Rochester Center for Economic Research

On Modeling the Natural Rate of Unemployment with Indivisible Labor

Greenwood, Jeremy and Gregory W. Huffman

University of
Rochester

ON MODELLING THE NATURAL RATE OF UNEMPLOYMENT
WITH INDIVISIBLE LABOR*

Jeremy Greenwood
The University of Western Ontario
and
Rochester Center for Economic Research

and

Gregory W. Huffman
The University of Western Ontario

Working Paper No. 93

July 1987

Abstract

This analysis investigates modelling the natural rate of unemployment in settings where labor's utilization has some lumpy aspect to it. Specifically, the introduction of various nonconvexities into tastes and technology lead to unemployment in general equilibrium. Equilibria can emerge where (i) those currently unemployed have higher probabilities of being unemployed in the future than those currently employed, (ii) some agents are jobless while others work overtime, and (iii) seniority rules seemingly arise whereby old workers cannot be laid off so long as either new workers are being hired or some workers are doing overtime. The relative welfare levels of employed and unemployed agents are analyzed.

Mailing Address

Jeremy Greenwood
Department of Economics
University of Western Ontario
London, Ontario
N6A 5C2   CANADA

Introduction

For about the last fifteen years, economists have been concerned with the microeconomic foundations of employment theory. Friedman (1968) defined the concept of the natural rate of unemployment as "the level that would be ground out by the Walrasian system of general equilibrium equations ..." Generating unemployment in Walrasian frameworks has proved to be difficult to date. While such paradigms can easily provide a determination of equilibrium employment, making the jump to modelling equilibrium unemployment has proven to be elusive. Introducing "frictions" to labor market trade would seem to be an obvious place to start when trying to formulate theories of equilibrium unemployment. This is more difficult than it seems, though, with many attempts along these lines having limited success in this regard.

As a case in point, consider the implicit labor contracting model in environments with asymmetric information as formulated by Azariadis (1983), Grossman and Hart (1983), and others--see Hart (1983) for a survey of this literature. Here workers contract with firms for a wage/employment package designed to stabilize labor income in the face of fluctuations in economic activity. Since only firms observe the underlying shocks affecting the economy, workers will believe entrepreneurs' declarations that a "bad" state-of-the-world has occured only if, at the time of wage cuts, an observable such as employment is also sufficiently reduced. This necessity for contracts to be written in a manner that ensures entrepreneurs truthfully report the state-of-the-world can result in the "underemployment" of labor relative to a world with symmetric information, in the sense that average hours per worker is lower.[1] It does not result in any agents being unemployed, though. Thus, while such contracting can lead to real wage

rigidity and underemployment, it does not in and of itself result in unemployment.

A notable exception which does bridge the hiatus between modelling employment and unemployment in Walrasian frameworks is the seminal equilibrium search model developed by Lucas and Prescott (1974). Here, an aggregate economy made up of many individual markets subject to idiosyncratic disturbances is constructed. Given the stochastic structure of the economy, a worker is continually undertaking calculations to decide whether or not it is in his best interest to quit his current job in a particular industry and enter a search process in pursuit of a higher return to work effort elsewhere. The model generates a natural rate of unemployment together with equilibrium distributions of wages, employment, and unemployment across industries.

An alternative approach to modelling unemployment in Walrasian frameworks has recently been advanced by Rogerson (1985). Here the gap between the notions of equilibrium employment and unemployment is bridged by introducing indivisibilities into economic agents' labor supply decisions. Such nonconvexities turn out to be capable of generating unemployment within the context of equilibrium models. Rogerson also shows how these indivisibilities can be handled by a simple extension to the standard competitive equilibrium construct. The fact that simple modifications of the standard competitive equilibrium model can be applied to model what many may view as intrinsically "non-market clearing" phenomena has recently been stressed by Prescott and Townsend (1984).

Drawing on Rogerson's work, Hansen (1985) simulates a stochastic growth model with indivisible labor. He finds that this type of model mimics certain U.S. labor market stylized facts, such as the large fluctuations in

aggregate hours worked relative to average productivity, quite well. Greenwood and Huffman (1987) borrow from Rogerson to construct a dynamic equilibrium model which is capable of explaining, in a theoretical sense, the covariance properties between unemployment and inflation--or Phillips curve correlations--both conditioned and unconditioned upon exogenous factors such as the current growth rate of the money supply, the level of productivity, etc.

A closer examination of modelling the natural rate of unemployment with indivisible labor is undertaken here. The analysis focuses on how different ways of introducing nonconvexities into the specification of taste and technology can produce various observed labor market phenomena. For instance, it is shown that this approach can generate appealing serial correlation properties in agents' employment histories. Specifically, those currently working can have higher (lower) probabilities of being (un)employed in the future than those currently not working. Also, it is demonstrated that the introduction of such nonconvexities into the economic environment allows for a simultaneous determination of both the total number of individuals working in the economy, and the amount of hours worked per employed agent. This permits an explanation of certain observed facts such as some labor force participants working overtime while others are unemployed. Additionally, seniority rights for workers can seemingly emerge in such environments. In particular, situations exist where old workers can never be laid off so long as new workers are being hired or some agents are working overtime. Finally, some discussion of the determinants of the relative consumption and welfare levels of the employed versus unemployed is undertaken.

The remainder of the paper is organized as follows. The second section contains a general description of the economic environment to be employed. In the third section the representative agent's optimization problem is cast.

The link between nonconvexities and nonseparable preferences on the one hand, and the serial correlation properties of agents' employment histories on the other, is studied in the fourth section. The relationship between employed and unemployed agents' consumption and welfare levels is examined in the subsequent section. In the sixth section it is shown how the introduction of nonconvexities into the economy's technology can allow for desirable properties in agents' employment histories as well as a determination of both the extensive and intensive margins of labor force participation. Conclusions are offered in the final section.

The Economic Environment

Consider the following model of a two-period economy inhabited by a continuum of identical agents distributed uniformly over the interval $[0,1]$. An agent's goal in life is to maximize the expected value of his lifetime utility as given by

$$E_0[\underline{U}(c_1, c_2, \ell_1, \ell_2)]$$

where c_t and ℓ_t denote his period-t consumption and work effort (for $t = 1,2$). The function $\underline{U}(\cdot)$ is assumed to be strictly concave and continuously twice differentiable. It is strictly increasing in its first two arguments and strictly decreasing in its last two. Furthermore, it is assumed that

$$\lim_{c \to 0} \underline{U}_1(c, \cdot, \cdot, \cdot) = \infty, \text{ and } \lim_{c \to 0} \underline{U}_2(\cdot, c, \cdot, \cdot) = \infty.$$

Aggregate output in period t is described by the constant-return-to-scale production process

$$y_t = f(L_t, K) \qquad\qquad t = 1,2$$

with L_t and K representing the aggregate amount of labor and capital employed in this period. It is assumed that the function $f(\cdot)$ is strictly increasing in both its arguments and additionally that it is twice continuously differentiable with

$$\lim_{L \to 0} f_1(L,K) = \infty, \quad \lim_{L \to \ell} f_1(L,K) = 0, \quad \text{and } f(0,K) = 0.$$

The ownership of capital is assumed to be completely diversified with each agent possessing K units of capital.

Each agent is assumed to face a nonconvexity in his labor supply decision. In particular, he either works the amount $\ell > 0$, or he does not work at all. It has long been recognized that the Pareto optimality property of a competitive equilibrium is not necessarily destroyed by the presence of nonconvexities in either agents' tastes or firms' production technologies--see Rothenberg (1960). As is shown by Rogerson (1985), however, an extension of agents' choice sets to allow for the possibility of lotteries over consumption and labor allocations, will in general improve individuals' (ex ante) welfare in the situation currently under consideration. Specifically, imagine the production process as being owned by a competitive firm who offers an individual an income-employment contract of the following form: In the first period the firm and the agent agree on the probability ϕ_1^w that the individual will be called in to work ℓ units. An individual who is chosen to be employed receives a wage-cum-dividend payment from the firm designed to allow him to undertake c_1^w units of consumption. The probability of being unemployed in the first period is $\phi_1^u = (1 - \phi_1^w)$, and in this state the individual receives income sufficient to provide for c_1^u units of consumption. Since all agents are identical ex ante and uniformly distributed over the unit interval,

the equilibrium first-period aggregate employment is given by $L_1 = \phi_1^w \ell$.

