}_i) + \tfrac{1}{2}U''(\bar{y}_i)(y_i^1 - \bar{y}_i)^2\} + \tfrac{1}{2}\{U(\bar{y}_i) + U'(\bar{y}_i)(y_i^2 - \bar{y}_i) + \tfrac{1}{2}U''(\bar{y}_i)(y_i^2 - \bar{y}_i)^2\}$$

$$\approx U(\bar{y}_i) + \tfrac{1}{2}U''(\bar{y}_i)(\Delta w/2)^2.$$

Using the same approximation method, the incentive compatibility constraint (9.2) reduces to the risk neutral form, $C'(\mu_i) = g(0)\Delta w$. Collecting these approximation results generates the following streamlined problem:

$$\text{(16.0)} \qquad \underset{\{w_1, w_2; \mu_i\}}{\text{Max}} \; EU_i \equiv U[(w_1 + w_2)/2 - C(\mu_i)] + \tfrac{1}{2}U''(\bar{y}_i)\left[\frac{w_1 - w_2}{2}\right]^2$$

subject to

$$\text{(16.1)} \qquad P\mu_i = (w_1 + w_2)/2$$

$$\text{(16.2)} \qquad C'(\mu_i) = g(0)(w_1 - w_2).$$

Substituting the constraints into the objective yields the unconstrained problem.

$$\text{(17)} \qquad \underset{\Delta w}{\text{Max}} \; EU_i = U[P\mu^*(g\Delta w) - C(\mu^*(g\Delta w))] + \tfrac{1}{2}U''(\bar{y}_i)(\Delta w/2)^2.$$

I ignore terms of order three in deriving the marginal condition

$$\text{(18)} \qquad \frac{\partial EU_i}{\partial \Delta w} = U'(\bar{y}_i)\cdot\left[\frac{P - C'}{C''}\right]\cdot g(0) + \frac{U''(\bar{y}_i)\cdot\Delta w}{4} = 0.$$

(Note that $d\mu^*/d\Delta w = g(0)/C''$.) Using the incentive compatibility condition to substitute $g(0)\Delta w$ for C', and the definition of the coefficient of absolute risk aversion $S \equiv -U''/U'$, the solution of equation (18) for the optimal prize spread is

$$\text{(19)} \qquad \Delta w^* = \frac{g(0)P}{g(0)^2 + SC''/4}.$$

The tournament's optimal prize spread is increasing in product price and decreasing in the degree of risk aversion and curvature of the cost of effort.[4] Since I have restricted g to be symmetric and unimodal, the smaller is g(0) the riskier is the distribution in the sense of a mean preserving spread. The riskiness of the distribution of the individual specific disturbances has an ambiguous effect on the prize spread. For large S and σ_ϵ^2, $d\Delta w^*/dg(0) > 0$, but if $S = 0$, $d\Delta w^*/dg(0) < 0$.

From equation (16.2), the solution for effort μ^* in the optimal tournament is implicitly defined by

$$\text{(20)} \qquad C'(\mu^*) = \frac{P}{1 + SC''/4g(0)^2}.$$

Effort is increasing in product price, and decreasing in the degree of risk

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[4]To simplify the comparative static analysis, I treat S and C" as constants.

aversion, the curvature of the cost of effort and the riskiness of the distribution function G.[5] Note also that with risk averse contestants ($S > 0$), the optimal effort level μ^* falls short of the first-best level. Risk averse contestants accept insufficient incentives in exchange for income insurance. Hence risk aversion narrows the prize spread which lowers effort.

Nalebuff and Stiglitz's parameterization of the two-contestant tournament is burdened by the same formidable algebraic complications. By employing a similar quadratic approximation the unconstrained problem is reduced to

$$(21) \qquad \max_{\Delta w} EU_i = U[P \cdot E(\eta\mu^*)] + \tfrac{1}{2}U''(\tilde{y})(\Delta w/2)^2 - E[V(\mu^*)],$$

with $\tilde{y} \equiv (w_1 + w_2)/2$. The solution for the optimal prize spread is

$$(22) \qquad \Delta w^* = \frac{g(0)P}{g(0)^2 + SC''/4(1 + \sigma_\eta^2)}.$$

Optimal effort μ^* is determined by

$$(23) \qquad C'(\mu^*) = \frac{P\eta}{1 + SC''/4g(0)^2(1 + \sigma_\eta^2)}.$$

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[5]Lazear and Rosen assume g is a normal density. Thus $g(0) = 1/\sqrt{2\pi}\,\sigma_\epsilon$, and equation (20) becomes

$$C'(\mu^*) = \frac{P}{1 + SC''\pi\sigma_\epsilon^2}.$$

Clearly, μ^* is decreasing in σ_ϵ^2.

The same qualitative results for P, S, C" and g(0) hold for the Nalebuff-Stiglitz parameterization; in addition, both the prize spread and effort are increasing in the variance of the common disturbance σ_η^2. Because the tournament differences out the η component for considerations of rank order, all that is left is the effect on the marginal product of effort. If the employees were to ignore η in determining their effort, they would reach the Lazear-Rosen solution independent of σ_η^2; therefore, not only does exploiting the η variation improve welfare, but <u>expected utility is increasing in σ_η^2</u> (Nalebuff and Stiglitz 1983, 29-30). Furthermore, for σ_η^2 sufficiently large, effort in the optimal tournament approaches the first-best level even under risk aversion. Nalebuff and Stiglitz (1983, 29) assert that μ^* can exceed the first-best, but this is not true in the quadratic approximation.

The Lazear-Rosen and Nalebuff-Stiglitz specifications differ in two ways: the forms of both the utility function and the output equation are different. However, the solutions to the two quadratically approximated problems reveal that it is the multiplicative disturbance in the output equation that drives the difference in results. In particular, setting $\sigma_\eta^2 = 0$ (with $\eta = 1$) in equations (22) and (23), the two solutions become identical even though the quadratically approximated preferences differ.

Comparison with Individualistic Contracts

Lazear and Rosen, and Nalebuff and Stiglitz compare their tournament models with some simple individualistic payment schemes, such as the linear piece rate and the quota. With a linear piece rate, the payment scheme Φ takes the form

(24) $$y_i = I + rq_i.$$

The contract parameters $R = \{I, r\}$ are a guarantee I and a piece rate r. For the Lazear-Rosen specification (augmented to include η), the problem is:

(25.0) $$\max_{\{I,r\}} EU_i = \int U[I + r\cdot(\mu_i + \epsilon_i + \eta) - C(\mu_i)]f(\epsilon_i)h(\eta)d\epsilon_i d\eta$$

subject to

(25.1) $$P\mu_i = I + r\mu_i$$

(25.2) $$C'(\mu_i) = r.$$

Again μ^* is independent of η. The marginal condition for the unconstrained version of this problem is

(26) $$\frac{\partial EU_i}{\partial r} = E\{U'\cdot[(P - C')\frac{d\mu^*}{dr} + (\epsilon_i + \eta)]\} = 0$$

$$= \left[\frac{P - C'}{C''}\right]\cdot EU' + E[U'\cdot(\epsilon_i + \eta)] = 0.$$

The second step follows from $d\mu^*/dr = 1/C''$ by equation (25.2). Take a Taylor series expansion of U' around $\bar{y}_i = I + r\mu_i$ and collect terms to obtain

(27) $$r^* = C'(\mu^*) = \frac{P}{1 + SC''\cdot(\sigma_\epsilon^2 + \sigma_\eta^2)}.$$

With respect to P, S, C" and σ_ϵ^2 the results are similar to those of the tournament. More curvature of utility or of the cost of effort, and a riskier distribution G, reduces effort; and if S = 0, then equation (27) reduces to $r^* = C'(\mu^*) = P$ --the first-best solution. But notice the effect of higher σ_η^2 is to reduce effort. This is in contrast with the Lazear-Rosen tournament in which σ_η^2 has no effect, and the Nalebuff-Stiglitz tournament in which effort is increasing in σ_η^2.[6]

A second simple individualistic contract is the quota or standard. The payment scheme takes the following form for the quota:

$$(28) \qquad \begin{aligned} y_i &= w_1 \text{ if } q_i > \bar{q} \\ &= w_2 \text{ otherwise,} \end{aligned}$$

where $\bar{q}$ is the level of the quota. Although the quota appears to be a simplification of the tournament since the former does not depend on q_j, it is actually more difficult to derive the solution to the optimal quota problem if agents are risk averse. The tournament exploits the Nash solution to set the probability of winning to 1/2, but this cannot be done for the quota. Without cranking through the model, I identify three important features of the quota: First, in the absence of a common disturbance, the optimal quota weakly dominates the optimal tournament because the quota does not subject agent i to

[6]The solution to the quadratic approximation to the linear piece rate problem under the Nalebuff-Stiglitz specification is messy. It depends on such terms as the covariance between $\eta\mu^*$ and η^2. See Nalebuff and Stiglitz (1983, 34-36) for the analysis without approximations.

agent j's individual-specific risk. Second, the optimal quota is not invariant to the distribution of the common disturbance; the quota does not wash out η. Third, under risk neutrality, the optimal quota obtains the first-best level of effort if the common distubance is additive (Lazear and Rosen 1981, 848), but not if the common disturbance is multiplicative (Nalebuff and Stiglitz 1983, 35).

