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I'M NOT A HIGH QUALITY FIRM— BUT I PLAY ONE ON T.V.: A MODEL OF SIGNALING PRODUCT QUALITY

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Introduction

The notion that advertising can be used to signal product quality originates from Nelson [1970,1974,1978]. In a series of articles during the seventies Nelson has repeatedly argued this point and collected empirical evidence to support his claim. Nelson's argument is based on a distinction between *search* and *experience* goods. Nelson [1974] defines a search good as a good whose quality "... can be determined by inspection prior to purchase of the brand." The quality of experience goods, on the other hand, can only be determined after purchase (barring of course, signaling by the firm).

Nelson [1974] argues that since the quality of search goods are verifiable upon inspection, "[t]his reduces considerably–but not entirely–incentives for misleading advertising." Hence, "For search [goods] advertising provides direct information about the characteristics of the brand."

For experience goods, however, there are incentives for the advertiser to make exaggerated or false claims about the product. Hence a rational consumer would be skeptical of the claims made by advertisers of experience goods. Put differently, the content of such advertising can contain only a "... [a] miniscule amount of direct information."

Nelson's insight is that, even though the content of such advertising is of no value, the *level* of advertising itself may convey useful information. In particular, expenditures on advertising may contain information about the quality of the product. If, on average, high-quality firms advertise more than low-quality firms, it might be optimal for consumers to respond to advertising. As Nelson points out, it does not matter whether consumers are fully aware of this correlation between advertising and quality. "Whatever their explicit reasons, the consumers' ultimate reason for responding to advertising is their self interest in so doing ... If it were not in consumer self interest to respond to advertising, then consumers' sloppy thinking about advertising would cost enough that they would reform their ways."

Hence, although the underlying rational for advertising is its informational value, a consumer's response may be individually justified by some other mechanism.

In order to close the circle one needs an explanation of why advertising and quality should be correlated. Nelson [1974] believes that repeat business can explain why high quality firms might advertise more. Suppose advertising has a direct effect on consumer demand, perhaps because, "Advertising increases the probability of a consumers' remembering the name of the brand." If high quality firms can expect more repeat purchases then they have a greater incentive to advertise than do low quality firms. Nelson also believes that high quality brands tend to deliver more utility per dollar to the consumer. If we pretend for a moment that the good being sold is a unit of utility, then "... a firm that has lower costs ... will find that it pays to expand its ouput by both increasing advertising expenditures and decreasing [the price]." In other words, advertising expenditures are not used explicitly by the firm to signal quality. Instead, they are used to increase initial demand which is more valuable to the high-quality firm.

A more direct way to close the circle is to argue that because of future sales, it is in the interest of a high-quality firm to try to distinguish itself from a low-quality firm by advertising. For this to be plausible, it must be too costly for the low-quality firm to mimic the actions of a high-quality producer. If consumers are aware of this fact then they can correctly infer product quality. The distinction between the two arguments is subtle, but important. In the first case, advertising is used to increase demand by informing or reminding people about a products existence. In the second case the high-quality firm makes a "conscious" decision to signal its quality via advertising. In this later case, advertising may or may not have direct effect on demand; it may be a purely dissipative signal (i.e. flushing money down the toilet to make a point).

Most models of product quality signaling extrapolate from the first effect and concentrate exclusively on the signaling aspect. The underlying assumption in this type of model is that all consumers already know that the product in question exists, they just don't know how good it is (i.e. the consumer is facing an experience good).

Milgrom and Roberts [1986], for example, study a signaling monopolist under a regime with dissipative advertising.

In this paper I study and compare a signaling monopolist under both regimes. The game with informative advertising can be viewed as an improvement over its precursor in that it provides an explanation of why both high and low-quality firms might advertise. This new innovation also calls into question the basic conclusion that advertising will be positively correlated with quality. In particular, it is shown that this result depends upon the nature of repeat business, the relationship between quality and marginal cost, and the size of the market. In other words, the correlation will depend on the manner in which advertising is informative and hence its direct effects on the monopolist's profit function.

Another reason why low-quality firms might advertise can be found by relaxing the assumption that signaling via advertising is perfectly deterministic. In this paper I show how the Milgrom and Roberts model might be generalized to encompass stochastic elements in the advertising dimension. I show that any advertising equilibria in such a model must involve advertising by both quality types, whereas in Milgrom and Roberts only the high quality type advertises. This approach also has the potential to eliminate their result that all advertising takes place in the initial period. This conclusion is deeply embedded in the nature of a separating equilibrium. For example, in an advertising game, if the two firms separate in period one, then no additional advertising in subsequent periods should be expected. The reason is that because the two firms initially choose different strategies, the consumers should become fully informed in period one. Unless consumers have a very short memory, theres nothing left to signal in period two. In other words, by the second period, consumers already know whether or not they are facing a high or low-quality monopolist. Hence, it seems advantageous to add a degree of uncertainty into the signaling process if we desire a model where advertising persists across several periods.

One way to model this uncertainty is to simply add noise into the signaling channel. Under this scheme, we have consumers observing an advertising expenditure which is perturbed by a stochastic process. The resulting equilibria might be richer in a variety of ways. First, the nature of repeat business might become more interesting and important. This is desirable since the role of repeat business in previous models has often been oversimplified. In some models, repeat purchases have no effect on the equilibrium level of advertising at all. Ramey [1987], for example, assumes that all customers are fully informed by the second period. This results in a de facto one-period model. Kihlstrom and Riordan [1984] make the same assumption and conclude from their model, " In virtually all cases in which advertising equilibria exist with repeat purchases, they also exist without repeat purchases. " Milgrom and Roberts [1986], however, do develop a model in which repeat business is important. They assume that only customers who purchase in the first period can purchase in subsequent periods. They also assume that all potential customers are fully informed by period two. Under these assumptions, an n-period model is cleverly reduced to a two period model. A closer look at repeat business is desirable since this might help us to better encompass the insights of Nelson within the model.