For the second period, the probability of an agent being called in to work is assumed to depend upon his previous employment history. Thus, the firm and the worker agree on the probability $\phi_2^w(w)$ that the individual will be chosen to work in the second period _conditional_ on the fact that he was _employed_ in the first period, and on the probability $\phi_2^w(u)$ that he will work in the second period _conditional_ on the fact that he was unemployed in the previous period. Hence, more formally, $\phi_2^w(w) = \text{prob}(\ell_2 = \ell | \ell_1 = \ell)$ and $\phi_2^w(u) = \text{prob}(\ell_2 = \ell | \ell_1 = 0)$. It immediately follows that $(1 - \phi_2^w(w))$ and $(1 - \phi_2^w(u))$ represent the probabilities of being unemployed in the second period conditional on whether the agent was working or not, respectively, during the first period. The unconditional probabilities of being employed, ϕ_2^w, and unemployed, ϕ_2^u, in the second period are given by $\phi_2^w = \phi_1^w \phi_2^w(w) + \phi_1^u \phi_2^w(u)$, and $\phi_2^u = \phi_1^w \phi_2^u(w) + \phi_1^u \phi_2^u(u)$. In the second period the firm makes the individual a wage-cum-dividend payment designed to provide for _one_ of four consumption possibilities, viz. $c_2^w(w)$, $c_2^u(w)$, $c_2^w(u)$, and $c_2^u(u)$, depending upon _both_ the worker's current and past employment status. For example, if the individual is currently working but was unemployed last period, the firm pays him income sufficient to allow for consumption in the amount, $c_2^w(u)$. The unconditional probability associated with this consumption possibility is $(1 - \phi_1^w)\phi_2^w(u)$. Finally, second-period aggregate employment is given by $L_2 = [\phi_1^w \phi_2^w(w) + (1 - \phi_1^w)\phi_2^w(u)]\ell$.

The Representative Agent's Optimization Problem

The decision-making of consumer-workers and firms in competitive equilibrium can be summarized by the outcome of the following "representative" agent/firm's programming problem with the choice variables being c_1^w, c_1^u, $c_2^w(w)$, $c_2^u(u)$, $c_2^u(w)$, $c_2^u(u)$, $\phi_1^w = (1 - \phi_1^u)$, $\phi_2^w(w) = (1 - \phi_2^u(w))$, and $\phi_2^w(u) = (1 - \phi_2^u(u))$, to maximize

$$E_0[\underline{U}(\cdot)] = \phi_1^w E[\underline{U}(\cdot)|\ell_1 = \ell] + (1 - \phi_1^w)E[\underline{U}(\cdot)|\ell_1 = 0] \tag{1}$$

$$= \phi_1^w[\phi_2^w(w)\underline{U}(c_1^w, c_2^w(w), \ell, \ell) + (1 - \phi_2^w(w))\underline{U}(c_1^w, c_2^u(w), \ell, 0)]$$

$$+ (1 - \phi_1^w)[\phi_2^w(u)\underline{U}(c_1^u, c_2^w(u), 0, \ell) + (1 - \phi_2^w(u))\underline{U}(c_1^u, c_2^u(u), 0, 0)]$$

s.t.

$$\phi_1^w c_1^w + (1 - \phi_1^w)c_1^u = f(\phi_1^w\ell, K) \tag{2}$$

$$\phi_1^w[\phi_2^w(w)c_2^w(w) + (1 - \phi_2^w(w))c_2^u(w)] + (1 - \phi_1^w)[\phi_2^w(u)c_2^w(u) + (1 - \phi_2^w(u))c_2^u(u)] \tag{3}$$

$$= f(\phi_1^w\phi_2^w(w)\ell + (1 - \phi_1^w)\phi_2^w(u)\ell, K)$$

$$0 \le \phi_2^w(w) \le 1 \tag{4}$$

$$0 \le \phi_2^w(u) \le 1. \tag{5}$$

The expected value of the agent's lifetime utility function, or the maximand, is given by the first equation, (1). The next two equations, (2) and (3), are the economy's resource constraints for the first and second periods. The last two equations bind the conditional probabilities $\phi_2^w(w)$ and $\phi_2^w(u)$ to have values in the unit interval. (Unlike the unconditional probabilities, there do not seem to be any natural restrictions on tastes and technology to ensure this outcome).

The solution to the above choice problem is completely summarized by the following set of efficiency conditions--in addition to equations (2) and (3)

$$E[\underline{U}_1(\cdot)|\ell_1 = \ell] = E[\underline{U}_1(\cdot)|\ell_1 = 0] \tag{6}$$

$$[\text{for } c_1^w, c_1^u]$$

$$\underline{U}_2(c_1^w, c_2^w(w), \ell, \ell) = \underline{U}_2(c_1^w, c_2^u(w), \ell, 0) = \underline{U}_2(c_1^u, c_2^w(u), 0, \ell)$$

$$= \underline{U}_2(c_1^u, c_2^u(u), 0, 0) \tag{7}$$

$$[\text{for } c_2^i(j): \quad i,j = u,w]$$

$$E[\underline{U}(\cdot)|\ell_1 = 0] - E[\underline{U}(\cdot)|\ell_1 = \ell] \tag{8}$$

$$= E[\underline{U}_1(\cdot)|\ell_1 = \ell][f_1(\phi_1^w\ell, K)\ell - (c_1^w - c_1^u)]$$

$$+ \underline{U}_2(\cdot)\{f_1(\phi_2^w\ell, K)[\phi_2^w(w) - \phi_2^w(u)]\ell - (E[c_2|\ell_1 = \ell] - E[c_2|\ell_1 = 0])\}$$

$$[\text{for } \phi_1^w]$$

$$\underline{U}(c_1^w, c_2^u(w), \ell, 0) - \underline{U}(c_1^w, c_2^w(w), \ell, \ell) + \psi_2^w/\phi_1 \qquad (9)$$

$$\geq \underline{U}_2(\cdot)[f_1(\phi_2^w \ell, K)\ell - (c_2^w(w) - c_2^u(w))]$$

[with equality if $\phi_2^w(w) > 0$]

$$\underline{U}(c_1^u, c_2^u(u), 0, 0) - \underline{U}(c_1^u, c_2^w(u), 0, \ell) + \psi_2^u/(1 - \phi_1^w) \qquad (10)$$

$$\geq \underline{U}_2(\cdot)[f_1(\phi_2^w \ell, K)\ell - (c_2^w(u) - c_2^u(u))]$$

[with equality if $\phi_2^u(w) > 0$]

$$\psi_2^w[1 - \phi_2^w(w)] = 0 \qquad (11)$$

$$\psi_2^u[1 - \phi_2^w(u)] = 0 \qquad (12)$$

where ψ_2^w and ψ_2^u are the Lagrange multipliers associated with the constraints (4) and (5). Note that equation (9) will hold strictly when $\phi_2^w(w) > 0$, and similarly when $\phi_2^u(w) > 0$ equation (10) will be binding.

By glancing at the above set of equations, it may appear that not much can be gleaned from the simple problem posed. It turns out that the structure of preferences plays a crucial role in determining the nature of the model's solution. By restricting preferences in various ways, the above system of equations can be used to gain insight on two issues: first, the serial correlation properties of agents' states of employment and second, the relative welfare levels of employed and unemployed workers.

Nonseparable Preferences and Serially Correlated Employment

It will now be investigated whether the above model is capable of generating serial correlation in agents' employment histories. More precisely, the question to be addressed is whether an agent is more likely to be employed in the second period if he was employed, rather than unemployed, in the first period. That is, can the model generate $\phi_2^w(w) > \phi_2^w(u)$ as a solution? In pursuit of this end, the utility function $\underline{U}(\cdot)$ will be specialized to

$$\underline{U}(\cdot) = U(c_1) + \beta U(c_2) + V(\ell_1, \ell_2)$$

The advantage of this restriction is that an agent's consumption in each period is independent of his employment history, a fact evident from (6) and (7), so that $c_1^w = c_1^u = c_1$, and $c_2^i(j) = c_2$ for $i, j = w, u$. The concavity and separability properties of the momentary utility function, $\underline{U}(\cdot)$, imply that agents desire to smooth (perfectly) consumption in a given period across their employment characteristics. A complete characterization of the model's solution in the current situation--that is a determination of c_1, c_2, ϕ_1^w, $\phi_2^w(w)$, $\phi_2^u(w)$--is given by the following analogues to (2), (3), (8), (9), and (10):

$$c_1 = f(\phi_1^w \ell, K) \tag{13}$$

$$c_2 = f(\phi_1^w \phi_2^w(w)\ell + (1 - \phi_1^w)\phi_2^w(u)\ell, K) \tag{14}$$

$$E[V(\cdot)|\ell_1 = 0] - E[V(\cdot)|\ell_1 = \ell] = U'(c_1)f_1(\phi_1^w \ell, K)\ell_1 \tag{15}$$

$$+ \beta U'(c_2)f_1(\phi_1^w \phi_2^w(w)\ell + (1 - \phi_1^w)\phi_2^w(u)\ell, K)[\phi_2^w(w) - \phi_2^w(u)]\ell$$

$$[V(\ell,0) - V(\ell,\ell)] + \psi_2^w/\phi_1^w \geq \tag{16}$$

$$\beta U'(c_2)f_1(\phi_1^w \phi_2^w(w)\ell + (1 - \phi_1^w)\phi_2^w(u)\ell, K)\ell$$

[with equality if $\phi_2^w(w) > 0$]

$$[V(0,0) - V(0,\ell)] + \psi_2^u/(1 - \phi_1^w) \geq \tag{17}$$

$$\beta U'(c_2)f_1(\phi_1^w \phi_2^w(w)\ell + (1 - \phi_1^w)\phi_2^w(u)\ell, K)\ell$$

[with equality if $\phi_2^w(u) > 0$].