Variations on the Tournament Theme

Nalebuff and Stiglitz (1983), and O'Keeffe, Viscusi and Zeckhauser (1984) offer various extensions on the basic two-contestant tournament. In this section, I present four variations: winning by a gap, payment by relative performance, endogenous precision, and unfair tournaments.

Winning by a gap introduces a third outcome to the tournament: a draw. With a gap of size γ, contestant i wins if $q_i - q_j > \gamma$, loses if $q_j - q_i > \gamma$, and if contestant i neither wins nor loses, the tournament is a draw paying both contestants $(w_1 + w_2)/2$. Clearly, the basic two-contestant tournament is nested within the tournament with a gap, namely $\gamma = 0$. Does the optimal tournament have a positive gap? To answer this question, one solves for the optimal gap γ^* to determine whether $\gamma^* > 0$. In the Nash equilibrium, the probability of winning is $1 - G(\gamma)$ which by the symmetry of g equals $G(-\gamma)$; thus a draw occurs with probability $1 - 2G(-\gamma)$. For the Lazear-Rosen specification, the problem is as follows:

$$(29.0) \qquad \underset{\{w_1, w_2, \gamma; \mu_i\}}{\text{Max}} \; EU_i = G(-\gamma)\{U[w_1 - C(\mu_i)] + U[w_2 - C(\mu_i)]\} + [1 - 2G(-\gamma)]\cdot U[(w_1 + w_2)/2 - C(\mu_i)]$$

subject to

$$(29.1) \qquad P\mu_i = (w_1 + w_2)/2$$

$$(29.2) \qquad C'(\mu_i) = \frac{g(-\gamma)[U(y_i^1) - U(y_i^2)]}{G(-\gamma)[U'(y_i^1) + U'(y_i^2)] + [1 - 2G(-\gamma)]U'(\bar{y}_i)}.$$

Since this problem is more complicated than the basic two-contestant tournament, I immediately restrict the analysis to the quadratic approximation. Using the same Taylor series expansions, the quadratically approximated problem is:

$$(30.0) \qquad \underset{\{w_1, w_2, \gamma; \mu_i\}}{\text{Max}} \; EU_i = U[(w_1 + w_2)/2 - C(\mu_i)] + G(-\gamma)U''(\bar{y}_i)(\Delta w/2)^2$$

subject to

$$(30.1) \qquad P\mu_i = (w_1 + w_2)/2$$

$$(30.2) \qquad C'(\mu_i) = g(-\gamma)\Delta w.$$

With $\gamma = 0$, equations (30.0)-(30.2) reduce to (9.0)-(9.2).

Substituting in the constraints and maximizing with respect to Δw and γ gives the following two marginal conditions:

$$(31.1) \qquad C'(\mu^*) = \frac{P}{1 + SC''G(-\gamma)/2g(-\gamma)^2}$$

$$(31.2) \qquad C'(\mu^{*}) = \frac{Pg'(-\gamma)}{g'(-\gamma) + SC''/4}.$$

Equations (31.1) and (31.2) are solved simultaneously to get γ^{*} as the root of $g(-\gamma)^2 = 2g'(-\gamma)G(-\gamma)$. To see that the optimal gap is positive for risk averse agents, note that $\gamma = 0$ does not satisfy this condition because $g'(0) = 0$ and $g(0)^2 > 0$. Also note that $\gamma < 0$ cannot satisfy this condition either because $g'(-\gamma) < 0$ for $\gamma < 0$. Therefore, if it exists, the optimum gap γ^{*} is positive. It turns out that effort is higher in the tournament with a gap. Because $S = 0$ for risk neutral contestants, equations (31.1) and (31.2) reduce to equivalent relations under risk neutrality; that is, one marginal condition is redundant. Any combination of the pair $(\Delta w, \gamma)$ satisfying $P = g(-\gamma)\Delta w$ is a first-best solution.[7]

Payment by relative performance, a scheme introduced by Nalebuff and Stiglitz (1983, 36-37), is a combination of a tournament and a piece rate. The scheme pays agent i

$$(32) \qquad y_i = I + r\cdot(q_i - q_j).$$

It is not an individualistic contract since pay depends on the performance of the other agent q_j; but the purely rank-order property of the tournament is lost because distance matters.

Using the symmetric Nash solution to simplify the objective and both

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[7]The solutions for the Nalebuff-Stiglitz specification are similar. The optimal gap γ^{*} is invariant to the multiplicative versus additive specifications of η, even though Δw^{*} and μ^{*} are not.

constraints, the framework for the payment-by-relative-performance problem under the Lazear-Rosen specification is:

$$\text{(33.0)}\qquad \underset{\{I,r;\mu_i\}}{\text{Max}}\ EU_i = \int U[I + r\cdot(\epsilon_i - \epsilon_j) - C(\mu_i)]f(\epsilon_i)f(\epsilon_j)d\epsilon_i d\epsilon_j$$

subject to

$$\text{(33.1)}\qquad P\mu_i = I$$

$$\text{(33.2)}\qquad C'(\mu_i) = r.$$

Notice that η differences out of the objective. Replacing I with $P\mu$ by the zero-profit constraint, and μ_i with μ^* by the incentive compatibility constraint, the marginal condition with respect to r is

$$\text{(34)}\qquad \left[\frac{P - C'}{C''}\right]\cdot EU' + E[U'\cdot(\epsilon_i - \epsilon_j)] = 0.$$

Since the second term is negative if agents are risk averse, $C'(\mu^*) < P$ follows immediately. The first-best effort level does not obtain. Using the familiar Taylor series expansion to approximate equation (34) yields

$$\text{(35)}\qquad r^* = C'(\mu^*) = \frac{P}{1 + 2SC''\sigma_\epsilon^2}.$$

The relative piece rate is less than--but is increasing in--P, and is falling in S, C" and σ_ϵ^2. These results are qualitatively consistent with the solution to the linear piece rate problem, but the latter is not invariant to the variance of the common disturbance. Relative to the pure piece rate, payment by relative performance accepts additional individual-specific risk in exchange for lower common risk. If $\sigma_\eta^2 > \sigma_\epsilon^2$, effort under the relative performance scheme is closer to the first-best, and the variance of income is reduced as well. Comparison of relative-performance contracts with tournaments is most direct if ϵ_i and ϵ_j are distributed normally. Under normality, the optimal tournament induces greater effort than the optimal relative- performance contract because $\pi > 2$ (see note 5, above). Under risk neutrality, $r^* = P$ and the first-best effort level obtains.[8]

Nalebuff and Stiglitz (1983, 29) recognize that the pure strategy, symmetric, Nash equilibrium, which is used in solving the tournament models above, can result in a fundamental nonconvexity. That is, the Nash solution need not be sustainable. Since the Nash solution to the basic two-contestant tournament gives a probability of winning equal to one-half, the equilibrium effort level μ^* could be deathly but the probability of winning remains one-half. Beyond some critical effort level $\hat{\mu}$ it is better to shirk (avoiding all the disutility of effort) and collect the bottom prize. The question turns on whether the Nash solution is a sustainable or global solution. O'Keeffe, Viscusi and Zeckhauser (1984) (hereafter, OVZ) take up this topic in more detail for the risk neutral case.

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[8] For Nalebuff-Stiglitz, the solution is

$$C'(\mu^*) = \frac{P\eta}{1 + 2SC''\sigma_\epsilon^2/(1 + \sigma_\eta^2)} .$$

with the additional implication that effort is increasing in σ_η^2.

Consider the simplest case first. Let the probability of winning while shirking (i.e., setting $\mu_i = 0$) be zero. The global incentives condition is satisfied if

$$(36.0) \qquad EU_i(\mu^*) \equiv (w_1 + w_2)/2 - C(\mu^*) > w_2,$$

which reduces to $C(\mu^*) < \Delta w^*/2$. The global decision compares the cost of effort with the expected benefit which is one-half the prize spread. Define $\hat{\mu}$ as the root of $C(\hat{\mu}) = \Delta w^*/2$, or the level of effort which equalizes the expected values of shirking and working. The agent's decision problem is reduced to: work if $\mu^* < \hat{\mu}$, and shirk otherwise. Consequently, $\mu_i = \mu^*$ if $\mu^* < \hat{\mu}$, and $\mu_i = 0$ otherwise. Since $d\hat{\mu}/d\Delta w^* = 1/2C' > 0$, the global incentives condition is less likely to be satisfied for tournaments with small prize spreads, or equivalently, with high g(0).

OVZ treat monitoring precision--summarized by g(0)--as endogenous. Consider the case in which Δw^* is too small to satisfy global incentives. By decreasing the level of monitoring precision and increasing Δw^* to preserve the marginal condition, the Nash solution can be sustained. μ^* is unchanged and $\hat{\mu}$ is increasing in Δw^*; therefore there always exits some prize spead Δw^* sufficiently large to satisfy global incentives.

OVZ (1984, 36, n. 10) argue that the global incentives condition "deals with the difficulty Lazear and Rosen (1981, 845) describe with the second-order conditions." These are two conceptually distinct issues. Here I consider the problem of the second-order condition, the existence of a local Nash solution and the role of global incentives with $G(\mu^*) \neq 1$.