A second motivation for the introduction of noisy signaling is that the learning process of the consumer might become more sophisticated. The consumer will be able to learn about product quality not only through advertising, but also through repeated consumption. The end result might be a multi-faceted Bayesian learning process. [1]

The paper is broken up into four parts. Section 2 explores a simple variant of the Milgrom and Roberts model without repeat business. It is shown that for "large" markets a unique separating equilibrium which satisfies an equilibrium refinement called "immunity to sequential elimination of dominated strategies," almost always exists. This equilibrium involves only price signaling by the high-quality firm and

[1] Horstman and MacDonald [1988] have also recognized the importance of the learning process. The authors make clever utilization of identical profit functions for both high and low-quality firms. The result is that whenever its beneficial for the high-quality firm to signal its quality, its also beneficial for the low-quality firm to mimic this signaling in order to fool the consumer, and be perceived as a high-quality firm as well. This implies that a first period separation is impossible and forces us to extent the learning process of the consumer. Hence advertising might exist beyond the initial period.

does not have any advertising. Quality may be signaled by a price higher or lower than the the "full information" price depending on whether marginal cost is directly or inversely related to quality.

Section 3 involves a simple two period model. In large markets, it is shown that this model will almost always have equilibria which involve advertising by the high-quality type firm. As in Section 1, an explicit formula for the equilibrium is presented and hence comparative statics can be done.

In Section 4 I describe a model where advertising is a stochastic process and define the relevant equilibrium concept. This model is interesting for several reasons. First, I show that any advertising equilibria must involve advertising by both types and at different levels. Second, because beliefs must be consistent with Bayesian updating, all these equilibria involve a price which is uncorrelated with quality. This conclusion is markedly different from the other models and is quite general in that it does not depend on the specific functional form of the signal loss function. The implication is that the results of Milgrom and Roberts can not be extended in a natural manner to the case with stochastic signal loss. It is, however, shown that that in a stochastic environment with repeat business, **advertising** will be positively correlated to quality provided that the market is sufficiently large. In other words the deviation from Milgrom and Roberts concerns whether or not price would be used to signal to quality and whether or not low quality firms advertise.

In Section 5, a model where advertising is also directly informative is presented. It is shown that under this regime both types are likely to advertise and at different levels. The correlation between quality and advertising may be weaker than in the previous model. When there are diminishing returns from the direct effects of advertising, it is shown that advertising may in fact be inversely correlated with quality. This example requires a small market where marginal cost is proportional to quality.

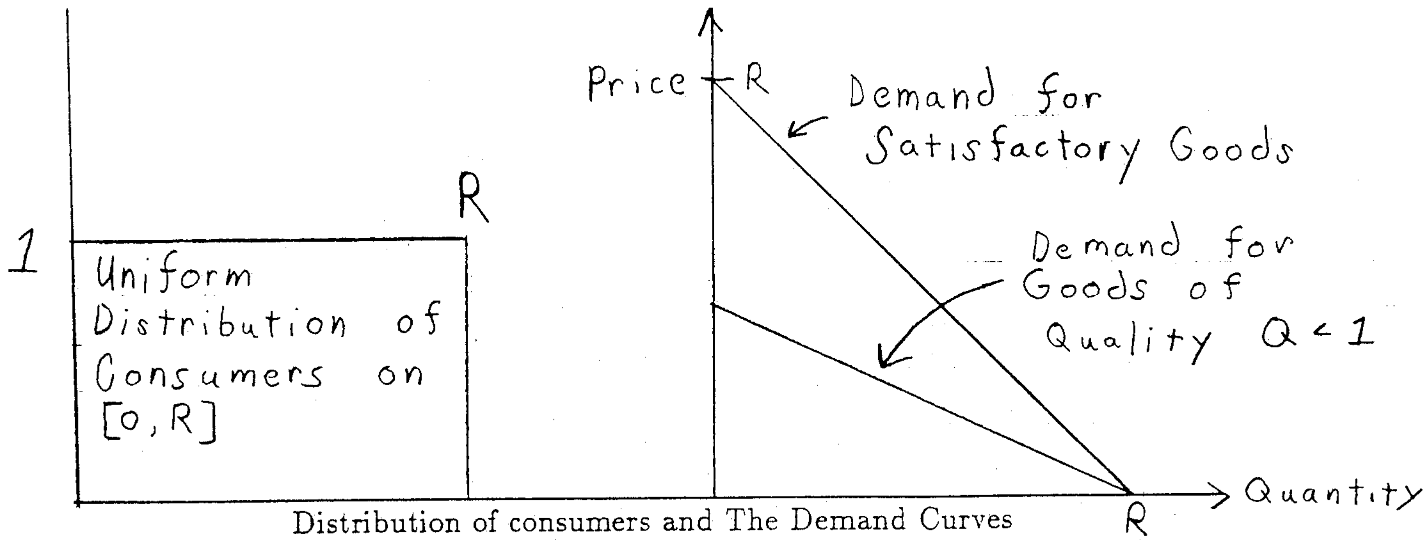
Section 4 and Section 5 can be viewed as an attempt to relax the assumption of Milgrom and Roberts that advertising expenditures are perfectly observable. If we think about television advertisements, this assumption seems unrealisitic.

Total advertising expenditures would equal the sum of production costs and air time. Consumers would need to know not only how much it costs to produce one commercial, but also how how many times the commercial was aired. Even is we grant that production costs are perfectly observable, the Milgrom and Roberts' assumption says, " **All** the consumers see **All** of the advertising." In Section 4 I weaken this by developing a model where, " **All** of the consumers see **some** of the advertising." In Section 5 the model is, " **Some** of the consumers see **All** of the advertising."

I want to stress that all four models consider only the marketing of a new product about which little is known. These theories say nothing about the advertising of established brands. Also, like Milgrom and Roberts, quality will be treated as exogenous and not as a choice variable, readers are encouraged to see Ramey [1987] for a model where the reverse is true. Although the main focus of the paper is on separating equilibria in the signaling game, I will argue at the end of Section 3 that when the prior belief by consumers of facing a high-quality firm is sufficiently high, a pooling equilibrium without advertising is perhaps more plausible. These comments will extend Milgrom and Roberts' brief discussion of these equilibria.