[The complementary slackness conditions (11) and (12) governing ψ_2^w and ψ_2^u also hold here.]

An examination of the above set of equations will now be conducted in order to learn more about the serial correlation properties of an agent's state of employment generated by the model. In particular, many would hold the view that a person's probability of being employed (unemployed) this period increases (decreases) if the individual was employed (unemployed) last period. Thus, the focus of the investigation will be on developing the set of necessary and sufficient conditions which allow for $0 \leq \phi_2^w(u) < \phi_2^w(w) \leq 1$ to emerge as a solution to the model. [Note that such a solution implies that $0 \leq \phi_2^u(w) < \phi_2^u(u) \leq 1$.] An analysis of equation (16) and (17) is central to the issue being pursued. The right-hand sides of these expressions

illustrate the marginal benefits of having a larger fraction of the labor force working. The left-hand sides (ignoring the ψ_2 terms) represent the expected utility losses associated with a rise in the probability of being called into work. An important observation to make is that the right-hand sides of these two conditions are identical. Thus, not surprisingly, in the current situation the marginal benefit from either increasing the employment rate of those who were previously employed or those who were unemployed is the same.

Proposition 1: Suppose that $[V(\ell,0) - V(\ell,\ell)] \neq [V(0,0) - V(0,\ell)]$. Then the conditional probabilities, $\phi_2^w(w)$ and $\phi_2^w(u)$, have values determined in the following manner

$$0 \leq \phi_2^w(u) < \phi_2^w(w) \leq 1 \tag{18}$$

if and only if

$$[V(\ell,0) - V(\ell,\ell)] < [V(0,0) - V(0,\ell)], \tag{19}$$

and

$$0 \leq \phi_2^w(w) < \phi_2^w(u) \leq 1 \tag{20}$$

if and only if

$$[V(0,0) - V(0,\ell)] < [V(\ell,0) - V(\ell,\ell)]. \tag{21}$$

<u>Proof</u>: Observe that $\phi_2^w = \phi_2^w(w) = \phi_2^w(u) = 1$ cannot be a solution to the model since this situation is precluded by the assumption that

$$\lim_{\phi^w \to 1} f_1(\phi^w \ell, K) = 0$$ which results in a violation of the necessary conditions

(16) and (17). Likewise, a solution of the form $\phi_2^w = \phi_2^w(w) = \phi_2^w(u) = 0$ cannot transpire since this event is ruled out by the assumption that

$f(0,K) = 0$ and $\lim_{c \to 0} U'(c) = \infty$, which again would contradict (16) and (17). The postulated relationship between (18) and (19) will now be demonstrated, with the proof of that between (20) and (21) being entirely analogous.

(Necessity). Suppose that (18) holds. Then equation (16) is binding because $\phi_2^w(w) > 0$. Also, since $\phi_2^w(u) < 1$ it happens that $\psi_2^u = 0$ [from (12)]. Therefore, since the right-hand sides of (16) and (17) are equal, $[V(\ell,0) - V(\ell,\ell)] + \psi_2^w/\phi_1^w \leq [V(0,0) - V(0,\ell)]$ which yields the desired result as $\psi_2^w/\phi_1^w \geq 0$.

(Sufficiency). Alternatively, assume that (19) holds. This implies that (16) must hold with equality. To see this, suppose that it didn't hold. Then (17) must hold with equality because $\phi_2^w = \phi_2^w(w) = \phi_2^w(u) = 0$ can't be a solution to the model. But then from (16) and (17) it follows that $[V(\ell,0) - V(\ell,\ell)] > [V(0,0) - V(0,\ell)] + \psi_2^u/(1 - \phi_1^w)$ which contradicts the initial assumption that (19) holds. Now, first, if (17) is also binding, then from (16), (17), and (19) it transpires that $\psi_2^w/\phi_1^w - \psi_2^u/(1 - \phi_1^w) = [V(0,0) - V(0,\ell)] - [V(\ell,0) - V(\ell,\ell)] > 0$. Since ψ_2^w and ψ_2^u can't both be strictly positive--for this would imply $\phi_2^w = \phi_2^w(w) = \phi_2^w(u) = 1$--it follows that here $0 \leq \phi_2^w(u) < \phi_2^w(w) = 1$. Second, if (17) is slack, then $0 = \phi_2^w(u) < \phi_2^w(w) \leq 1$. These two inequalities yield (18).

□

The next natural question to ask is under what conditions is $[V(\ell,0) - V(\ell,\ell)] \underset{>}{<} [V(0,0) - V(0,\ell)]$? This question is answered by the following proposition.

Proposition 2: Suppose that $V_{12}(\cdot)$ is either <u>strictly</u> positive or negative for <u>all</u> values of $\ell_1,\ell_2 \in [0,\ell]$. Then

$$[V(\ell,0) - V(\ell,\ell)] \underset{>}{<} [V(0,0) - V(0,\ell)]$$

if and only if

$$V_{12}(\cdot) \underset{<}{>} 0.$$

Proof: By the Fundamental Theorem of Calculus, the first statement is equivalent to

$$\int_0^\ell V_2(\ell,x)dx \underset{<}{>} \int_0^\ell V_2(0,x)dx,$$

which occurs as

$$V_{21}(\cdot) \underset{<}{>} 0.$$

$\square$

Thus, the situation described in statement (18) <u>can</u> happen when the sign of $V_{21}(\cdot)$ is positive, while that characterized by (20) can arise when $V_{21}(\cdot)$ is negative. This makes intuitive sense. Consider the case where $V_{21}(\cdot) > 0$ which, as was just demonstrated, is a necessary condition for $0 \leq \phi_2^w(u) < \phi_2^w(w) \leq 1$. (A maintained hypothesis for the rest of the analysis in this section will be that leisure in adjacent periods are Edgeworth-Pareto complements.) Here more (less) work in the first period lowers (increases) the marginal disutility of working in the second. This is equivalent to leisure in the two periods being net complements in the Edgeworth-Pareto

sense. Not surprisingly then, for an optimal labor contract to draw mostly from the pool of individuals who were employed in the first period, it must happen that this set of agents has the lowest marginal disutility of working. In fact, it will be shown next that in this situation the firm will always fully exhaust the pool of individuals who were employed in the first period before it hires any agents who were unemployed then, i.e., if $\phi_2^w(u) > 0$ then $\phi_2^w(w) = 1$.

Proposition 3: Suppose $V_{12}(\ell_1, \ell_2) > 0$ for all $\ell_1, \ell_2 \in [0,\ell]$. Then

$$\phi_2^w(u) = 0 \text{ if } \phi_2^w(w) < 1.$$

Proof: Let $\phi_2^w(w) < 1$. Then from Propositions 1 and 2 it is known that $0 \leq \phi_2^w(u) < \phi_2^w(w) < 1$. This implies from (11) and (12) that $\psi_2^u = \psi_2^w = 0$. Also, it must be the case that equation (16) is binding. Thus $\phi_2^w(u) = 0$, because equation (17) must be slack since it has an identical right-hand side to (16) but a larger left-hand one by Proposition 2 and $\psi_2^u = \psi_2^w = 0$.

□

"Old" workers seemingly have seniority in that they will never laid off $[\phi_2^w(w) < 0]$ so long as "new" workers are being hired $[\phi_2^w(u) > 0]$.