Using Lazear and Rosen's incentive compatibility condition under risk neutrality, it is straightforward to derive the slope of agent i's reaction function, and to sign the expression based on the properties of g:

$$\frac{d\mu_i}{d\mu_j} = \frac{g'(\mu_j - \mu_i)\Delta w}{C'' + g'(\mu_j - \mu_i)\Delta w} \tag{37.0}$$

$$d\mu_i/d\mu_j = 0 \text{ for } \mu_i = \mu_j \tag{37.1}$$

$$0 < d\mu_i/d\mu_j < 1 \text{ for } \mu_i > \mu_j \tag{37.2}$$

$$d\mu_i/d\mu_j < 0 \text{ for } \mu_i < \mu_j \tag{37.3}$$

$$\operatorname{sign}(d^2\mu_i/d\mu_j^2) = \operatorname{sign}(g''). \tag{37.4}$$

For $\mu_i < \mu_j$, there might be a problem with the second-order conditions as exhibited in the denominator of equation (37.0). With $\mu_j - \mu_i$ positive, the density might be steep enough to send the denominator to zero and eventually negative, in which case the extremum is not a maximum. If the density is bell-shaped, its slope falls (in absolute value) beyond an inflection point, therefore the reaction function might be defined for even larger $\mu_j - \mu_i$. For a given Δw, the properties in equations (37.0)-(37.4) are collected to depict contestant i's reaction function in Figure 1. A symmetric (to the 45° line) reaction function can be superimposed for contestant j. Note that by the properties of g, specifically that $g'(0) = 0$, a unique Nash equilibrium is guaranteed to exist locally.

The next step is to examine the role of global incentives in this model with $G(\mu^*) \neq 1$, that is, with the probability of winning while shirking positive. Abstracting from the possibility of multiple interior equilibria, the shirking solution is at $\mu_i = 0$. Recalling that the value of the distribution function G indicates the probability that contestant i loses, the global incentive condition becomes

$$C(\mu^*) < \Delta w^* \cdot [G(\mu^*) - \tfrac{1}{2}]. \tag{38}$$

Define the indifference relation $\hat{\mu}(\mu_j)$ for this problem by

$$\hat{\mu}(\mu_j) = \{\mu_i \mid C(\mu_i) = \Delta w^* \cdot [G(\mu_j) - \tfrac{1}{2}]\}. \tag{39}$$

$\hat{\mu}(\mu_j)$ is an increasing concave function approaching an asymptote $\hat{\mu}$. $\hat{\mu}(\mu_j)$ is superimposed on Figure 1. If $\mu^* < \hat{\mu}(\mu^*)$, the contestant works, otherwise he shirks. Of course, if g(0) is subject to control, the firm can decrease g(0) and increase Δw^* (maintaining μ^*) in order to shift up $\hat{\mu}(\mu_j)$ to satisfy global incentives.

The global incentives and second-order conditions problems are conceptually distinct. For densities with nice properties the local Nash solution is guaranteed to exist and to be unique, but it might not be sustainable globally.

OVZ consider another variation on the basic two-contestant tournament by examining unfair tournaments. A tournament is unfair if the probability of winning is not symmetric in the effort levels of the agents. For the risk neutral case, OVZ's conditions for effort in an unfair tournament with additive η are

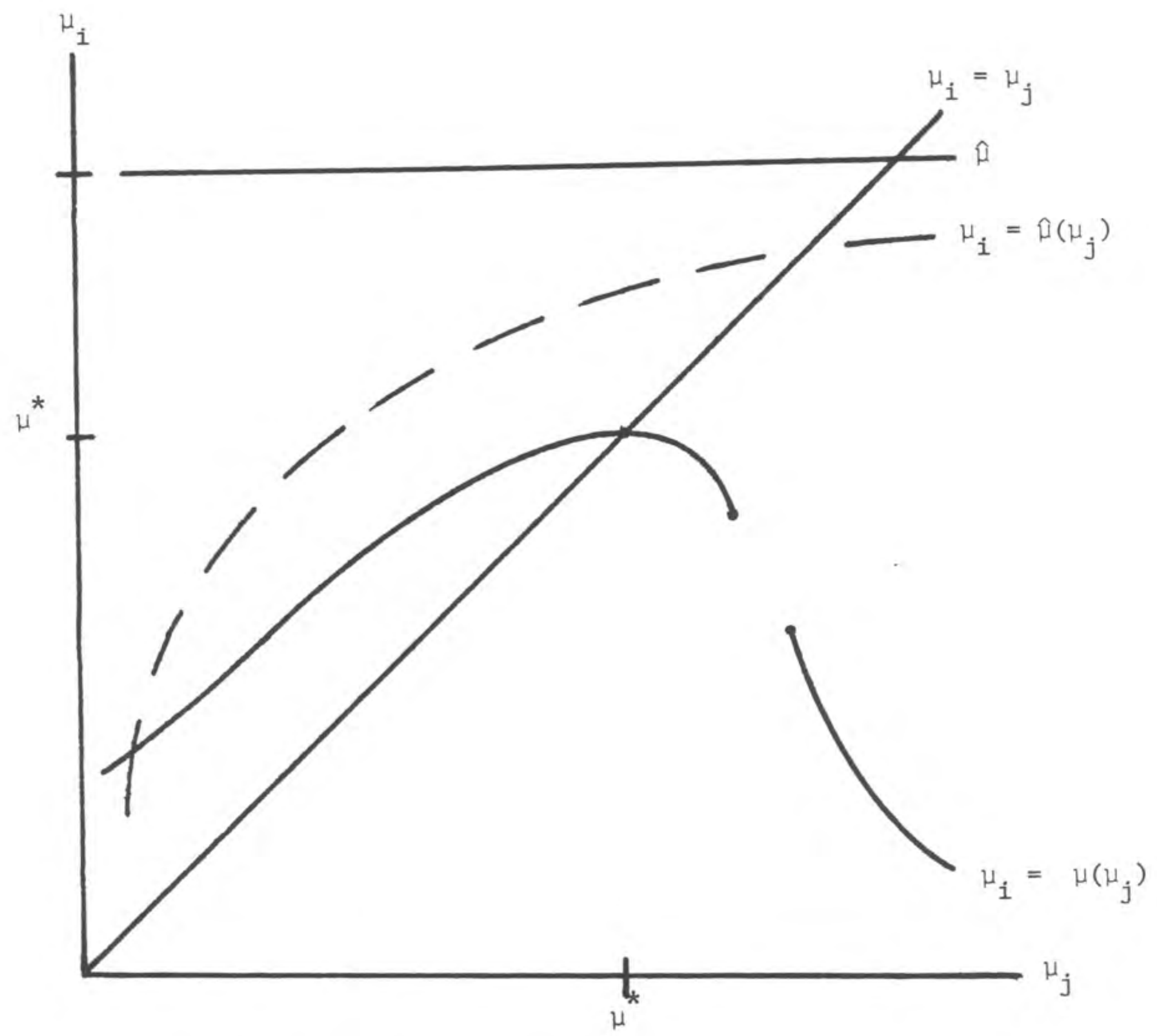

Figure 1. Nonconvexities and Global Incentives

$$g_i(\mu_i, \mu_j) = g_j(\mu_i, \mu_j) = P/\Delta w, \tag{40}$$

where g_i and g_j are the density functions for i and j respectively. If $g_i(\mu^*, \mu^*) = g_j(\mu^*, \mu^*)$, then the marginal conditions generate the first-best level of effort. Clearly, the fair tournament with $G_i = 1 - G(\mu_j - \mu_i)$ and $G_j = G(\mu_j - \mu_i)$ satisfies this condition, but equation (40) allows more general forms. To see this, consider the following variation on the Lazear-Rosen model: i wins if $q_i > q_j - h$, where the handicap h is a constant discrimination factor which favors i. The probability that i wins is $G_i = 1 - G(\mu_j - \mu_i - h)$, so $g_i = g(\mu_j - \mu_i - h)$ and $g_j = g(\mu_j - \mu_i - h)$. The tournament favors contestant i, but the marginal conditions are unaffected. Therefore the local solution is unaltered. The global condition is more difficult to satisfy if monitoring precision is exogenous. If the probability that contestant i loses while shirking, $G(\mu^* - h)$, equals one, then the global conditions for contestants i and j are

$$C(\mu^*) < [1 - G(-h)]\Delta w^* \quad \text{for i} \tag{41.0}$$

$$C(\mu^*) < G(-h)\Delta w^* \quad \text{for j.} \tag{41.1}$$

If $h > 0$, $G(-h) < ½$; therefore, contestant i's (j's) global condition is less (more) restrictive than if the handicap were set to zero. Only condition (41.1) is more restrictive than the global condition for the fair tournament. The more restrictive condition determines the minimal prize spread; with endogenous precision the minimal Δw^* is larger, so monitoring must be less precise.

II. TOURNAMENTS WITH MANY CONTESTANTS

The n-contestant tournaments analyzed by Nalebuff and Stiglitz (1983, 32-33, 37-38) and Green and Stokey (1983) offer conceptual extensions on the basic two-person tournament. The inquiry is extended in two directions: the first is to specify the effect the number of contestants n has on the prize structure, effort, and global incentives in n-contestant tournaments with a single prize. Although somewhat more complicated, the solutions exhibit the same basic structure; this allows a direct comparison of the n-person and two-person tournaments. The second extension analyzes the ability of multiple prize tournaments to approximate the optimal nonlinear contract.