1. The Basic Model

I assume that there are a continuum of consumers uniformly distributed along the interval [0,R]. A consumer's address tells us how much he/she would value a product which performs satisfactorily. The quality of the product is operationalized as the probability that the good in question will perform satisfactorily in each period, for a randomly selected consumer. Throughout the paper I make the simplifying assumption that consumers get no utility from an unsatisfactory product. I also assume that one purchase is sufficient for each consumer to determine whether or not the good is satisfactory. Hence, consumers are not directly concerned with the product quality, but instead would use the quality as an imperfect measure of



Distribution of consumers and The Demand Curves

potential satisfaction. The assumption that consumers become completely informed after the initial purchase seems useful since the paper intentionally abstracts from a detailed description of repeat business. To a degree these assumptions might be relevant to the pharmaceutical industry where benefits from a new drug are often discrete in nature and frequently distributed at random among patients. An alternative example might be found in the prepared food industry or in the marketing of over-the-counter health and beauty aids. In any case, these are the same basic assumptions of Milgrom and Roberts [1986].

The end result of the above assumptions is a linear demand curve for a satisfactory product whose slope is -1 and whose price and quantity intercepts are both R. The demand curve for a product whose expected quality is Q<1 can then be derived from this by rotating the original demand curve counter-clockwise around the point R on the quantity axis. The new demand curve will have a slope of -Q. (See the diagram).

I assume that there is a monopoly which manufactures either a high- or low-quality good. Nature randomly selects between two possible quality values H and L according to some prior probability distribution. $(1 \geq H > L > 0)$. I use ρ to denote the prior probability of a High-quality monopolist. Hence, we are modeling a game of incomplete-information as a game of imperfect information. The high(low)-quality firm is assumed to face constant marginal costs $C_H(C_L)$.

Consumers will purchase a maximum of one unit in each period, whenever the expected value of the product exceeds the market price. For simplicity, I will assume that all consumers have identical prior beliefs. Because there is a continuum of agents, this assumption is equivalent to assuming that prior beliefs are independent of an individual's marginal valuation (i.e their address).

A strategy for the monopolist is a function $M(Q) : [H, L] \rightarrow (P, A)$, which translates the actual quality of the firm into a nonnegative price–advertising pair. We have already assumed that each consumer will purchase one unit of the good, provided that its expected marginal valuation exceeds the market price. Hence, only the beliefs of the consumer are relevant. In particular, the beliefs of all consumers are described by $B(P, A) : R^2_{++} \rightarrow [0, 1]$ which translates an observed price–advertising pair into an expectation of quality. (When $L < B(P, A) < H$, I will sometimes use the notation $EQ(\cdot, \cdot, \cdot)$). I define the profits of the firm which sets price P and advertises at level A and whose actual quality is Q as:

$$\Pi(P, Q, B(P, A)) - A = (R - (P/(B(P, A))(P - C_Q) - A$$

Hence, $\Pi(P, Q, B)$ represents the one period profit function (net of advertising) for a monopolist which is believed to be of quality B and faces marginal cost C_Q. We are now ready to define the equilibrium concept.

Definition. A Sequential equilibrium for this model is a strategy $M(Q)$ for the firm, and a system of beliefs for the consumers $B(P, A)$ such that:

(1) $M(L)$ maximizes $\Pi(P, L, B(P, A)) - A$ given the beliefs $B(P, A)$

(2) $M(H)$ maximizes $\Pi(P, H, B(P, A)) - A$ given the beliefs $B(P, A)$

(3) $B(P, A)$ is computed using Bayes Rule along the equilibrium path.

(4) Both P and A are nonnegative

2. No Repeat Business

The first step in looking for an equilibrium is to solve for the complete information equilibrium price and profits for both the high- and low-quality firms. This is required in order to derive necessary conditions for a separating equilibrium. Since advertising serves only as a dissipative signal, under complete-information neither firm will advertise. We denote by $\Pi(P, Q, B)$ the profit function for a firm which is believed to be of quality B and actually is of quality Q (actual quality comes first, then the perceived quality). P_B^Q is used to denote the profit maximizing price of such a firm. Complete-information prices and profits can now be easily derived:

$$\Pi(P, H, H) = (R - P/H)(P - C_H) \qquad for \quad 0 < P < RH$$

$$\partial\Pi/\partial P = R - 2P/H + C_H/H = 0$$

$$\text{solving for} \quad P_H^H \quad yields \quad P_H^H = (1/2)(RH + C_H)$$

$$\text{substitution then reveils that:} \quad \Pi(P_H^H, H, H) = \frac{(RH - C_H)^2}{4H}$$

Doing the same for the low-quality firm then gives us:

$$\Pi(P, L, L) = (R - P/L)(P - C_L) \qquad for \quad 0 < P < RL$$

$$P_L^L = (1/2)(RL + C_L) \quad \text{and} \quad \Pi(P_L^L, L, L) = \frac{(RL - C_L)^2}{4L}$$

We also need P_L^H and $\Pi(P_L^H, H, L)$ the relevant price and profits for a high-quality firm perceived as a low-quality as well as P_H^L and $\Pi(P_H^L, L, H)$ which are similarly defined:

$$P_L^H = (1/2)(RL + C_H) \quad \text{and} \quad \Pi(P_L^H, H, L) = \frac{(RL - C_H)^2}{4H}$$

$$P_H^L = (1/2)(RH + C_L) \quad \text{and} \quad \Pi(P_H^L, L, H) = \frac{(RH - C_L)^2}{4L}$$

Definition. A natural separation is said to occur when in equilibrium both types of firms employ their complete-information strategies: $(P_H^H, 0)$ and $(P_L^L, 0)$ for the high- and low-types respectively. [2]

The separation is natural in the sense that the high quality firm is able to signal its quality to the consumer without making a conscious deviation from its complete-information strategy. We wish to be explicit about the lack of advertising since, as we shall later see, when $C_H = C_L$ there exists a separating equilibrium which involves complete-information prices and positive advertising. For a natural separation to occur it must be the case that the low-quality firm has no incentive to charge P_H^H and be thought of as a high-quality firm. If this were not true, then the low-quality firm would deviate from the proposed equilibrium and thereby overturn it. It should also be easy to see that when the low-quality firm has no incentive to charge P_H^H neither type has any incentive to advertise. We now provide a formal statement of the first remark:

Lemma 2.0. A necessary and sufficient condition for a natural separation to occur is that: $\Pi(P_L^L, L, L) \geq \Pi(P_H^H, L, H)$

Intuitively, this just says that even if the low-quality firm could masquerade as a high type by charging P_H^H, he prefers his profits at the complete- information price P_L^L. Beliefs which can sustain the equilibrium are: $B(P_H^H, 0) = H$; $B(P_L^L, 0) = L$; $B(P, A) = L$ for all $A > 0$ and for all $P \neq (P_L^L, P_H^H)$. Given these beliefs, which are consistent with the equilibrium strategies $M(H) = (P_H^H, 0)$ and $M(L) = (P_L^L, 0)$, it is clear that neither type has an incentive to defect, since defection would result in the belief that quality was L. Type L is already maximizing his profits subject to this belief, and so has no desire to deviate. Type H is receiving the highest profits he can possibly hope to receive and likewise would not deviate. It should be noted that this is the same equilibrium which would emerge in a game of complete information where knowledge about quality was not privately held by the firm, but also available to consumers. After solving explicitly for $\Pi(P_H^H, L, H)$ we can rewrite

[2] (P, A) is used to denote the strategy as a price-advertising pair.

the condition above as:

$$(3) \qquad \frac{(RL - C_L)^2}{4L} > \frac{R^2H^2 + 2C_L(C_H - RH) - C_H^2}{4H}$$

Lemma 2.1. For sufficiently large R (relative to C_H, C_L, H,and and L) or for sufficiently small marginal costs, natural separation will not occur. For sufficiently high C_H (Relative to C_L and R) natural separation will occur.

Proof: Fix C_H, C_L, H, L then as $R \to \infty$ equation (3) reduces to $R^2L - 2RC_L > R^2H - 2RC_L$ in the limit. This inequality is clearly false since we assumed that $H > L$. Alternatively, fix H,L, and R, and let $C_H, C_L \to 0$. Then once again inequality (3) reduces to $R^2L > R^2H$, which is false. Finally, as C_H approaches R, the right-hand side approaches zero. Hence, for a fixed C_L the inequality can be made to hold with a high C_H.

Lemma 2.0 naturally leads us to consider other kinds of separating sequential equilibria. In particular we consider equilibria where the two types pick different strategies, but not necessarily their complete-information strategies. At this point a proposition of Milgrom and Roberts is useful:

Proposition 1 (Milgrom and Roberts). There exists a separating sequential equilibrium if and only if, for some $(P^*, A^*) \geq 0$:

(1) $\Pi(P^*, H, H) - A^* \geq \Pi(P_L^H, H, L)$
(2) $\Pi(P_L^L, L, L) \geq \Pi(P^*, L, H) - A^*$.

When the conditions for natural separation are met, then clearly $(P_H^H, 0)$ is a price–advertising pair which satisfies the proposition. Inequality (1) says that the high-quality firm would prefer to pick strategy (P^*, A^*) to the alternative of defecting from this and being perceived as a low-quality firm. Clearly the optimal such defection is $(P_L^H, 0)$, so if inequality (1) holds for P_L^H it clearly holds for all possible defections. Inequality (2) states that the low-quality firm, prefers his complete-information profits to the profits he could get by selecting (P^*, A^*) and masquerading as a high-quality type. If beliefs are given by: $B(P^*, A^*) = H$;

$B(P_L^L, 0) = L$; $B(P' \neq P^*, A) = L$ for all $A \in [0, \infty]$, then neither type has an incentive to deviate from their respective equilibrium strategies: $M(H) = (P^*, A^*)$ and $M(L) = (P_L^L, 0)$. Also $B(P, A)$ trivially satisfies Bayes' Rule. This demonstrates sufficiency.

If inequality (1) is not satisfied, then type H has an incentive to defect to $(P_L^H, 0)$. Similarly, (2) provides the incentive for a type L not to defect from $(P_L^L, 0)$. Hence the above conditions are both necessary and sufficient.

Any equilibria, however, where inequality (2) holds *strictly* can be regarded as completely unrealistic. In particular, it would be sustained by the belief that a low-quality type would mimic the strategy of a high-quality type even when this results in profits lower than the complete-information profits. A more realistic formulation would require that $B(P, A) = H$ whenever $A \geq \Pi(P, L, H) - \Pi(P_L^L, L, L)$. Intuitively, this just means that consumers should never expect the low-quality firm to play a strategy which is dominated by $(P_L^L, 0)$. Since deviations from $(P_H^H, 0)$ are costly for type H, the high quality firm would set advertising at $A = \Pi(P, L, H) - \Pi(P_L^L, L, L)$ since (for a given price) this achieves the required separation at minimum cost. Hence with the elimination of dominated strategies, advertising as a function of price is pinned down. In the future when I say sequential equilibrium, I mean one which satisfies this property.

Of course, this still leaves us with a continuum of potential equilibrium price–advertising pairs, each parametrized by the price. From this set, the high-quality firm would choose the price–advertising pair which maximizes profits (i.e. achieves the required separation at minimun cost). In other words, in any sequential equilibria, the high-quality firm would maximize:

$$\Pi(P, H, H) - [\Pi(P, L, H) - \Pi(P_L^L, L, L)]$$

with respect to price, where the term in brackets represents the minimum amount of advertising required to separate. The above formulation only makes sense when the optimal separation involves advertising. If the optimal separation involves zero advertising then the objective above may not even have a maximum. Noting that the last term $\Pi(P_L^L, L, L)$ is not a function of P leads us naturally to Milgrom and Roberts next result:

Proposition 2 (Milgrom and Roberts). If (P^*, A^*) is a sequential equilibrium with $A^* > 0$ then P^* solves:

$$\max_P \quad [\Pi(P,H,H) - \Pi(P,L,H) \quad \text{subject to:} \quad \Pi(P,L,H) - \Pi(P_L^L, L, L) > 0]$$

The proposition basically says that type H achieves as much separation as desirable in price, and that this separation is inadequate to deter mimicry (i.e. the constraint). Hence, type H advertises at the level indicated by the constraint and thereby achieves the required separation at minimum cost. When the objective has no maximum or has a maximum where the constraint is not satisfied, then all the separation will be done by price alone.