Given the maintained hypothesis that $V_{12}(\ell_1, \ell_2) > 0$ for all $\ell_1, \ell_2 \in [0,\ell]$, it is known from Propositions 1, 2 and 3 that the equilibrium solution for $\phi_2^w(u)$ and $\phi_2^w(w)$ must lie in one of two mutually exclusive sets as expressed below.

$$(\phi_2^w(u), \phi_2^w(w)) \in \{\{(x,y): x=0,\ 0<y\leq 1\} \cup \{(x,y): 0<x<y=1\}\} \tag{22}$$

Note that if $\phi_2^w(u)$ and $\phi_2^w(w)$ have values in the first set then the second-period aggregate employment rate is less than or equal to the first-period one since $\phi_2^w = \phi_1^w \phi_2^w(w) \leq \phi_1^w$. Alternatively, if the

values for $\phi_2^w(u)$ and $\phi_2^w(w)$ lie in the second set then the aggregate employment rate in period two exceeds that in the first period as $\phi_2^w = \phi_1^w + (1-\phi_1^w)\phi_2^w(u) > \phi_1^w$. The feasibility of the latter possibility characterizing solutions to the model would appear to be related to the notion of time discounting. On the one hand, with a positive rate of time preference, agents, loosely speaking, prefer to consume goods today rather than tomorrow. On the other hand, they would rather work tomorrow than today. Obviously, aggregate consumption and employment are directly related to one another through the goods market clearing conditions [(13) and (14)]. It might be expected intuitively that if the representative agent's rate of time preference for goods is higher than for leisure, then the equilibrium time profile for consumption will be downward sloping while the corresponding one for leisure will slope upward.

To investigate this conjecture more closely, let the representative consumer/worker's rate of time preference be restricted in the following manner

$$\frac{V_1(x,y)}{V_2(y,x)} \le \frac{1}{\beta} \equiv 1 + \rho \qquad \text{for all } x,y \in [0,\ell]. \tag{23}$$

That is, the marginal disutility associated with working x units today (along with y tomorrow) is less than the marginal disutility of working x units tomorrow (and y today) multiplied by a factor of $1/\beta$. For constant labor effort and consumption profiles across time, the agent's (gross) rate of time preference for leisure is bounded above by his one for consumption, $1 + \rho$. Simply put, individual's rate of impatience for consuming leisure cannot exceed that for goods. Clearly, this restriction holds for separable

utility functions of the form $V(\ell_1, \ell_2) = W(\ell_1) + \beta W(\ell_2)$. The standard

Cobb-Douglas utility function $V(\ell_1, \ell_2) = A(\bar{\ell} - \ell_1)^\alpha (\bar{\ell} - \ell_2)^\delta$, defined for

$\ell_1, \ell_2 \in [0, \ell]$ where $\ell < \bar{\ell}$ and $\alpha > \delta$, also exhibits this property together with

Edgeworth-Pareto complementarity if α and δ are chosen so that

$$(\alpha/\delta)[\bar{\ell}/(\bar{\ell} - \ell)]^{\alpha - \delta} < 1/\beta.$$

Proposition 4: If $V_{12}(x,y) > 0$ and $V_1(x,y)/V_2(y,x) \leq 1/\beta$ for all $x, y \in [0, \ell]$

then

$$0 = \phi_2^W(u) < \phi_2^W(w) \leq 1.$$

Proof : From (22) it suffices to show that equilibria where

$(\phi_2^W(u), \phi_2^W(w)) \in \{(x,y): 0 < x < y = 1\}$ cannot exist. Suppose to the contrary that

they can. Then situations occur where $\phi_2^W > \phi_1^W$ so that

$U'(f(\phi_2^W \ell, K)) f_1(\phi_2^W \ell, K)\ell < U'(f(\phi_1^W \ell, K)) f_1(\phi_1^W \ell, K)\ell$. For this possibility to be

realized, however, it must happen that [from (15), and (17)]^2

$$[V(0,0) - V(0,\ell)] < \beta[V(0,\ell) - V(\ell,\ell)]. \tag{24}$$

Now by the Fundamental Theorem of Calculus, the above condition occurs if and

only if

$$\int_0^\ell [V_2(0,x) - \beta \frac{V_1(x,\ell)}{V_2(\ell,x)} V_2(\ell,x)]dx > 0. \tag{25}$$

Next, note that the restriction (23) imposed on agent's rate of time

preference for leisure implies that

$$[V_2(0,x) - V_2(\ell,x)] \geq [V_2(0,x) - \beta \frac{V_1(x,\ell)}{V_2(\ell,x)} V_2(\ell,x)]. \tag{26}$$

Since $V_{12}(\cdot) > 0$, then $V_2(0,x) < V_2(\ell,x)$ and, by (26), the left side of (25) will be negative, which is the required contradiction.

<div align="right">□</div>

Observe that structural unemployment in its severest form has been generated here. All those agents who were unemployed in the first period remain so in the second.

Employment and Welfare

An examination into the relationship between an individual's state of employment on the one hand and his levels of consumption and welfare on the other will now be undertaken. Clearly, in an _ex ante_ sense the labor contract outlined in the third section improves agents' welfare since it maximizes their expected utility. The issue of what happens to workers' welfare, _ex poste_, once the state of their employment is known is unclear. Most would have the prejudice that the welfare levels of unemployed workers should be less than those of the employed. The model does not guarantee this outcome, though. In fact, in the previous section unemployed agents realize a higher welfare level than the employed. This result obtained because all individuals had the same level of consumption regardless of their state of employment.

Since utility is a decreasing function in work effort, those who are employed are worse off than those who aren't. This feature is shared by the models of Rogerson (1985), Hansen (1985), and Greenwood and Huffman (1987).

The goal of this section is to discover whether unemployed workers can have lower levels of consumption and utility than employed ones in the model. A conjecture made here is that having a momentary utility function which is nonseparable in consumption and labor effort is central to this issue. To investigate this hunch let the utility function be written as

$$\underline{U}(\cdot) = U(c_1, \ell_1) + \beta U(c_2, \ell_2).$$

The virtue of the above form of an agent's lifetime utility function is that it is time separable. This implies that his second-period level of consumption and probability of being employed will be determined independently of his state of employment in the first period [see (7), and (9) and (10)] so that formally $c_2^i(i) = c_2^i(j) \equiv c_2^i$, and $\phi_2^i(i) = \phi_2^i(j) = \phi_2^i$ for $i, j = w,u$. The model's solution with this restricted form of the utility function is given by the following analogue to the equation system described by (2), (3), and (6) to (10):

$$\phi_t^w c_t^w + (1 - \phi_t^w)c_t^u = f(\phi_t^w \ell, K) \qquad \forall\, t = 1,2 \qquad (27)$$
$$[\text{cf. (2) and (3)}]$$

$$U_1(c_t^w, \ell) = U_1(c_t^u, 0) \qquad \forall\, t = 1,2 \qquad (28)$$
$$[\text{cf. (6) and (7)}]$$

$$U(c_t^u, 0) - U(c_t^w, \ell) \qquad\qquad\qquad (29)$$

$$= U_1(\cdot t)[f_1(\phi_t^w \ell, K)\ell - (c_t^w - c_t^u)]$$

$$\forall\, t = 1,2$$
$$[\text{cf. (8), (9) and (10)}].$$

The key to learning about the relationship between consumption and employment within a period is given in the proposition presented below.

Proposition 5: Suppose that $U_{12}(c_t, \ell_t)$ is either strictly positive or negative for all values of $c_t \in R_+$ and $\ell_t \in [0, \ell]$. Then

$$c_t^w \underset{<}{>} c_t^u \text{ as } U_{12}(\cdot t) \underset{<}{>} 0.$$

Proof: Define $c(\ell_t)$ by the equation $U_1(c(\ell_t), \ell_t) = \tilde{U}_1$. Thus

$$\left. \frac{dc_t}{d\ell_t} \right|_{U_1 = \tilde{U}_1} = -\frac{U_{12}(c(\ell_t), \ell_t)}{U_{11}(c(\ell_t), \ell_t)} \underset{<}{>} 0 \text{ as } U_{12}(c(\ell_t), \ell_t) \underset{<}{>} 0. \tag{30}$$

Now set $\tilde{U}_1 = U(c_t^u, 0)$. The desired result then follows from the observation made below [cf. (28)]

$$c_t^w = c_t^u + \int_0^\ell \left[\left. \frac{dc_t}{d\ell_t} \right|_{U_1 = \tilde{U}_1} \right] d\ell_t.$$

$\square$

Suppose that consumption and work effort (leisure) are net complements (substitutes) in the Edgeworth-Pareto sense so that $U_{12}(\cdot t) > 0$—this will be a maintained hypothesis for the rest of this section. Here, working increases the marginal utility of consumption, which is not implausible, with the optimal labor contract therefore assigning a higher level of consumption to employed agents.