Consider the multiple contestant tournament with n prizes $\{w_1, \ldots, w_n\}$, where n is the top rank. In an n-contestant tournament, the probability of placing *kth* from the bottom, given that each of the n-1 other contestants supply effort μ^* is

$$(42)\qquad \bar{F}_n(k) = \frac{(n-1)!}{(n-k)!(k-1)!} \int F(\epsilon_i+\mu_i-\mu^*)^{k-1} \, [1 - F(\epsilon_i+\mu_i-\mu^*)]^{n-k} \, f(\epsilon_i)d\epsilon_i,$$

recalling that $F(\epsilon_i + \mu_i - \mu^*) = \Pr[q_i > q_j]$ is the distribution function for each (i.i.d.) ϵ_j, evaluated at $\epsilon_i + \mu_i - \mu^*$. Since contestant i's objective function under the Lazear-Rosen specification is

$$(43)\qquad EU_i = \sum_{k=1}^{n} U[w_k - C(\mu_i)]\bar{F}_n(k),$$

the resultant incentive compatibility condition at the Nash solution is

$$(44.2) \qquad C'(\mu_i) = \frac{\sum_{k=1}^{n} U(y_i^k)\cdot \bar{f}_n(k)}{\frac{1}{n}\sum_{k=1}^{n} U'(y_i^k)}.$$

with $y_i^k \equiv w_k - C(\mu_i)$. In writing equation (44.2), the term $\partial\bar{F}_n/\partial\mu_i \equiv \bar{f}_n(k)$ is evaluated at $\mu_i = \mu^*$. Also note that $\bar{F}_n(k) = 1/n$ for all k in the Nash equilibrium. Collecting these results gives the general form for the multiple prize, n-contestant tournament:

$$(44.0) \qquad \underset{\{\{w_k\};\mu_i\}}{\text{Max}} \quad EU_i = \frac{1}{n}\sum_{k=1}^{n} U[w_k - C(\mu_i)]$$

subject to

$$(44.1) \qquad P\mu_i = \frac{1}{n}\sum_{k=1}^{n} w_k,$$

and equation (44.2).

<u>Single Prize</u>

Nalebuff and Stiglitz (1983, 32-33) work with a problem of similar form, but with only one prize. In this case, n contestants race for a single top prize w_n; the multiple prize tournament in equations (44.0)-(44.2) reduces to the n-contestant tournament problem with a single prize:

$$(45.0) \qquad \underset{\{w_1, w_n;\mu_i\}}{\text{Max}} \quad EU_i = [1 - \frac{1}{n}]\cdot U[w_1 - C(\mu_i)] + \frac{1}{n}\cdot U[w_n - C(\mu_i)]$$

subject to

$$(45.1) \qquad P\mu_i = [1 - \frac{1}{n}]\ w_1 + \frac{1}{n}\ w_n$$

$$(45.2) \qquad C'(\mu_i) = \frac{[U(y_i^n) - U(y_i^1)]\cdot \bar{f}_n(n)}{\frac{1}{n}\ U'(y_i^n) + (1 - \frac{1}{n})U'(y_i^1)}.$$

From equation (42), the probability of winning or placing *nth* in an n-contestant tournament is

$$(46.0) \qquad \bar{F}_n(n) = \int F(\epsilon_i + \mu_i - \mu^*)^{n-1}\ f(\epsilon_i)d\epsilon_i;$$

consequently, at the symmetric Nash equilibrium

$$(46.1) \qquad \bar{f}_n(n) = (n-1) \int F(\epsilon_i)^{n-2}\ f(\epsilon_i)^2 d\epsilon_i.$$

Note that if $n = 2$, $\bar{f}_n(n) = \int f(\epsilon_i)^2 d\epsilon_i = g(0)$. The second equality follows because the density of the difference between two i.i.d. random variables takes this form if evaluated at $\xi = 0$ (see Mood, Graybill, and Boes 1974, 185).

Using the Taylor series method to approximate the problem, the following optimal prize spread and effort for the n-contestant tournament are derived:

$$(47.0) \qquad \Delta w^* = \frac{\bar{f}_n(n)P}{\bar{f}_n(n)^2 + SC''/4}$$

$$(47.1) \qquad C'(\mu^*) = \frac{P}{1 + SC''/4\bar{f}_n(n)^2}.$$

The comparative static results with respect to product price, the degree of risk aversion and curvature of the cost of effort are qualitatively consistent with the $n = 2$ case. The point of interest here, however, is how the solutions are affected by tournament size. Although μ^* is not affected by n under risk neutrality (i.e., $S = 0$), the risk-neutral prize structure is not invariant to n. The prize spread is increasing in the number of contestants and in the limit $\Delta w^* \to \infty$. With n large, a marginal increase in effort has a negligible effect on the probability of winning. Hence a big prize spread is required to induce effort. More formally, the limit of $\bar{f}_n(n)$ as n goes to infinity is zero.[9] Under risk neutrality $\Delta w^* = P/\bar{f}_n(n)$, thus in the limit the prize spread is infinite.

Although the first-best effort level obtains in the symmetric Nash solution under risk neutrality, there might be a problem with global

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[9]Several conditions are sufficient to establish that the limit of $\bar{f}_n(n)$ is zero. (I thank John Boyd III for showing me the result which is derived using Lebesgue's Dominated Convergence Theorem.) Three sufficient conditions are that the derivitive of f is integrable, the limit of f as $\epsilon_i \to \infty$ is zero, and the support of ϵ_i is from $-\infty$ to ∞. Therefore the result does not necessarily hold for distributions of ϵ_i with bounded support. For example if ϵ_i is distributed uniformly over [a, b], then $\bar{f}_n(n) = 1/(b-a)$ and is independent of n. I have not been able to establish that $\bar{f}_n(n) \leq \bar{f}_n(n-1)$ for all $n > 2$.

incentives as the size of the tournament increases. Since effort is invariant to n, but the probability of winning the prize is declining in n, one might expect that the incentive to "give-up" is increasing in the number of contestants. Under risk neutrality the condition for the sustainability of the Nash solution is $C(\mu^*) < \Delta w^*/n$. Since the left-hand side is independent of n, $\Delta w^*/n$ must go to zero in the limit to support the conjecture that contestants give up in large tournaments. This would require that the limit as n goes to infinity of the product $n\bar{f}_n(n)$ is infinite. This result has not been established.

For the more general case of risk averse contestants, that $\bar{f}_n(n)$ decreases in the number of contestants implies that effort decreases in n and in the limit goes to zero. The limiting result for the prize spread is fragile: under risk aversion, the optimal prize spread goes to zero in the limit.

Nalebuff and Stiglitz (1983, 33-34) introduce a tournament with penalties. In this scheme, the top n-1 contestants receive the prize w_n, with the "rotten egg" being penalized. From equation (42), the probability of placing 1st or losing an n-contestant tournament is

$$(48.0) \qquad \bar{F}_n(1) = \int [1 - F(\epsilon_i + \mu_i - \mu^*)]^{n-1} f(\epsilon_i)d\epsilon_i;$$

at the symmetric Nash equilibrium

$$(48.1) \qquad \bar{f}_n(1) = -(n-1) \int [1 - F(\epsilon_i)]^{n-2} f(\epsilon_i)^2 d\epsilon_i < 0.$$

The tournament-with-penalty problem under the Lazear-Rosen specification is

$$(49.0) \qquad \max_{\{w_1, w_n; \mu_i\}} EU_i = U[w_n - C(\mu_i)] - \bar{F}_n(1)\{U[w_n - C(\mu_i)] - U[w_1 - C(\mu_i)]\}$$

subject to

$$(49.1) \qquad P\mu_i = \frac{1}{n} w_1 + (1 - \frac{1}{n})w_n$$

$$(49.2) \qquad C'(\mu_i) = \frac{- [U(y_i^n) - U(y_i^1)]\cdot\bar{f}_n(1)}{(1 - \frac{1}{n})U'(y_i^n) + \frac{1}{n} U'(y_i^1)}.$$

The solution to the quadratic approximation to this problem is

$$(50.0) \qquad \Delta w^* = \frac{- \bar{f}_n(1)P}{\bar{f}_n(1)^2 + SC''/4}$$

$$(50.1) \qquad C'(\mu^*) = \frac{P}{1 + SC''/4\bar{f}_n(1)^2}.$$

The qualitative results with respect to P, S and C" are unchanged. A penalty is the same as a prize if $n = 2$: $-\bar{f}_n(1) = g(0)$. Like $\bar{f}_n(n)$, the limit of $\bar{f}_n(1)$ as n goes to infinity is zero. Therefore, the limiting results for effort and the prize spread under both risk neutrality and risk aversion are the same for penalties as for prizes.

Nalebuff and Stiglitz introduce the penalty model to circumvent the global incentives problem. Under risk neutrality, the global condition with a penalty is $C(\mu^*) < (1 - 1/n)\Delta w^*$. Since the limit as n goes to infinity of (1

$- 1/n)\Delta w^*$ is infinite, the penalty model can be used as an alternative to the prize model if the latter suffers from the global incentives problem.[10]

Without considering the details of the problem, note that OVZ (1984, 38-39) introduce an n-contestant tournament with several winners of the prize. This is an intermediate case of the n-contestant prize and penalty models. The derivations are complicated and are omitted. The modification to the global condition is that $1/n$ is replaced by α, the fraction of prizes awarded. Of course, Δw^* depends on α not just n.