We are now in a position to discover the sequential equilibria in the one period game. The proposition says that if there is advertising then $\frac{\partial \Pi}{\partial P}(P,H,H) = \frac{\partial \Pi}{\partial P}(P,L,H)$ which is equivalent to $R - (2P/H) + C_H/H = R - (2P/H) + C_L/H$. Except when $C_H = C_L$, this condition fails and there are no separating equilibria with advertising. Intuitively, when the condition fails this just means that some price adjustments are always cheaper for type H than for type L. Hence H will always choose to separate in price alone. This implies that there are no separating equilibria with advertising.

When $C_H > C_L$, $P_H^H > P_H^L$ separation will involve raising the price above P_H^H, while if $C_H < C_L$ the reverse is true. (Recall that P_H^L is the maximizer of $\Pi(P, L, H)$, the profit function for a low-quality firm believed to be of higher quality.) This can be seen by considering the following inequality:

$$R + C_H/H - 2P/H > R + C_L/H - 2P/H$$

$$\frac{\partial \Pi}{\partial P}(P,H,H) > \frac{\partial \Pi}{\partial P}(P,L,H)$$

If $C_H > C_L$, then the left-hand side is always greater. At P_H^H the left hand side is zero and so the right-hand side is negative. At any price above P_H^H both sides are negative, but the left side is less negative. Hence a dollar increase in price above P_H^H results in a smaller loss in profits for the high-quality firm than for the low-quality firm. In other words, a dollar's worth of separation is achieved at a fraction of a dollar. The alternative for H is to spend one dollar on advertising for each dollar of separation desired. Obviously, this option is more costly and hence left unemployed. When there exists a separating equilibrium where neither type advertises, we'll refer to this as a *price-separating equilibrium* .

We are now in a position to solve for the unique price-separating equilibrium which is immune from the sequential elimination of dominated strategies. Setting $A = 0$ in proposition (1) leads us to the following lemma:

lemma 2.2. $\Pi(P, H, H) \geq \Pi(P_H^L, L, H)$ **if and only if**

(3) $P_H^H - (1/2)\sqrt{G(R^2HL - C_H^2} < P < P_H^H + (1/2)\sqrt{G(R^2HL - C_H^2}$

$\Pi(P_L^L, L, L) \geq \Pi(P, H, L)$ **if and only if**

(4) $P \geq P_H^L + (1/2)\sqrt{G(R^2HL - C_L^2}$ or (5) $P \leq P_H^L - (1/2)\sqrt{G(R^2HL - C_L^2)}$

where $G = \frac{H(H-L)}{HL}$ and $P_H^H = \frac{RH+C_H}{2}$ and $P_L^L = \frac{RL+C_L}{2}$.

Proof: see the mathematical appendix.

The first inequality says that when P is relatively close to P_H^H, then type H is willing to signal his quality. The second inequality says that when P is too far from P_L^L, then type L prefers his complete-information profits to $\Pi(P, H, L)$ and, hence, won't overturn the equilibrium.

lemma 2.3. In any price separating equilibrium the unique price strategies are given by:

$$P_H = (1/2)\left((RH + C_L) \pm \sqrt{G(R^2HL - C_L^2)}\right).$$

$$P_L = (1/2)(RL + C_L) = P_L^L \qquad \Pi(P_H, L, H) = \Pi(P_L^L, L, L)$$

Proof: From the previous two lemmas we know that the equilibrium price must simultaneously solve inequalities (3) and (4) or (3) and (5). (3) and (5) will be satisfied whenever :

$$(RH + C_H) - \sqrt{G(R^2HL - C_H^2)} < (RH + C_L) - \sqrt{G(R^2HL - C_L^2)}$$

$$(C_H - C_L) < \sqrt{G}\left(\sqrt{R^2HL - C_H^2} - \sqrt{R^2HL - C_L^2}\right)$$

This last line will be true for R sufficiently large and $C_H < C_L$. Alternatively, (3) and (4) will be satisfied if:

$$C_H - C_L > \sqrt{G}(\sqrt{R^2HL - C_L^2} - \sqrt{R^2HL - C_H^2})$$

This last line will be true for R sufficiently large and $C_H > C_L$ and signaling would be done by type H setting a price above P_H^H. We also note that:

$$\frac{\partial \Pi(P,H,H)}{\partial P} = R - 2P/H + C_H/H < 0 \iff P > \frac{RH + C_H}{H} = P_H^H \quad .$$

Hence, since profits are decreasing above P_H^H, the high-quality firm picks the lowest price above P_H^H which satisfies (4). This is P_H. QED.

In summary, a one-period game involves no advertising. When the market is large enough there exists a unique separating equilibrium where the high-quality type sets its price at P_H, just high or low enough to deter mimicry by the low-quality firm. The low type, therefore, charges its complete information price P_L^L. In equilibrium consumers will always know the quality of the product . As is typical of separating equilibria, the strategies M(H) and M(L) are invariant with respect to ρ, the prior probability of facing a high-quality firm. At the end of the next section I argue that this equilibrium may not be plausible when ρ is very close to one.