The next issue to address is whether or not employed workers have higher levels of welfare than unemployed ones. Observe from equation (29) that

$$U(c_t^w, \ell) \gtrless U(c_t^u, 0) \text{ as } f_1(\phi_t^w \ell, K)\ell - (c_t^w - c_t^u) \lessgtr 0 \qquad (31)$$

Therefore, the employed can only have a higher welfare level than the unemployed when $c_t^w > c_t^u$, which necessitates that $U_{12}(\cdot t) > 0$. Now for employed agents actually to have a higher welfare level than unemployed ones, it must be the case that the higher level of consumption they receive more than compensates them for the loss in their leisure.

One would like to know whether or not this possibility can arise in the model. The answer seems to be a definite, but qualified, yes. As can be seen from (30), the amount of compensation an employed agent receives for his labor effort appears to be directly related to the magnitude of $-U_{12}(\cdot)/U_{11}(\cdot)$. There are reasonable limits, however, to how large this quantity can be made. In particular, if leisure is to remain a normal good then the following restriction on this magnitude must hold: $-U_{12}(\cdot)/U_{11}(\cdot) < -U_2(\cdot)/U_1(\cdot)$ or equivalently $U_{11}(\cdot)[-U_2(\cdot)/U_1(\cdot)] + U_{12}(\cdot) < 0$.

It will now be investigated whether or not this factor is likely to be an important consideration when comparing the welfare levels of employed and unemployed agents.

<u>Proposition 6</u>:

$$U(c_t^w, \ell) > U(c_t^u, 0)$$

only if

$$U_{11}(c_t, \ell_t)\left[\frac{-U_2(c_t, \ell_t)}{U_1(c_t, \ell_t)}\right] + U_{12}(c_t, \ell_t) > 0 \text{ for some } c_t \in [c_t^u, c_t^w] \text{ and } \ell_t \in [0, \ell].$$

<u>Proof</u>: As in Proposition 5 define the function $c(\ell_t)$ by the equation

$$U_1(c(\ell_t),\ell_t) = \tilde{U}_1. \quad \text{Again set } \tilde{U}_1 = U_1(c^u,0). \quad \text{Then } c_t^w = c(\ell) \text{ and}$$

$c_t^u = c(0)$ so that $U(c_t^w,\ell) = U(c(\ell),\ell)$ and $U(c_t^u,0) = U(c(0),0)$. Thus, by the Fundamental Theorem of Calculus

$$U(c_t^w,\ell) - U(c_t^u,0) = \int_0^\ell [U_1(c(\ell_t),\ell_t) \left.\frac{dc_t}{d\ell_t}\right|_{U_1=\tilde{U}_1} + U_2(c(\ell_t),\ell_t)]d\ell_t$$

$$= \int_0^\ell [-U_1(c(\ell_t),\ell_t) \frac{U_{12}(c(\ell_t),\ell_t)}{U_{11}(c(\ell_t),\ell_t)} + U_2(c(\ell_t),\ell_t)]d\ell_t$$

[using (30)].

Consequently, for $U(c_t^w,\ell) > U(c_t^u,0)$ it must be the case that

$-U_{11}(c_t,\ell_t)U_2(c_t,\ell_t)/U_1(c_t,\ell_t)+U_{12}(c_t,\ell_t) > 0$ for some $c_t \in [c_t^u,c_t^w]$ and $\ell_t \in [0,\ell]$.

□

Thus, if employed individuals are to have a higher level of welfare than unemployed ones, the representative agent's utility function <u>must exhibit inferiority of leisure over some range</u> in the space $[c_t^u,c_t^w] \times [0,\ell]$. The above result does not seem to imply, though, that leisure has to be an inferior good for either employed or unemployed agents at their equilibrium levels of consumption and work effort.

Some further light can be shed upon the relationship between the welfare levels of employed and unemployed agents. In particular, Rogerson and Wright (1987) have extended the above results to show that employed agents are better (worse) off than unemployed ones if and only if an exogenous lump-sum

increase in the economy's income causes the aggregate employment rate to rise (fall).

<u>Proposition 7</u> (Rogerson and Wright, 1987): Let $f(\phi_t^w \ell, K) = g(\phi_t^w \ell, K) + I_t$.

$$U(c_t^w, \ell) \underset{<}{\overset{>}{}} U(c_t^u, 0) \text{ as } d\phi_t^w / dI_t \underset{<}{\overset{>}{}} 0.$$

<u>Proof</u>: Displacing the system of equations (27), (28), and (29) with respect to I_t yields

$$\frac{d\phi_t^w}{dI_t} = \frac{U_{11}(c_t^w, \ell) U_{11}(c_t^u, 0)[U(c_t^w, \ell) - U(c_t^u, 0)]}{\Omega_t}$$

where $\Omega_t \equiv U_1(c_t^w, \ell)\{[U(c_t^w, \ell) - U(c_t^u, 0)]^2 U_{11}(c_t^w, \ell) U_{11}(c_t^u, 0)/U_1(c_t^w, \ell)^2$

$+ U_1(c_t^w, \ell) f_{11}(\phi_t^w \ell, K)[\phi_t^w U_{11}(c_t^w, \ell) + (1 - \phi_t^w) U_{11}(c_t^u, 0)]\} > 0.$

$\square$

Finally, another observation regarding the comparison between the welfare levels of those working and not working can be drawn. Recall from (31) that $U(c_t^w, \ell) \underset{<}{\overset{>}{}} U(c_t^u, 0)$ as

$$f_1(\phi_t \ell, K)\ell - (c_t^w - c_t^u) \underset{>}{\overset{<}{}} 0.$$

Note by multiplying both sides of the above statement by ϕ_t^w and rearranging it can be equivalently expressed in the following manner:

$$c_t^u \underset{>}{\overset{<}{}} \phi_t^w c_t^w + (1 - \phi_t^w) c_t^u - f_1(\phi_t \ell, K)\ell \phi_t^w.$$

Next by utilizing (27) this can be rewritten in the form

$$c_t^u \underset{>}{\overset{<}{}} f(\phi_t^w \ell, K) - f_1(\phi_t^w \ell, K)\ell \phi_t^w,$$

which in turn can be further simplified through use of the constant-returns-to-scale assumption to obtain

$$c_t^u \underset{>}{\overset{<}{}} f_2(\phi_t^w \ell, K)K \equiv \pi_t.$$

Thus, in the situation where employed agents have a higher (lower) level of welfare, $U(c_t^w, \ell)$ than the unemployed, $U(c_t^u, 0)$, the latter are being taxed (subsidized) in the sense that their consumption, c_t^u, is less (greater) than their share of firms' profits, π_t--recall that the ownership of firms was assumed to be distributed uniformly across agents.

Ex ante, it is clearly beneficial for all agents to enter into the labor contract specified in the third section. Ex post, though, some subset of agents in the economy is made worse off in the sense that this collection of individuals is subsidizing the living standard of others. This draws into question whether the income transfers prescribed by the labor contract are likely to be enforcable.

Modifications of Technology

In the fourth and fifth sections it was shown that certain restrictions on preferences in conjunction with indivisible labor could help to produce behavior of particular labor market variables which were consistent with casual observation. Some people may object to this line of inquiry, however, on the grounds that much of the behavior of these labor market variables could be realistically attributable to the characteristics of technology rather than preferences. For instance, the fact that the probability of an employed agent remaining employed is greater than the probability of an unemployed agent attaining employment could be attributed to the higher productivity of agents with better employment histories, a technological consideration, rather than the form of his preferences. To these and other issues surrounding the specification of technology, attention will now be turned.[3]

To begin with, consider the effect of introducing the nonconvexities into the economic environment via lump-sum hiring costs. Specifically, assume a two period economy with inter- and intra-temporally separable preferences described by equation (32) below, in order to abstract from the influence of effects described in previous sections. The production technology is the same as that used earlier except that firms must incur once-and-for-all hiring costs, γ, associated with employing a worker. Now the firms and workers must choose the probability of working in each period $(\phi_1^w,\ \phi_2^w(w),\ \phi_2^w(u))$ as well as the quantity of effort to be supplied by each agent who is employed.

$$\underline{U} = U(c_1) + V(\ell_1) + \beta[U(c_2) + V(\ell_2)] \tag{32}$$

The competitive equilibrium for such an environment is characterized by the solution to the following programming problem where the choice variables are $c_1,\ c_2,\ \phi_1^w,\ \phi_2^w(w),\ \phi_2^w(u),\ \ell_1,\ \ell_2$.