Multiple Prizes

Without solving a multiple prize, n-contestant tournament problem like the one given by equations (44.0)-(44.2), Green and Stokey (1983) present a series of lemmas and propositions which uncover a number of properties of these tournaments under risk aversion. In what follows, I state the main results and sketch the logic of the proofs.

In the absence of the common disturbance η, the optimal individualistic contract dominates the optimal tournament (Green and Stokey 1983, 356-58, Proposition 1 and Corollary 1). The proof is in two steps. First, a multiple quota contract can always be designed to replicate any feasible tournament in terms of expected payoffs. But by Jensen's inequality in conjunction with the concavity of the utility function, this multiple quota is preferred; it is not impaired by the variance of the other contestants' specific disturbances.

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[10]To get the solutions to the Nalebuff-Stiglitz specification of the model with a single prize or penalty, just replace g(0) with, respectively, $\bar{f}_n(n)$ or $\bar{f}_n(1)$ in equations (22) and (23).

Since there exists a multiple quota which dominates any tournament, there exists one which dominates the optimal tournament. Of course, this quota is potentially suboptimal. Therefore, the optimal individualistic contract, a fortiori, dominates the optimal tournament.

For a sufficiently diffuse density $h(\eta)$, the optimal tournament with n contestants yields an expected utility exceeding that of the optimal individualistic contract (Green and Stokey 1983, 358-59, Proposition 2). Two preliminary results are critical. First, for $h(\eta)$ sufficiently diffuse, the optimal individualistic contract exhibits no incentives. Agents fully insure, leaving no incentives. This is apparent in equation (27) above for the linear piece rate case. Effort is falling in σ_η^2, and in the limit goes to zero. In contrast, the solution to the tournament problem is invariant to $h(\eta)$ (Green and Stokey 1983, 356, Lemma 1), <u>if η is additive</u>. This is clear from the solutions to the Lazear-Rosen specification of the tournament problems; σ_η^2 is absent from, e.g., equations (19) and (20). Since the optimal tournament delivers at least as much expected utility as any no-incentive contract (or tournament), the optimal tournament eventually dominates.

As $n \to \infty$, the optimal tournament dominates the optimal individualistic contract (Green and Stokey 1983, 363, Proposition 3). To prove this proposition, Green and Stokey employ a number of intermediate results. First, any contract can be approximated arbitrarily closely by a piecewise continuous step function (Green and Stokey 1983, 360, Lemma 2). If n is high, there exists a tournament which approximates these step functions (Green and Stokey 1983, 362, Lemma 3). Therefore, there exists a tournament which approximates the η-known contract, and since tournaments are unaffected by $h(\eta)$, this tournament is independent of $h(\eta)$. But if η is not known, the optimal individualistic contract falls short of the η-known contract. Therefore, for large n, the optimal tournament dominates the optimal contract and approaches the optimal η-known contract.

As Nalebuff and Stiglitz (1983, 38) note, this last result depends critically on the additive nature of η since the tournament is not invariant to $h(\eta)$ for the multiplicative specification.

III. TOURNAMENTS WITH HETEROGENEOUS CONTESTANTS

The final extension of the tournament model is in relaxing the assumption of identical contestants. Generalizing the analysis to allow for heterogeneous contestants is rich with potentially refutable implications. The discussion is in two parts. First, asymmetries in the knowledge of abilities are shown to entail inefficiencies because contestants do not self-sort into their own leagues. Entry credentials and bigger prize spreads in the major leagues are two implications. In the second part, with full knowledge of abilities, uneven tournaments cause incentive problems. Handicapping and prize structures indexed by ability (e.g., up-front money to high-quality contestants) are implications generated by a model with competition from segregated tournaments (all high quality, and all low quality). Following Lazear and Rosen (1981, 857-63) and OVZ (1984, 42-46), the analysis is limited to the risk-neutral case with an additive common disturbance η. Throughout, differential ability is modeled as differential marginal costs of effort.

Self-Sorting

Each contestant knows his own ability, but has no information about the ability of his opponent. In a straightforward exercise, Lazear and Rosen (1981, 859-60) show that if heterogeneous contestants do not self-sort into their own leagues, the resultant mixed tournament with random assignments is inefficient. Taking this as given, I analyze Lazear and Rosen's second result on this topic: contestants do not self-sort into their own leagues. Specifically, <u>minor leaguers infiltrate the major leagues</u>.

Let a denote high ability, and b low ability, so $C_a'(\mu) < C_b'(\mu)$ for all μ. Let k index an individual's ability and ℓ ability level of a league. (For notational ease, I suppress the (i, j) subscripts denoting individual contestants.) The expected utility function of a contestant of type k in a tournament for ability ℓ contestants is

$$(51) \qquad EU_{k\ell}(\mu) = w_2^{\ell} + [1 - G(\mu_{\ell}^{*}-\mu)](w_1^{\ell} - w_2^{\ell}) - C_k(\mu)$$

where μ_{ℓ}^{*} is the root of $C_{\ell}'(\mu) = P$; therefore $\mu_a^{*} > \mu_b^{*}$. Assuming P and g(0) do not depend on the ability of the contestants, the optimal prize spread $\Delta w^{*} = P/g(0)$ is invariant across leagues. This result and the zero expected profit condition are used to rewrite equation (51).

$$(52) \qquad EU_{k\ell}(\mu) = P\mu_{\ell}^{*} + \frac{P}{g(0)}\,[1/2 - G(\mu_{\ell}^{*}-\mu)] - C_k(\mu)$$

$$\equiv R_{\ell}(\mu) - C_k(\mu).$$

The proof of the proposition that minor leaguers infiltrate the senior circuit is based on the properties of the expected revenue function $R_{\ell}(\mu)$. Taking a third-order Taylor series expansion of $G(\mu_{\ell}^{*}-\mu)$ around $G(0) = 1/2$, (53.0) $$R_\ell(\mu) = P\mu - \frac{Pg''(0)}{6g(0)} (\mu_\ell^* - \mu)^3.$$

Because $g''(0) < 0$, it follows immediately that

(53.1) $$R_\ell(\mu) \lesseqgtr P\mu \quad \text{for all} \quad \mu \lesseqgtr \mu_\ell^*.$$

$R_\ell(\mu)$ is an increasing function with an inflection point at $\mu = \mu_\ell^*$, since

(54) $$R_\ell'(\mu) = \frac{P}{g(0)} g(\mu_\ell^* - \mu).$$

Therefore, $R_\ell'(\mu_\ell^*) = P$ and $R_\ell'(\mu) < P$ for all $\mu \neq \mu_\ell^*$.[11] These results are collected to illustrate $R_a(\mu)$ and $R_b(\mu)$ in Figure 2. The expected revenue functions do not cross.

Without regard to costs, it is shown that minor leaguers climb to the majors. For any effort level the cost of effort for a minor leaguer is invariant to the league in which he plays. Therefore, the difference in his expected utility at $\mu = \mu_b^*$ is just $R_a(\mu_b^*) - R_b(\mu_b^*) > 0$ by the result illustrated in Figure 2.[12] Of course, a minor leaguer playing in the major

[11]It also holds that $R_a'(\mu) = \frac{P}{g(0)} g(\mu_a^* - \mu) = R_b'[\mu - (\mu_a^* - \mu_b^*)]$.

[12]By a third-order Taylor series expansion,

$$EU_{ba}\Big|_{\mu_b^*} - EU_{bb} = R_a(\mu_b^*) - P\mu_b^* = \frac{-Pg''(0)}{6g(0)} (\mu_a^* - \mu_b^*)^3 > 0,$$

since $\mu_a^* > \mu_b^*$.