3. A Two-Period Game

In this section we consider the standard Milgrom and Roberts [1986] model, but with a simplified two-period profit function. Following Milgrom and Roberts we abstract from a complete strategic formulation and consider a reduced-form profit function which represents total profits as a function of the initial price and advertising levels. Implicitly we are assuming that the initial actions of the monopolist induce a unique pattern of play (and hence unique profits) for the rest of the game (see Milgrom and Roberts [1986]). The easiest way to reduce the model is to restrict the strategy space of the monopolist. In particular, the monopolist will not be permitted to alter its price in period two. A similar restriction is placed on advertising. We can either imagine that all the advertising takes place in period one, or that it is equally divided between the two-periods. In both cases we consider only the aggregate level. Under these assumptions we can express the profit function (net of advertising) as:

$$\Pi(P,Q,B) = (1+Q)(R-(P/B))(P-C_Q) \quad . \tag{3.0}$$

The additional term simply reflects the fact that Q% of the consumers would be satisfied after their initial purchase and would continue to purchase at the same price. A full strategic model would also incorporate the possibility that the monopolist could raise its price in period two since a satisfactory product is worth more to the consumer. Hence, although the model says very little about the optimal second period strategy the conclusions for the initial period are likely to be at least qualitatively correct. This assumption is therefore consistent with our basic scheme of abstracting from a detailed description of repeat business. The simplified expression could also be justified by nontrivial transaction costs (i.e. changing prices every period might be expensive). In any case, it enables us to avoid a consideration of various "kinks" in the profit function. Since, however, these kinks would only emerge for certain parameter values, even if one is uncomfortable with restrictions on the strategy space, the results are at least "conditionally" true. This approach contrasts with Milgrom and Roberts where restrictions are incorporated into the consumer's strategy space. They assume that customers who do not purchase the good in period one can never purchase the good in future periods.

We are now ready to solve for the unique separating equilibrium which is immune to the elimination of dominated strategies. First we note that because we have done nothing but multiply the profit functions by (1+H) and (1+L), the results from lemmas 2.2 and 2.3 are still relevant. When R is large there will exist a price-separating equilibrium. Hence there is at least one price-separating equilibrium which will satisfy Proposition 1. We can show, however, that with the elimination of dominated strategies, it is cheaper for H to separate with a mixture of both price and advertising signals by an application of Proposition 2. We find that if $\partial\Pi/\partial P(P^T, H, H) = \partial\Pi/\partial P(P^T, L, H)$ then:

$$P^T = \frac{(H^2 - HL)R + (1+H)C_H - C_L(1+L)}{2(H-L)}$$

$$P^T = \frac{RH}{2} + \frac{C_H H - C_L L + (C_H - C_L)}{2(H-L)} \quad .$$

Also recall that:

$$P_H^H = \frac{RH}{2} + \frac{C_H H - C_H L}{2(H-L)}$$

$$P^T \begin{matrix} < \\ > \end{matrix} \frac{RH + C_L}{2} \begin{matrix} + \\ - \end{matrix} 1/2\sqrt{\frac{(H^2 - HL)R^2 - C_L^2 H(H-L)}{HL}} \quad .$$

Here P^T refers to the price where the two profit functions have the same slope (i.e. the isoprofit curves in price-advertising space are tangent). We can see that P^T is greater than or less than P_H^H as C_H is greater or less than C_L. This means that the "direction" of price signaling is consistent with the results form the previous section.

The last inequality will be true for large R. The right hand side is the highest price ($C_L < C_H$) that a low-quality firm would be willing to charge if so doing would convince consumers that it were a high-quality type (see lemma 2.2). This means that the optimal price separation is inadequate to achieve the required separation and the high-quality firm must advertise at $A^T = \Pi(P^T, L, H)$ $-\Pi(P_L^L, L, L)$. Clearly (P^T, A^T) achieves greater profits for the high-quality type than price-separating at $(P_H, 0)$, since a dollars worth of separation costs more than a dollar for all price increases beyond P^T. This then establishes (P^T, A^T) as the relevant separating sequential equilibrium for the Milgrom and Roberts game with the profit function given in (3.0). We now summarize the results in a lemma.

lemma 3.0. In the two-period Milgrom and Roberts' game with the profit function given in (3.0) the separating equilibrium strategy for each type is given by:

$$M(H) = (P^T, A^T) \quad \text{where} \quad A^T = \Pi(P^T, L, H) - \Pi(P_L^L, L, L)$$

$$M(L) = (P_L^L, 0) \quad \text{where} \quad P_L^L = (1/2)(RL + C_L)$$

Although we have considered only separating equilibria, it can be argued that when the prior belief by consumers of facing a high-quality firm "ρ" is sufficiently high, then a pooling equilibrium might be more reasonable. In particular, when ρ is near one, the separating equilibrium is sustained by the belief that defections are more likely to come from a low type. Since both types could potentially benefit from such a defection, its not obvious that this belief is plausible.

For example, we can show that when ρ is near one both types prefer a pooling equilibrium to the separating equilibrium at $[M(H) = (P^T, A^T), M(L) = (P_L^L, 0)]$ Let $\gamma = \rho H + (1-\rho)L$. Then as $\rho \to 1$: $\gamma \to H$, $P_\gamma^H = 1/2(R\gamma + C_H) \uparrow$ $1/2(RH + C_H) = P_H^H$. Also by the continuity of the profit function:

$$(3.0) \qquad \Pi(P_\gamma^H, H, \gamma) \longrightarrow \Pi(P_H^H, H, H) > \Pi(P^T, H, H) - A^T \quad \text{as} \quad \rho \to 1$$

Also, noting that natural separation will not occur for large R and that $|P^H_\gamma - P^L_L| < |P^H_H - P^L_L|$ indicates that $\Pi(P^H_\gamma, L, \gamma) > \Pi(P^L_L, L, L)$ so the low-quality firm would also prefer the profits from pooling at this equilibrium to his complete-information profits.

This line of reasoning, however, will not get us to a unique price pooling equilibrium. In particular, when ρ is near one, there will exist a continuum of price pooling equilibria which type H prefers to $M(H) = (P^T, A^T)$. A similar set exists for type L. The intersection of these two sets is nonempty since P^H_γ is in both sets. It can be argued that a price pooling equilibrium, with the price from this intersection is more plausible since both types would prefer this to the separating equilibrium. In particular, both types would prefer to pool at any price in the interval $[P^L_\gamma, P^H_\gamma]$. If not already there, both types could benefit by moving to a price inside this set, and hence, anything outside this set is suspect. Within this set L prefer P^L_γ while type H prefers P^H_γ. This means no further improvements are possible. [2]

One final remark seems in order. When equation (3.0) has the inequality going in the reverse direction, then the high type prefers separating with the strategy $M(H) = (P^T, A^T)$ to the best possible pooling equilibria (i.e., $M(Q) = (P^H_\gamma, 0)$). This implies that no pooling equilibria can exist unless equation (3) is satisfied. We can therefore conclude that except for the unique value of ρ where (3) holds with equality, either pooling equilibria **or** separating equilibria, are plausible, but not both.