Max $U(c_1) + \phi_1^w V(\ell_1) + (1-\phi_1^w)V(0) + \beta U(c_2) + \beta\{\phi_1^w[\phi_2^w(w)V(\ell_2) +$

$\qquad (1-\phi_2^w(w))V(0)] + (1-\phi_1^w)[\phi_2^w(u)V(\ell_2) + (1-\phi_2^w(u))V(0)]\}$

s.t.

$$c_1 = f(\phi_1^w \ell_1, K) - \gamma\phi_1^w \tag{33}$$

$$c_2 = f(\phi_2^w \ell_2, K) - \gamma(1-\phi_1^w)\phi_2^w(u) \tag{34}$$

$$0 \le \phi_2^w(w) \le 1 \tag{35}$$

$$0 \le \phi_2^w(u) \le 1. \tag{36}$$

$$[\text{Recall } \phi_2^w = \phi_1^w\phi_2^w(w)+(1-\phi_1^w)\phi_2^w(u)].$$

The first-order necessary conditions--in addition to (34) and (35) and the requisite complementary slackness conditions--associated with the above problem are:

$$V(0)-V(\ell_1) + \beta\{E[V(\cdot 2)|\ell_1 = 0] - E[V(\cdot 2)|\ell_1 > 0]\} \tag{37}$$

$$= U'(c_1)[f_1(\phi_1^w \ell_1, K)\ell_1 - \gamma] + \beta U'(c_2)\{f_1(\phi_2^w \ell_2, K)\ell_2[\phi_2^w(w) - \phi_2^w(u)] + \gamma\phi_2^w(u)\}$$

$$[\text{for } \phi_1^w]$$

$$- V'(\ell_t) = U'(c_t)f_1(\phi_t^w \ell_t, K) \tag{38}$$

$$[\text{for } \ell_t]$$

$$\beta[V(0)-V(\ell_2)]+\psi_2^w/\phi_1^w \geq \beta U'(c_2)f_1(\phi_2^w \ell_2, K)\ell_2 \tag{39}$$

$$[\text{for } \phi_2^w(w); \text{ with equality if } \phi_2^w(w) > 0]$$

$$\beta[V(0)-V(\ell_2)] + \psi_2^u/(1-\phi_1^w) \geq \beta U'(c_2)[f_1(\phi_2^w \ell_2, K)\ell_2 - \gamma] \tag{40}$$

$$[\text{for } \phi_2^w(u); \text{ with equality if } \phi_2^w(u) > 0]$$

where ψ_2^w and ψ_2^u are the Lagrange multipliers associated with the constraints (35) and (36).

Note that in general the above programming problem provides a solution for _both_ the fraction of agents employed and hours worked per employed agent for both periods. In general, however, not much can be said about the properties of the solution. One fact is fairly immediate though. Individuals who were employed in the first period have seniority in the second period in the sense that they will never be unemployed if there are _any_ new recruits working. In other words the firm will only hire individuals to work in the second period who were previously unemployed when the pool of first-period employed agents has been totally exhausted. This occurs because there is a lump-sum hiring cost attached to the former but not the latter set of agents.

<u>Proposition 8</u>: $\phi_2^w(u) > 0$ only if $\phi_2^w(w) = 1$.

<u>Proof</u>: Suppose $\phi_2^w(u) > 0$. Then from (40), $\beta[V(0)-V(\ell_2)] + \psi_2^u/(1-\phi_1^w) = \beta U'(c_2)[f_1(\phi_2^w\ell_2,K)\ell_2 - \gamma]$. Using this fact in (39), generates $-\beta U'(c_2)\gamma - \psi_2^u/(1-\phi_1^w) + \psi_2^w/\phi_1^w \geq 0$. This can only be true if $\psi_2^w/\phi_1^w > 0$, which in turn implies that $\phi_2^w(w) = 1$.

$\square$

A numerical example will now be presented in illustration of how equilibrium employment, hours worked, and consumption are potentially related to the lump-sum costs of employment. To construct the example, let

$U(c) = \ell n(c)$, $V(\ell) = -\ell^2$, $f(\phi_t^w\ell_t,K) = (\lambda_t\phi_t^w\ell_t)^{.7} - \kappa_t$ where $\kappa_1 = 0$, $\lambda_1 = 1$, $\kappa_2 = .85$, $\lambda_2 = 2$, and $\beta = .5$. Given this representation of the economy, Table 1 reports the values of the seven endogenous variables for various values of the lump-sum employment costs. It is interesting to note that as the lump-sum costs of hiring increase the equilibrium employment rate falls, while the number of hours worked per employed agent rises. On the one hand, as lump-sum hiring costs increase there is clearly a <u>substitution</u> effect which will operate to reduce the number of employees hired by increasing the number of hours worked by the remaining workers. On the other hand, since $V(\cdot)$ is a decreasing concave function, there are limits to the firm's ability to persuade a few workers (or one worker) to do all the production in the economy. This latter observation is reflected in the fact that consumption falls as the lump-sum hiring costs are increased implying that it isn't optimal to make the remaining employed agents pick up enough extra hours to compensate for the cut in the labor force. Thus a natural trade-off arises between the equilibrium employment rate and hours worked per employee due to the assumed nonconvexity in the labor hiring decision.

Table 1

γ	ϕ_1^w	ℓ_1	c_1	$\phi_2^w(w)$	$\phi_2^w(u)$	ℓ_2	c_2
.45	1.00	.84	.44	1.00	*	.90	.66
.50	.90	.90	.41	1.00	1.00	.92	.63
.55	.77	.97	.393	1.00	1.00	.96	.60
.60	.69	1.03	.375	1.00	.67	1.03	.56
.65	.62	1.09	.36	1.00	.50	1.11	.53
.70	.56	1.15	.35	1.00	.40	1.18	.50

*Value not relevant here

Recall that in the previous sections the amount of labor an agent could supply was restricted to be an element in the two point set $\{0,\ell\}$. This assumed indivisibility in labor a la Rogerson (1985) may be thought of as representing a crude approximation to a production technology which is not concave in labor input over the domain $[0,\ell]$, such as represented in Figure 1. The nonconcave region of the production function could reflect "starting or warming-up" costs associated with the production activity. This type of phenomena, however, is perhaps better characterized by the approximate technology illustrated in Figure 2; either an agent works at _least_ ℓ or he doesn't work at all. Such an extension can be a fruitful ingredient in modelling labor market adjustment along both the extensive and intensive margins. Another useful ingredient in modelling both margins of labor force participation is to incorporate some notion of "on-the-job training." It may be the case that an individual's productivity is directly affected by work experience. In particular, in the current two period setting it will be assumed that agents who worked in period one are more productive in the second period by a factor of $\lambda > 1$—the skill factor—over those who don't have work experience.[4]

Incorporating these two ingredients into the assumed economic environment, while ignoring the previously described hiring costs, leads to a competitive equilibrium which is described by the solution to the following programming problem

$$\text{Max } U(c_1) + \phi_1^w V(\ell_1) + (1-\phi_1^w)V(0) + \beta U(c_2) + \beta\{\phi_1^w[\phi_2^w(w)V(\ell_2(w)) + (1-\phi_2^w(w))V(0)]$$
$$+ (1-\phi_1^w)[\phi_2^w(u)V(\ell_2(u)) + (1-\phi_2^w(u))V(0)]\}$$

s.t.

$$c_1 = f(\phi_1^w\ell_1, K), \quad c_2 = f(\phi_1^w\phi_2^w(w)\lambda\ell_2(w) + (1-\phi_1^w)\phi_2^w(u)\ell_2(u), K)$$