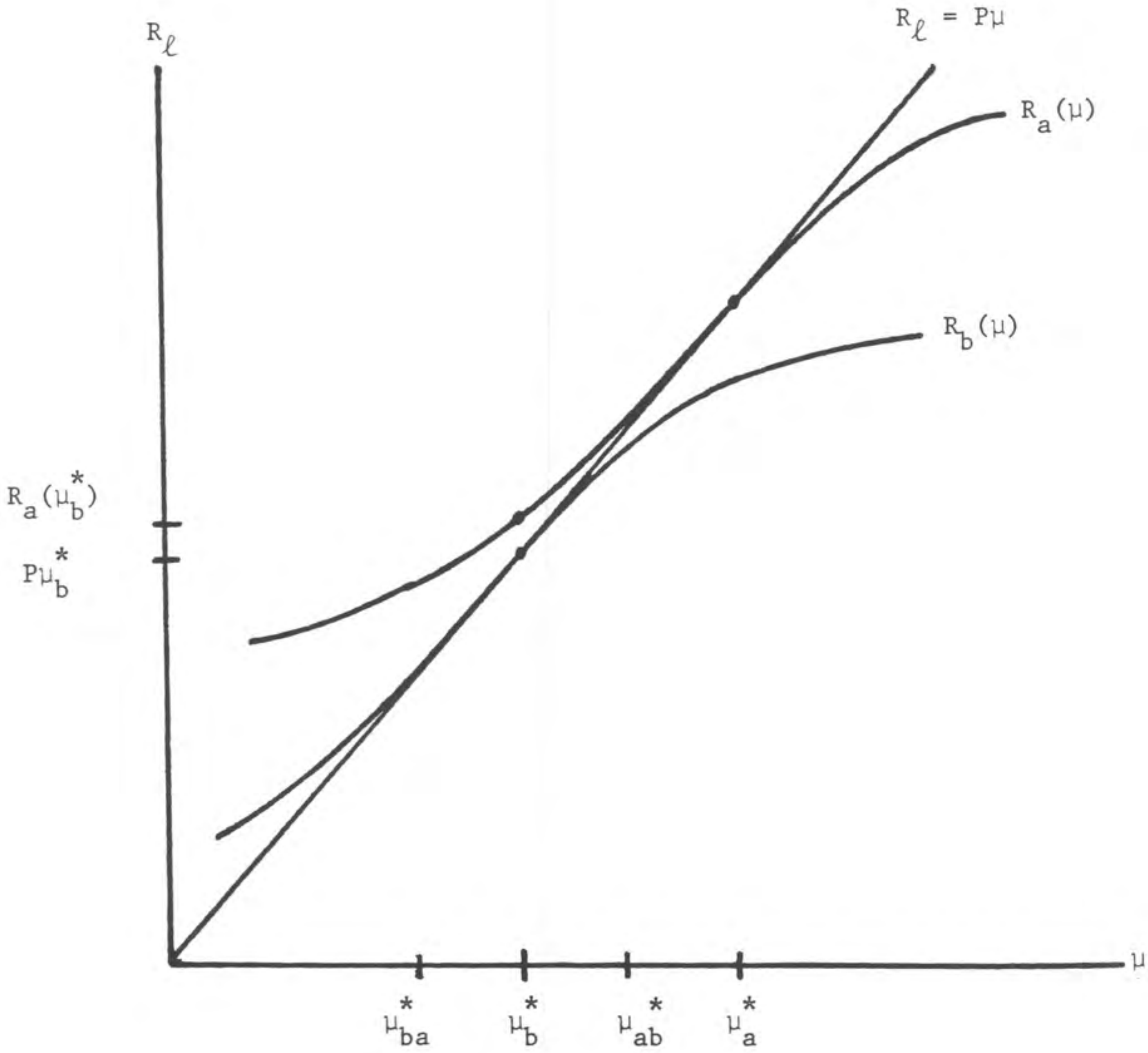

Figure 2. Expected Revenue Functions and Adverse Selection

leagues would not set his effort equal to μ_b^*. $\mu_{ba} = \mu_b^*$ is in general suboptimal for the climber, so a fortiori the expected utility is increased by climbing.

Major leaguers do not slum to the minors. At the optimal effort of a slumming major leaguer μ_{ab}^*, expected utility in the majors exceeds that in minors by the same argument as made above for climbers. A fortiori, $EU_{aa}(\mu_a^*) > EU_{ab}(\mu_{ab}^*)$.

OVZ (1984, 52-53, Proposition III.1) provide a method to prevent the slumming of major leaguers which, by this analysis, would be irrelevant. But OVZ allow G to differ across leagues. With distribution functions indexed by league, slumming can occur if $g_a''(0)$ is close to zero and $g_b''(0)$ is sufficiently negative. Although the expected revenue functions can cross in this case, the intersection must be at some $\mu < \mu_b^*$, because $R_a(\mu) > P\mu > R_b(\mu)$ for $\mu_b^* < \mu < \mu_a^*$. This runs counter to OVZ's conjecture that a major leaguer slums to the minors to get a sure win. If a major leaguer does slum, his average output in the minors is less than that of his minor league competition.

The inefficiencies due to adverse selection generate a demand for information. One form this demand might take is a requirement of entry credentials as signals of contestant quality. Another way to obtain information on contestant quality is to induce self-sorting. Lazear and Rosen (1981, 860) argue that by making the major league prize spread Δw^a sufficiently large, minor leaguers self-sort into their own league, but at the expense of excess effort by major leaguers. Adding a constant ρ to w_1^a and subtracting it from w_2^a results in the modified expected revenue function

$$\tilde{R}_a(\mu) = P\tilde{\mu}_a + \left[\frac{P}{g(0)} + 2\rho\right]\cdot[\tfrac{1}{2} - G(\tilde{\mu}_a - \mu)], \tag{55}$$

where $\tilde{\mu}_a > \mu_a^*$ since $\tilde{\mu}_a$ solves $C_a'(\mu) = g(0)(\Delta w + 2\rho)$ rather than $C_a'(\mu) = g(0)\Delta w$. $\tilde{R}_a$ is steeper than R_a for all μ: $\tilde{R}_a'(\mu) - R_a'(\mu) = 2\rho g(\tilde{\mu}_a - \mu) > 0$.

An immediate conclusion is that if $\tilde{\mu}_a = \mu_a^*$,[13] then $\tilde{R}_a(\mu_a^*) = R_a(\mu_a^*)$ and $\tilde{R}_a$ is steeper; therefore, $\tilde{R}_a$ is less than R_a for all $\mu < \mu_a^*$. For sufficiently large ρ, $\tilde{R}_a$ crosses R_b.[14]

Handicapping and Up-Front Money

In this section, a contestant knows the abilities of his competitors as well as his own ability, so there is no problem of adverse selection. My interest is in establishing the properties of mixed tournaments--major leaguers playing minor leaguers--in the presence of competition from "segregated" tournaments. The problem for the firm running a mixed tournament is to provide the appropriate marginal incentives and to be competitive with the segregated leagues. That is, the mixed tournament must be both efficient and able to attract contestants of both skill levels.

In a mixed tournament with an optimal segregated prize spread $\Delta w^* = P/g(0)$, a major leaguer has an expected output exceeding that of the minor leaguer because effort levels of the two type of contestants satisfy

$$(56) \qquad C_a'(\mu_a) = \frac{P}{g(0)}\, g(\mu_b - \mu_a) = C_b'(\mu_b).$$

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[13]This can occur if g(0) is endogenous; it is used in OVZ's proofs.

[14]That $\tilde{R}_a$ intersects R_b is necessary but not sufficient for sorting. Presumably, ρ can be large enough to outweigh the cost reduction due to low effort in the majors. Also note that for $\tilde{\mu}_a > \mu_a^*$, third-order effects can mitigate the result. As well as pivoting, the function shifts northeast.

The second equality follows from the symmetry of g. Let $\bar{\mu}_k$, k = a, b, denote the Nash solution for k's effort in a mixed tournament. Since $g(\bar{\mu}_b - \bar{\mu}_a) < g(0)$, both contestants in the mixed tournament work less than their respective efficient levels: $\bar{\mu}_a < \mu_a^*$ and $\bar{\mu}_b < \mu_b^*$.

Can a handicap overcome this problem and raise effort to the efficient levels? Consider the handicap h which was introduced in the discussion of unfair tournaments. The major leaguer must beat his minor league opponent by h units in order to win. That is, the probability that a type-*a* contestant wins is $1 - G(\mu_b + h - \mu_a)$. By letting the optimal prize spread in the mixed tournament be $\overrightarrow{\Delta w}^* = P/g(\mu_b^* + h - \mu_a^*)$, the correct marginal incentives obtain for any value of h. The problem of attracting contestants depends on the choice of h.

Consider three solutions for the handicap h. First, let h = 0. This prize spread incorporates the correct marginal incentives, but it is not competitive. Type-*b* contestants are not attracted from the minor leagues to the mixed tournament. Although average quality of play and hence the average reward is higher in mixed play than in the segregated minors, the lower probability of winning dominates (see Lazear and Rosen 1981, 862-63). To attract the minor leaguers, a mixed tournament without a handicap must offer a reward structure indexed by contestant quality. The solution is, in effect, to offer minor leaguers up-front money to take on the "big boys."

A second solution is to set the handicap h equal to the difference $\mu_a^* - \mu_b^*$; with this handicap, $g(\mu_b^* + h - \mu_a^*) = g(0)$ and $\overrightarrow{\Delta w}^* = \Delta w^*$. Here the mixed tournament is fair--major leaguers win half the contests--and efficient but it is not competitive. Major leaguers prefer the segregated senior circuit. The reason is that they contribute more than the minor leaguers to the average output of the mixed tournament, but receive the same expected utility. The talented contestants are bid away to their own segregated tournament. Up-front money to the major leaguers is used to attract them to the mixed tournament by equalizing expected utility across the tournaments.

The final solution to be considered is the competitive handicap. This is the value of h which equalizes expected utility across tournaments in the absence of ability-specific prize structures including up-front money. Lazear and Rosen (1981, 862) show that the competitive handicap is approximately $(\mu_a^* - \mu_b^*)/2$; therefore, major leaguers preserve an advantage--they win more than half the contests--in the competitive equilibrium.