[2] The reader should not be mislead into believing that all $P \in [P^L_\gamma, P^H_\gamma]$ are pooling equilibria. When R is small, the set of mutual improvements over the separating equilibrium will in general be smaller than this. Anything which is not an improvement over the separating equilibrium can not be sustained as a pooling equilibrium.

4. Stochastic Advertising

In this section we view advertising as a stochastic process. We will no longer assume that consumers see either all or none of the firm's advertising. This assumption is somewhat unreasonable in light of the fact that much advertising takes place over various electronic media to which not everyone is "tuned in." A more reasonable model would encompass the possibility that consumers might see less advertising than the monopolist has purchased.

The main result of this section is that the results of Milgrom and Roberts can not be extended in a natural way to encompass signal loss. In particular, even as the probability of a significant signal loss converges to zero, the set of potential equilibria does not include equilibria which are *close* to those found by Milgrom and Roberts. The only possible pure strategy equilibria involve both types setting the same price and both types advertising. Hence, *only advertising will be used to signal quality.* This differs from Milgrom and Roberts where both **price** and **advertising** are used to signal quality.

In this section I will assume that all consumers experience a proportional signal loss. In particular we multiply the two-period profit function $\Pi(P, Q, B)$ by a signal loss term $g(t)$, and integrate over all possible signal losses. The model has two interpretations. The function $g(t)$ can either be viewed as the likelihood all consumers will experience a signal loss of t, or can be viewed as a measure of how many consumers would experience a signal loss of $t \in [0, 1]$. In the first case all consumers will see the same signal loss and $g(t)$ is a probability density function. Integration over t yields an expected profit function. In the second case there is a continuum of signal losses experienced by different consumers and profits are deterministic. I will assume that $g(t)$ has positive density on $(0, 1)$. This will enable me to abstract from the possibility that a high level of advertising can overcome the largest possible signal loss and guarantee separation for the high type.

Consumer expectations of quality will, in both cases, be the result of Bayesian updating, consistent with the strategies of the high and low types. For example, if both types charged the same price, and the observed level of advertising was $A < A_L, A_H$, then Bayes rule would be used to compute an expectation of quality; while if the signal loss was smaller we might have $A_L < A < A_H$, in which case the consumer knows he/she is facing a high-quality firm. The next proposition tells us that in any equilibrium where $A_Q > 0$ for $Q = H, L$ the price will not be used to infer product quality.

lemma 4.0. Let $M(H) = (P_H, A_H)$ and $M(L) = (P_L, A_L)$ be equilibrium strategies in the stochastic signaling game with $A_H \neq 0$ or $A_L \neq 0$. Then $P_H = P_L$.

Proof: Let t be the type with nonzero advertising. Suppose that $P_t \neq P_s$. If consumer's belief are consistent with Bayes' rule then $B(P_t, A) = t$ for all $A \in [0, A_t]$, since this observation is consistent with t's strategy but not with s's. In particular, this observation could be rationalized by a signal loss of $t = (A_t - A)/A_t$ since then $(1-t)A_t = A$. Given these beliefs however, $\epsilon < A_t$ will induce the same consumer beliefs but increase profits by $(A_t - \epsilon)$. This proves that A_t could not have been optimal. The basic idea is that if all the information is contained in the price, advertising is an unnecessary expense.

With this behind us, the (expected) profit function, net of advertising costs, when the consumer believes the firms strategy is $[M(H) = (P, A_H),\ M(L) = (P, A_L)]$, would be given by:

$$(5.0)\qquad E\Pi(P, Q, EQ(A_H, A_L, A)) = \int_0^1 g(t)\Pi(P, Q, EQ(A_H, A_L, (1-t)A))$$

$$EQ(A_H, A_L, A) = \frac{\rho H\ g(\frac{A_H - A}{A_H}) + (1-\rho)L\ g(\frac{A_L - A}{A_L})}{\rho\ g(\frac{A_H - A}{A_H}) + (1-\rho)\ g(\frac{A_L - A}{A_L})} \quad \text{for} \quad A \leq A_L, A_H$$

$$EQ(A_H, A_L, A) = H \quad \text{if} \quad A \in [A_L, A_H] \quad EQ(A_H, A_L, A) = L \quad \text{if} \quad A \in [A_H, A_L] \quad .$$

Hence, we just multiply the probability of a signal loss, t, by the profits that would result from the corresponding expectations and integrate over all possible signal losses. This formulation assumes that an individual's signal loss is uncorrelated with his address. The expected quality encorporates Bayes' rule in the relevant region. Intuitively, Bayes' rule compares the relative likelihood of the two possible signal losses which could explain the observation A (i.e., $(A_H - A)/A_H$ if the original signal came from the high type and $(A_L - A)/A_L$ if the monopolist were really low-quality; note that $(1-t)A_H = (1-(A_H - A)/A_H)A_H = A$). Using this formula for profits we can define an equilibrium in a manner analogous to Section 1. In equilibrium, the expected profit function can be broken up into two integrals. The first integral where signal loss is small would encompass the situation where the second expected quality formula is in effect. While the second integrand would encompass the higher signal loss region where Bayes' Rule is in effect. We can now show that if there exists an advertising equilibrium, then both types of firms must advertise and at different levels.

lemma 4.1. Let $M(H) = (P, A_H)$ and $M(L) = (P, A_L)$ be an equilibrium with advertising, then $A_H > 0$, $A_L > 0$ and $A_L \neq A_H$.