$$\ell_1, \ell_2(w), \ell_2(u) \geq \ell, \quad 0 \leq \phi^w(j) \leq 1 \text{ for } j = w, u,$$
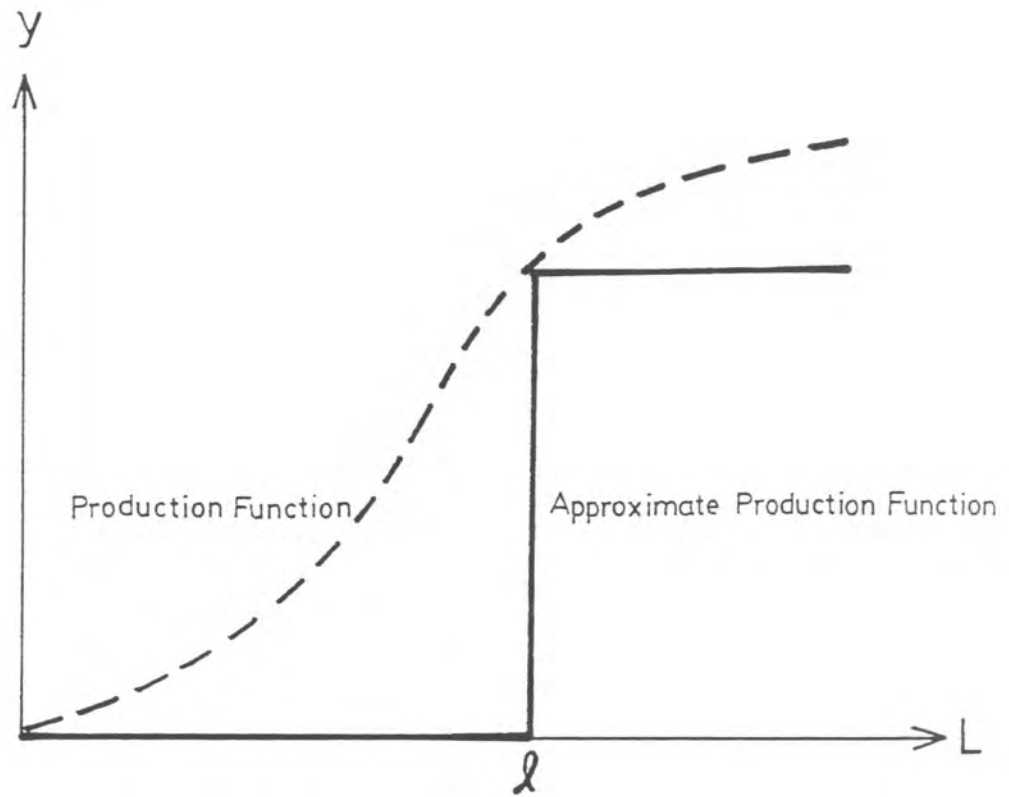
Figure 1
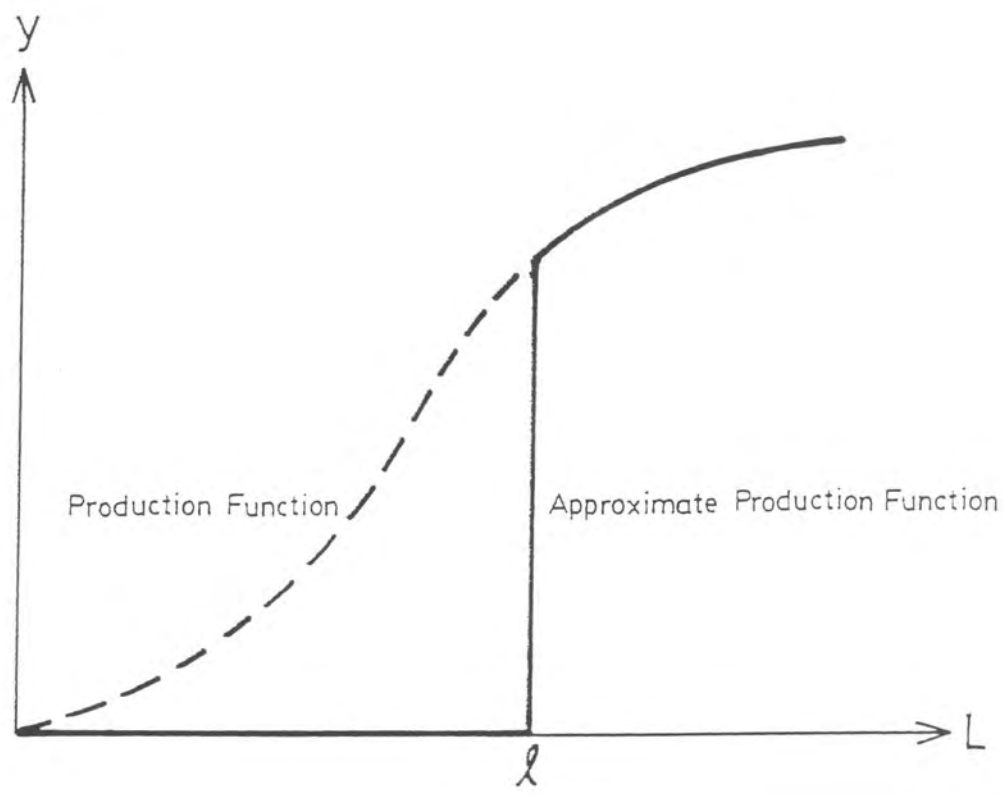
Figure 2

where $\ell_2(w)$ and $\ell_2(u)$ are the amounts of labor effort by employed skilled and unskilled workers in the second period, respectively, and the control variables are c_1, c_2, ϕ_1^w, $\phi_2^w(w)$, $\phi_2^w(u)$, ℓ_1, $\ell_2(w)$, $\ell_2(u)$.

Once again, in general it is difficult to say much about the solution to the above optimization problem. The above problem does permit an interesting phenomena to emerge. That is, it is possible to have competitive equilibria where in the second period some agents, the skilled ones, are working overtime (i.e., $\ell_2(w) > \ell$) while some unskilled agents are unemployed. Also, it is fairly easy to establish that experienced workers have seniority in the sense that first, they will never be unemployed so long as less experienced workers are employed and second, they will never be laid off while other experienced workers are doing overtime. This fact is stated more formally in the proposition below.

Proposition 9: $\phi_2^w(w) < 1$ only if $\phi_2^w(u) = 0$ and $\ell_2(w) = \ell.$[5]

Finally, to establish that it is possible to have skilled workers doing overtime while unskilled ones are unemployed, Table 2 reports the solution values to the above problem for the five labor market control variables over a range of values for the skill factor when the economy is represented as follows: $U(c) = c$, $V(\ell) = -\ell^2$, $f(L,K) = (L)^{.7}$, $\beta = .5$, and $\ell = 1.45$. Note that in this example a seemingly implacable form of structural unemployment is displayed; there are two types of agents in the economy here, those who are permanently employed and those who are permanently unemployed.

Table 2

λ	ϕ_1^w	ℓ_1	$\phi_2^w(w)$	$\phi_2^w(u)$	$\ell_2(w)$	$\ell_2(u)$
7.2	.61	1.45	1.00	0	1.45	*
8.2	.73	1.45	1.00	0	1.49	*
9.2	.87	1.45	1.00	0	1.52	*
10.2	1.00	1.45	1.00	0	1.56	*

*Value not relevant here

Conclusions

An equilibrium model of unemployment was presented here. The economic environment was postulated to be such that there were nonconvexities present in either tastes or technology. It was shown that environments with such nonconvexities were capable of displaying interesting labor market phenomena. Given the adopted setting, optimal labor contracting always resulted in a certain fraction of the population being unemployed. It was demonstrated that equilibria could be generated where those currently not working had higher probabilities of being unemployed in the future than those currently employed. In fact, a phenomena resembling structural unemployment could occur where those currently unemployed remained permanently so. Such nonconvexities also could allow for a simultaneous determination of both the extensive and intensive margins of labor force participation. It was possible to have certain agents working overtime while others were unemployed. Additionally, equilibria exist where old workers have seniority in the sense that are never laid off so long as either new workers are being hired or any agents are working overtime.

Footnotes

[1]A standard result in implicit labor contracting theory for environments with symmetric information is that all employment decisions are undertaken efficiently in the standard Walrasian manner--see Hart (1983).

[2]In more detail, from equation (17) it is known that $[V(0,0)-V(0,\ell)]=\beta U'(f(\phi_2^w \ell,K))f_1(\phi_2^w \ell,K)\ell$. Using this information in (15) together with the fact that $\phi_2^w(w) = 1$ yields $[V(0,\ell)-V(\ell,\ell)] = U'(f(\phi_1^w \ell,K))f_1(\phi_1^w \ell,K)\ell$. The desired result follows immediately.

[3]The two models presented in this section could easily be appended onto the general framework presented in the second section.

[4]This appears to be the simplest method of employing an intertemporal heterogeneity in the technology. More complicated formulations could be imagined.

[5]The fact that $\phi_2^w(w) < 1$ only if $\phi_2^w(u) = 0$ is easy to show by utilizing arguments similar to those employed in establishing some of the earlier propositions. That $\phi_2^w(w) < 1$ only if $\ell_2(w) = \ell$ essentially follows from the observations made below. First, the efficiency condition for $\phi_2^w(w)$ dictates that if $\phi_2^w(w) < 1$ then $[V(0) - V(\ell_2(w))] = U'(\cdot 2)f_1(\cdot 2)\lambda \ell_2(w)$. Second, the efficiency condition $\ell_2(w)$ necessitates that $-V'(\ell_2(w)) \geq U'(\cdot 2)f_1(\cdot 2)\lambda$, with this equation holding with equality whenever $\ell_2(w) > \ell$. Third, since $V(\cdot)$ is a concave function so that $[V(0) - V(\ell_2(w))] < - V'(\ell_2(w))\ell_2(w)$, both of the above equations can only hold simultaneously if the latter one remains slack.