IV. SOME TROUBLE WITH TOURNAMENTS

The purpose of this section is to evaluate the efficacy of rank-order tournaments as optimal labor contracts, and to identify some troubles with tournaments. It is useful to collect the results to this point in comparing tournaments with individualistic contracts. Tournaments successfully elicit effort especially if the contestants are risk neutral. Under risk aversion, the clearest comparison is between tournaments and piece rates with normally distributed individual-specific disturbances (see footnote 5 and equation (27)). If there is no common shock (i.e., if $\sigma_\eta^2 = 0$), the tournament elicits less effort than the piece rate contract.[15] However, a principal advantage of

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[15]Also, tournaments contain more risk than a similar individualistic compensation scheme, the quota. Of course, welfare comparisons must be based on expected utilities. See Lazear and Rosen (1981, 853-855) for such an analysis.

tournaments regarding risk is that contestants in a tournament are insulated from common risk. This is not true of an individualistic scheme. Indeed, with risk averse agents, the more risky the common disturbance, the better is the tournament. If $\sigma_{\eta}^{2} > (1 - \pi)\sigma_{\epsilon}^{2}$, the optimal tournament elicits more effort than the optimal piece rate contract and comes closer to the first-best level of effort.

One advantage of tournaments which has not been identified to this point is that tournaments are less likely to operate if it is more costly to measure output cardinally than ordinally: it is cheaper to see which of two piles of coal is bigger than to weigh both. Similarly, it is less costly to determine which associate in a law or accounting firm, or assistant professor, or tennis player, is doing better than their respective counterpart than to determine the precise value of each one's output.

The analysis to this point has also uncovered several troubles with tournaments. The first, the problem of global incentives, is easily remedied if the principal can relax the monitoring standard. The analysis also indicates a problem in inducing effort in uneven tournaments, but this is remedied with prize structures indexed by contestant quality, or with handicaps. The real problem of tournaments with heterogenous contestants arises if the contestants' types cannot be identified. Since tournaments which are mixed ex post do not induce optimal effort, and it is costly to induce self-sorting of contestants into their respective leagues, the outcome is not efficient. Of course, a piece rate system is not subject to this inefficiency.

Some additional troubles have been identified in the literature.

Consider first a case in which agents work two periods with pay in the second period dependent on rank in the first period. Dye (1984) points out that if losers have the option to sort into firms paying a piece rate in the second period, there is insufficient downside risk: the option value generates suboptimally low effort in the first period. This is not true in the risk neutral case if firms have rational expectations regarding the losers' behavior in the second period. Ex ante, firms offering the benchmark tournament would be surprised by a queue of workers because the option value makes the benchmark tournament too valuable to be competitive. Consequently, the level of pay is bid down. Under risk neutrality, the firm increases the prize spread to raise--relative to its level in the absence of turnover--effort in the first period to the first-best level. With risk averse workers, the only problem presented by the turnover of losers is the additional risk which must be borne to achieve the optimal second-best effort defined by equation (20), above. The severity of the problem is mitigated by the presence of firm-specific human capital which makes the workers' best alternative employment relatively less attractive. This suggests an alternative contract parameter to reduce the turnover of losers: severance pay such as nonvested pensions and forfeited stock options could reduce turnover without undermining the basic solution to the tournament.[16]

Dye (1984) also indicates that tournaments are subject to problems of collusion on the part of the contestants to reduce effort, and sabotage to

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[16] One should not assume that the firm acts to reduce the turnover of losers. Applications of the tournament model to promotion in law and accounting firms and in academia treat the "out" element of "up-or-out" policies as the penalty of losing. See Spurr (1986).

reduce the other contestants' output. Both are inefficient. To the extent such behavior cannot be neutralized by penalties to detected collusion or sabotage, it places a severe limitation on tournaments. Lazear (1986) shows that the optimal tournament in the presence of sabotage compresses the prize spread reducing incentives. Since individualistic payment schemes are not subject to sabotage, this places tournaments at a disadvantage.

Bronars (1986) uncovers another trouble. If contestants are allowed to choose not only the effort level, that is the mean of the output distribution, but also the riskiness of their activities, they choose too little risk. More precisely, Bronars shows that risk neutral contestants allocate too much time to safe projects and too little time to risky projects. Of course, if the firm operates a separate tournament for each project, with project-specific prize spreads, it can induce an optimal allocation of effort to each project.

Bronars (1986) also identifies the problem of intermediate information. Consider a single, two-stage tournament. Output is realized (independently) in each of two stages, and a single prize is awarded to the contestant with the higher combined output. A key insight is that a multi-stage tournament cannot be even, in general, at each stage even if the workers are identical ex ante. At the end of the first stage, one contestant is the leader, the other the trailer. Consequently, the tournament in the second stage is uneven and both the leader and the trailer shirk even under risk neutrality.[17] Furthermore, the firm cannot costlessly overcome this problem by introducing handicaps in the second stage, since this carries incentive repercussions to

[17] Combining these last two troubles, Bronars (1986) shows that trailers take suboptimally higher risks.

the first stage. Handicapping the second stage reduces effort in the first. This suggests as a survival property that tournaments suppress intermediate information.

However, one should not ignore the possibility of payoffs for each stage. Does the problem of intermediate information remain if a single, multi-stage tournament is replaced by a sequence of single-stage tournaments? The answer is no. Rosen (1986) analyzes sequential, single-elimination tournaments (e.g., a tennis ladder). The prize structure is as follows. A player who loses in the first round receives the lowest wage. Winners of each round are paid, in effect, the loser's wage <u>in the next round</u> plus the expected value of continued play. Assume the prize spread is the same across rounds, meaning the loser's wage rises by a constant from round to round. With a constant prize spread, how does effort evolve across rounds? That the game has a finite horizon generates declining effort through the rounds. Effort is lowest in the finals! The reason is that with a fixed prize spread, the value of continued winning goes down as one approaches the final round; simply put, the number of rounds available to win goes down. Rosen shows that to equalize effort across rounds, the prize spread must grow throughout the tournament to give the illusion of an infinite horizon. In fact, the wage grows linearly with rank through rank two, then it takes a distinct jump up for the top prize winner.

The analysis of this section suggests that tournaments are not perfect compensation schemes. But the tournament form has distinct advantages as well as some troubles. In many instances, the troubles can be overcome if the tournament is coupled with another mechanism such as severance pay, or penalties to detected sabotage.

V. TESTING TOURNAMENTS

The theoretical results of the previous sections are rich in structural implications governing both the incentive compatibility of effort and the prize structure of optimal tournaments. These include how effort and the prize spread vary with product price, the marginal cost of effort, the variances of the individual-specific and common disturbances, and the degree of risk aversion. To emphasize just how rich the implications are, begin by assuming that effort μ, the slope of its marginal cost C", the prize spread Δw, and the variances $(\sigma_{\epsilon}^2, \sigma_{\eta}^2)$ are all observable.[18]

Turn first to the incentive compatibility condition associated with Lazear and Rosen's (1981) tournament (see equation (16.2)). With quadratic costs of effort given by $C(\mu_i) = \lambda\mu_i^2/2$, and with normally-distributed individual-specific disturbances,

$$(57) \qquad \mu^* = \frac{\Delta w}{\sqrt{2\pi}\cdot\lambda\sigma_{\epsilon}}.$$

Taking logs and allowing for proportional optimization and measurement errors,

$$(58) \qquad \log \mu = -\tfrac{1}{2} \log 2\pi + \log \Delta w - \log \lambda - \log \sigma_{\epsilon} + u_{\alpha}$$

$$= \alpha_0 + \alpha_1 \log \Delta w + \alpha_2 \log \lambda + \alpha_3 \log \sigma_{\epsilon} + u_{\alpha}.$$

[18] Effort need not be observable; it is sufficient that effort be proxied by a variable with measurement error uncorrelated with the determinants of optimal effort μ^*.

With estimates of the four parameters, one can test whether $\alpha_0 = -\frac{1}{2} \log 2\pi$, and whether $\alpha_1 = -\alpha_2 = -\alpha_3 = 1$.

Next consider the solution for the optimal prize spread (see equation (19)). Again assuming quadratic cost of effort and normality,

$$(59) \qquad \Delta w^* = \frac{\sqrt{2\pi}\; P\; \sigma_\epsilon}{1 + S\lambda\pi\sigma_\epsilon^2}.$$

Taking logs and introducing an error term,

$$\begin{aligned} (60) \qquad \log \Delta w &= \tfrac{1}{2} \log 2\pi + \log P + \log \sigma_\epsilon - \log(1 + S\lambda\pi\sigma_\epsilon^2) + u_\beta \\ &\approx \tfrac{1}{2} \log 2\pi + \log P + \log \sigma_\epsilon - S\lambda\pi\sigma_\epsilon^2 + u_\beta \\ &\approx \beta_0 + \beta_1 \log P + \beta_2 \log \sigma_\epsilon + \beta_3\; \lambda\sigma_\epsilon^2 + u_\beta, \end{aligned}$$

assuming that the degree of risk aversion is small relative to the variance of the individual-specific disturbance and the curvature of the cost of effort. One can use estimates from such a regression to test whether $\beta_0 = \frac{1}{2} \log 2\pi$, and whether $\beta_1 = \beta_2 = 1$. Also, one can use the estimate of β_3 to identify the degree of risk aversion.

The form of equation (58) is satisfactory if the prize spread Δw is given exogenously. However, with Δw endogenous, one might prefer to estimate the determinants of effort choice by combining the incentive compatibility condition with the condition for the optimal prize spread (see equation 20 and note 5). Equivalently, by substituting equation (60) into equation (58) for Δw, one obtains

$$\text{(61)} \qquad \log \mu = \log P - \log \lambda - S\lambda\pi\sigma_{\epsilon}^{2} + u_{\alpha} + u_{\beta}$$

$$= \gamma_0 + \gamma_1 \log P + \gamma_2 \log \lambda + \gamma_3 \lambda\sigma_{\epsilon}^{2} + u_{\gamma}$$

With estimates of the regression coefficients, one can test the hypotheses $\gamma_0 = 0$, and $\gamma_1 = -\gamma_2 = 1$, and one can estimate the degree of risk aversion from the estimate of γ_3. Comparison with the estimate of β_3 from equation (60) provides a simple test across equations.[19]

Setting up such regressions and tests is not an esoteric exercise since experimental data are available on behavior in tournaments (Bull, Schotter, and Weigelt 1987). However, the regressions have not been estimated. Rather, Bull, Schotter, and Weigelt (hereafter, BSW) confront their experimental data with several requirements summarized by the following questions:

(i) Do experimental subjects choose the correct level of effort in every case, or at least on average?

(ii) Is the chosen effort level invariant to simultaneous variation in σ_{ϵ}^{2} and λ such that the optimal level of effort is unchanged?

(iii) Are the subjects' choices of effort levels "better" under a piece rate contract which generates the same optimal effort level?

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[19]The only modification required by the Nalebuff-Stiglitz tournament is to replace $\lambda\sigma_{\epsilon}^{2}$ with $\lambda\sigma_{\epsilon}^{2}/(1 + \sigma_{\eta}^{2})$ in equations (60) and (61). Comparison of the parameter estimates across the two specifications would be sufficient to access the importance of a multiplicative common disturbance if naturally generated data were available.

BSW limit their analysis to incentive compatibility: they do not investigate choice of the prize spread. The optimal effort level in BSW's experiments is derived from equation (16.2) above, so it follows from the Lazear-Rosen specification with quadratic utility[20] and cost of effort, and in particular with a uniform distribution of the individual-specific disturbances.

BSW's experimental results offer strong support for the incentive compatibility of tournaments. Experimental subjects choose effort levels which on average are quite close to the optimal level. BSW also find support for the invariance of effort supply to optimal-effort-preserving variation in $g(0)$ and λ. However, the variance of the responses is quite high, and is higher in tournaments than under the piece rate scheme.

One should not be surprised by the high variance of effort in the tournament or that this variance is higher in tournaments than under piece rate contracts. The tournament problem is challenging and precise results are too much to demand from any data. Indeed, to solve the BSW's experimental tournament requires computing $g(0)$ with g the probability density function of the difference of two uniformly distributed random variables. That is, the experimental subject must know that g is the density function of a triangular distribution. One might expect law associates, assistant professors, and boxers to solve their effort supply problems with some degree of precision--much is at stake, and information on the results of prior

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[20]BSW (1987) set up the model under risk neutrality and note that this weakens their results. However, equation (16.2) above clearly establishes that BSW's model holds exactly for quadratic utility as well.

tournaments is plentiful--but precise choice from a difficult problem is too much to ask from experimental subjects.

However, one should not discount the finding that the variance of effort is higher in the tournament than under the piece rate scheme. Perhaps the key insight of the experimental results is that the optimization errors induced by computational costs put tournaments at a disadvantage relative to the simpler piece rate contract.

The tournament model gives a parsimonious explanation for some observed compensation schemes. Perhaps as challenging as the structural tests of regression coefficients or the satisfaction of BSW's requirements are tests of some simple implications. The analysis of the preceeding section establishes that tournaments are more likely to be employed the more diffuse is the common disturbance, the bigger the cost differential between ordinal and cardinal measurement, the greater the firm-specific human capital, the easier the detection of sabotage and collusion, and the more easily the type of worker can be identified. Constructing scalar measures of these determinents would be formidable. Nevertheless, some evidence has been provided regarding specific implications.

Bronars (1986) presents some tests of his implications regarding the level of effort in multi-stage tournaments, and contestant choice of risk. Again, the analysis is limited to incentive compatibility. Bronars uses data on U.S. Congressional election campaign expenditures to measure each candidate's (contestant's) effort in an election (tournament) to test whether the effort of each candidate (contestant) is highest in close races.[21]

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[21]Does the tournament model apply to elections? The electorate is the principal and the politician is the agent. The electorate runs a tournament

Bronars finds that the expected closeness of the election significantly reduces expenditures (effort) of candidates who expect to lose, but has little effect on the expenditures (effort) of frontrunners.

The implication that trailers take excessively high risks (see note 17, above) is supported by casual evidence from sporting contests: toward the end of the game, trailing teams in basketball attempt three-point shots and foul intentionally; in hockey, the goalie is pulled from the net; in baseball, a pinch hitter bats for a pitcher who is throwing well; in football, plays are run without huddles and more passes are thrown. All these strategies are examples of risks which trailing teams take for a shot at the lead. Bronars quantifies this phenomenon for passing plays in football. Using the frequency of passing plays as a measure of team's risk-taking behavior, he finds that teams trailing at halftime pass in the second half at a rate which significantly exceeds their average passing rate over the season.

VI. CONCLUSION

Several themes recur throughout the current paper. By using a simple method to solve the many variations on the basic model (e.g., utility function and output equation specifications; risk neutral versus risk averse agents;

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to reveal the candidates policy positions, qualifications, stamina, etc.. Furthermore, there is a winner and one or more losers. Nevertheless, Stigler (1972) argues that the winning margin or vote spread matters; that is, elections are not "winner take all". Although output of the campaign is not measured cadinally, the observed vote spread is sufficient to support payment by relative performance. Payment by relative performance is not subject to the problem of intermediate information. Therefore, if Stigler's hypothesis is correct, Bronars' test is inappropriate. Alternatively, Bronars results might shed light on the efficacy of Stigler's hypothesis.

tournaments versus simple individualistic contracts; gaps, relative performance, etc.), the solutions take forms similar enough that direct comparison is a simple task. The comparative static results with respect to product price, degree of risk aversion, curvature of the cost of effort, and variance of the individual-specific disturbance are qualitatively invariant to the model and specifications. One important difference is the result with respect to the variance of the common disturbance. For this variable, the results are sensitive to the type of compensation scheme and the details of the specification. The analysis of an n-contestant tournament with a single prize also fits into the simple structure, hence the effect of tournament size on the prize structure and effort in the optimal tournament can be identified by direct comparison.

Tournaments do not dominate individualistic compensation schemes or contracts. In terms of positive analysis, this is comforting. Over all, tournaments are not common. However, elements of the tournament model are very important in some occupations and payment based on rank order is the principal form of compensation in most professional sports. Why? The analysis in sections IV and V catalogues the market conditions under which tournaments are most likely to be observed. The empirical challenge is to determine whether tournaments are employed where the theory predicts.

APPENDIX

In this appendix, I sketch a proof of the sufficiency side of the proposition in the text. First note that with $q_i = q^1(\mu_i, \eta) + q^2(\epsilon_i, \eta)$, the first-best level of effort solves $C'(\mu_i) = P \cdot \partial q^1(\mu_i, \eta)/\partial \mu_i$, which is independent of ϵ_i.

The incentive compatibility condition for contestant i is

$$\text{(A.1)} \qquad \mu_i = \mu^*(\eta) \in \underset{\{\mu_i\}}{\text{argmax}} \left[[1 - \tilde{G}(\tilde{\xi})] \cdot w_1 + \tilde{G}(\tilde{\xi}) \cdot w_2 - C(\mu_i) \right]$$

with $\tilde{G}$ denoting the cumulative distribution function of the random variable $\tilde{\xi} \equiv q^1(\mu_j, \eta) - q^1(\mu_i, \eta)$. The symmetric Nash equilibrium solves

$$\text{(A.2)} \qquad C'(\mu_i) = \tilde{g}(0) \cdot \frac{\partial q^1}{\partial \mu_i}(\mu_i, \eta) \cdot (w_1 - w_2).$$

The zero expected-profit condition is

$$\text{(A.3)} \qquad P \cdot E[q^1(\mu_i, \eta) + q^2(\epsilon_i, \eta)] = (w_1 + w_2)/2.$$

Under the conditions of the proposition, the objective function is

$$\text{(A.4)} \qquad EU_i = (w_1 + w_2)/2 - E[C(\mu_i)].$$

Substituting in the constraints (equations (A.2) and (A.3)), the unconstrained problem is to

$$\text{(A.5)} \qquad \max_{\Delta w} \; EU_i = P \cdot E[q^1(\mu^*, \eta) + q^2(\epsilon_i, \eta)] - E[C(\mu^*)]$$

with μ^* a function of the prize spread Δw from the incentive compatibility constraint. Using equation (A.3) to replace C' in the necessary condition,

$$\text{(A.6)} \qquad [P - \tilde{g}(0) \cdot \Delta w] \cdot E\left[\frac{\partial q^1}{\partial \mu_i}(\mu^*, \eta) \cdot \frac{d\mu^*}{d\Delta w}\right] = 0.$$

One can establish that the expectation is not in general zero, hence the optimal prize spread is $\Delta w^* = P/\tilde{g}(0)$. Substituting the optimal prize spread into the incentive compatibility condition establishes that the optimal effort level is first best: $C'(\mu^*) = P \cdot \partial q^1(\mu^*, \eta)/\partial \mu_i$.

Q.E.D.

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