References

Azariadis, Costas (1983) "Employment with Asymmetric Information"
Quarterly Journal of Economics XCIII, 157-172.

Friedman, Milton (1968) "The Role of Monetary Policy" American Economic
Review LVIII, 1-17.

Greenwood, Jeremy and Gregory W. Huffman (1987) "A Dynamic Equilibrium Model
of Inflation and Unemployment". Journal of Monetary Economics 19,
203-228.

Grossman, Sanford J. and Oliver D. Hart (1983) "Implicit Contracts Under
Asymmetric Information" Quarterly Journal of Economics XCIII, 123-156.

Hansen, Gary D. (1985) "Indivisible Labor and the Business Cycle", Journal of
Monetary Economics 16, 309-327.

Hart, Oliver D. (1983) "Optimal Contracts under Asymmetric Information:
An Introduction" Review of Economic Studies L, 3-35.

Lucas, Robert E. Jr. and Edward C. Prescott (1974) "Equilibrium Search and
Unemployment" Journal of Economic Theory 7, 188-209.

Prescott, Edward C. and Robert M. Townsend (1984) "Pareto Optimal and
Competitive Equilibrium with Adverse Selection and Moral Hazard"
Econometrica 52, 21-45.

Rogerson, Richard (1985) "Indivisible Labor, Lotteries, and Equilibrium",
Working Paper No. 10, RCER, Department of Economics, University of
Rochester.

Rogerson, Richard and Randall Wright (1987) "'Involuntary Unemployment' in
Economies with Efficient Risk Sharing", Unpublished Paper, Department of
Economics, Cornell University.

Rothenberg, Jerome (1960) "Non-Convexity, Aggregation, and Pareto
Optimality" Journal of Political Economy LXVIII, 435-468.

Rochester Center for Economic Research
University of Rochester
Department of Economics
Rochester, NY 14627

1986-87 DISCUSSION PAPERS

WP#46   FORMULA BUDGETING: THE ECONOMICS AND ANALYTICS OF FISCAL POLICY
UNDER RULES, by Eric A. Hanushek, June 1986

WP#48   EXCHANGE RATE POLICY, WAGE FORMATION, AND CREDIBILITY
by Henrik Horn and Torsten Persson, June 1986

WP#49   MONEY AND BUSINESS CYCLES: COMMENTS ON BERNANKE AND RELATED
LITERATURE, by Robert G. King, July 1986

WP#50   NOMINAL SURPRISES, REAL FACTORS AND PROPAGATION MECHANISMS
by Robert G. King and Charles I. Plosser, Final Draft: July 1986

WP#51   JOB MOBILITY IN MARKET EQUILIBRIUM
by Glenn M. MacDonald, August 1986

WP#52   SECRECY, SPECULATION AND POLICY
by Robert G. King, (revised) August 1986

WP#53   THE TULIPMANIA LEGEND
by Peter M. Garber, July 1986

WP#54   THE WELFARE THEOREMS AND ECONOMIES WITH LAND AND A FINITE NUMBER OF
TRADERS, by Marcus Berliant and Karl Dunz, July 1986

WP#55   NONLABOR SUPPLY RESPONSES TO THE INCOME MAINTENANCE EXPERIMENTS
by Eric A. Hanushek, August 1986

WP#56   INDIVISIBLE LABOR, EXPERIENCE AND INTERTEMPORAL ALLOCATIONS
by Vittorio U. Grilli and Richard Rogerson, September 1986

WP#57   TIME CONSISTENCY OF FISCAL AND MONETARY POLICY
by Mats Persson, Torsten Persson and Lars E. O. Svensson,
September 1986

WP#58   ON THE NATURE OF UNEMPLOYMENT IN ECONOMIES WITH EFFICIENT RISK
SHARING, by Richard Rogerson and Randall Wright, September 1986

WP#59   INFORMATION PRODUCTION, EVALUATION RISK, AND OPTIMAL CONTRACTS
by Monica Hargraves and Paul M. Romer, September 1986

WP#60   RECURSIVE UTILITY AND THE RAMSEY PROBLEM
by John H. Boyd III, October 1986

WP#61   WHO LEAVES WHOM IN DURABLE TRADING MATCHES
by Kenneth J. McLaughlin, October 1986

WP#62   SYMMETRIES, EQUILIBRIA AND THE VALUE FUNCTION
by John H. Boyd III, December 1986

WP#63   A NOTE ON INCOME TAXATION AND THE CORE
by Marcus Berliant, December 1986

WP#64   INCREASING RETURNS, SPECIALIZATION, AND EXTERNAL ECONOMIES: GROWTH AS DESCRIBED BY ALLYN YOUNG, By Paul M. Romer, December 1986

WP#65   THE QUIT-LAYOFF DISTINCTION: EMPIRICAL REGULARITIES by Kenneth J. McLaughlin, December 1986

WP#66   FURTHER EVIDENCE ON THE RELATION BETWEEN FISCAL POLICY AND THE TERM STRUCTURE, by Charles I. Plosser, December 1986

WP#67   INVENTORIES AND THE VOLATILITY OF PRODUCTION by James A. Kahn, December 1986

WP#68   RECURSIVE UTILITY AND OPTIMAL CAPITAL ACCUMULATION, I: EXISTENCE, by Robert A. Becker, John H. Boyd III, and Bom Yong Sung, January 1987

WP#69   MONEY AND MARKET INCOMPLETENESS IN OVERLAPPING-GENERATIONS MODELS, by Marianne Baxter, January 1987

WP#70   GROWTH BASED ON INCREASING RETURNS DUE TO SPECIALIZATION by Paul M. Romer, January 1987

WP#71   WHY A STUBBORN CONSERVATIVE WOULD RUN A DEFICIT: POLICY WITH TIME-INCONSISTENT PREFERENCES by Torsten Persson and Lars E.O. Svensson, January 1987

WP#72   ON THE CONTINUUM APPROACH OF SPATIAL AND SOME LOCAL PUBLIC GOODS OR PRODUCT DIFFERENTIATION MODELS by Marcus Berliant and Thijs ten Raa, January 1987

WP#73   THE QUIT-LAYOFF DISTINCTION: GROWTH EFFECTS by Kenneth J. McLaughlin, February 1987

WP#74   SOCIAL SECURITY, LIQUIDITY, AND EARLY RETIREMENT by James A. Kahn, March 1987

WP#75   THE PRODUCT CYCLE HYPOTHESIS AND THE HECKSCHER-OHLIN-SAMUELSON THEORY OF INTERNATIONAL TRADE by Sugata Marjit, April 1987

WP#76   NOTIONS OF EQUAL OPPORTUNITIES by William Thomson, April 1987

WP#77   BARGAINING PROBLEMS WITH UNCERTAIN DISAGREEMENT POINTS by Youngsub Chun and William Thomson, April 1987

WP#78   THE ECONOMICS OF RISING STARS by Glenn M. MacDonald, April 1987

WP#79   STOCHASTIC TRENDS AND ECONOMIC FLUCTUATIONS by Robert King, Charles Plosser, James Stock, and Mark Watson, April 1987

To order copies of the above papers complete the attached invoice and return to Christine Massaro, W. Allen Wallis Institute of Political Economy, RCER, 109B Harkness Hall, University of Rochester, Rochester, NY 14627. <u>Three (3) papers per year</u> will be provided free of charge as requested below. Each additional paper will require a $5.00 service fee which <u>must be enclosed with your order.</u> For your convenience an invoice is provided below in order that you may request payment from your institution as necessary. Please make your check payable to the **Rochester Center for Economic Research.** <u>**Checks must be drawn from a U.S. bank and in U.S. dollars**</u>.

W. Allen Wallis Institute for Political Economy

Rochester Center for Economic Research, Working Paper Series

OFFICIAL INVOICE

Requestor's Name _____

Requestor's Address _____

_____

_____

_____

Please send me the following papers free of charge (**Limit: 3 free per year**).

WP# _____   WP# _____   WP# _____

I understand there is a $5.00 fee for each additional paper. Enclosed is my check or money order in the amount of $ _____. Please send me the following papers.

WP# _____   WP# _____   WP# _____

WP# _____   WP# _____   WP# _____

WP# _____   WP# _____   WP# _____

WP# _____   WP# _____   WP# _____