Proof: Suppose that $A_t > 0$ while $A_s = 0$. Then once again, if beliefs are formed rationally we must have: $B(P, A) = t$ for all $A \in [0, A_t]$. This implies as before that $M(H) = (P, \epsilon)$ with $\epsilon < A_t$ will cause consumers to have the same beliefs but increase profits by $(A_t - \epsilon)$. Hence A_t could not have been optimal given consumer beliefs.

If $A_H = A_L$, then consumer beliefs must in equilibrium be $B(P, A) = \rho H + (1-\rho)L$ for all $A \in [0, A_H]$, where ρ is the prior probability of facing a high-quality firm. This implies that $M(H) = (P, \epsilon)$ with $\epsilon < A_H$ will again increase profits by $(A_H - \epsilon)$ without affecting consumer beliefs. Hence $A_L = A_H$ cannot be optimal for either type. The basic idea behind these results is that because of the "noise", the monopolist can defect from his strategy without being observed. In some sense, the presence of the signal loss limits the number of "out-of-equilibrium" beliefs. This prevents us from assigning an equilibrium to the game by defining the "out-of-equilibrium" beliefs in such a way as to discourage defections.

Although neither result indicates whether in equilibrium $A_H < A_L$ or $A_H > A_L$, it can be argued that the later is more likely. The former case implies that $EQ(A_H, A_L, A) = L$ for $A \in [A_H, A_L]$. Since expected profits are increasing in expected quality this would seem to provide an incentive for type L to deviate down to at least (P, A_H). The other incentive would be the reduced cost from reduced advertising. Whether or not the incentives to defect and overturn the equilibrium are strong enough will depend on the exact curvature properties of $g(t)$. This is because the gains to such a defection may be offset by losses which result from altering the "profit distribution" on the observed range where $A \in [0, A_H]$. We can show that when repeat purchases make the high-quality firm more profitable, then $A_H > A_L$ is the only possibility.

Theorem 4.0. Suppose $(1+H/1+L)(P-C_H/P-C_L) > 1$; then in any sequential equilibrium $[M(H) = (P, A_H),\ M(L) = (P, A_L)]$ we must have $A_H > A_L$.

Proof: In order for A_H, A_L to be optimal for types H and L respectively, (given that the consumers believe the firms strategy is [M(H),M(L)] as above) we must have:

$$A_H \in \arg \max_{A \in [A_H, A_L]} \left[E\Pi(P, H, EQ(AH, AL, A)) - A \right] \tag{5.1}$$

$$A_L \in \arg \max_{A \in [A_H, A_L]} \left[E\Pi(P, L, EQ(AH, AL, A)) - A \right] \tag{5.2}$$

If this were not the case then it would be optimal for one or both types to defect from A_Q to some other $A \in [A_H, A_L]$. By factoring out the (1+Q) and $(P - C_H)$ terms which are not functions of t, we can see that:

$$E\Pi(P, H, EQ(A_H, A_L, A)) = \left(\frac{1+H}{1+L}\right)\left(\frac{P-C_H}{P-C_L}\right) E\Pi(P, L, EQ(A_H, A_L, A)) \quad .$$

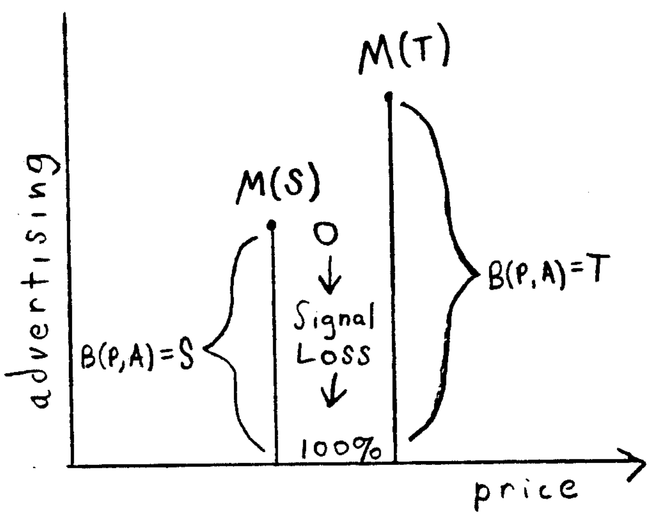
Since $\partial E/\partial A(P,Q,EQ(A_H,A_L,A)) > 0$, when $(1+H/1+L)(P-C_H/P-C_L) > 1$ we must have (5.1)>(5.2) (i.e. $A_H > A_L$). In other words, when we increase the "positive" term in (5.2) we must increase the argmax of the entire expression. We should also note that in the limit as $R \to \infty$ (everything else constant):

$$\lim_{R\to\infty} P_B^Q = \frac{RB + C_Q}{2} \longrightarrow \infty$$

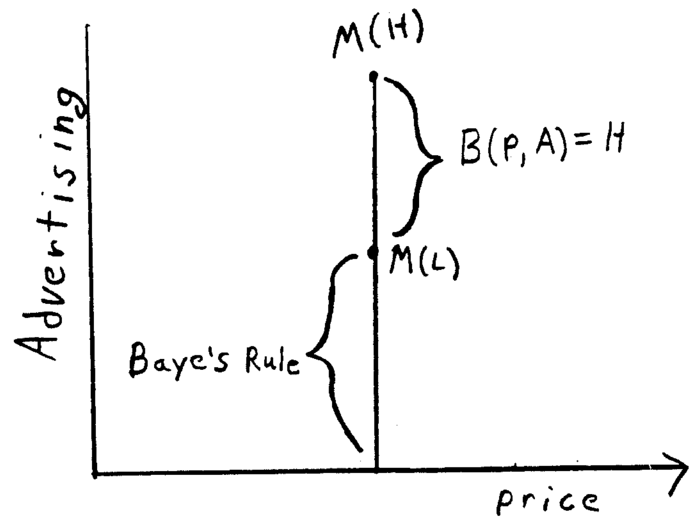
If we believe that the equilibrium strategies will be weakly efficient in the sense of Section 3, then the equilibrium price must also approach infinity. This is because P_B^Q, the optimal price for any quality type, under any belief will approach infinity. Hence, both types will want the equilibrium price to rise with R. This then implies that $(P-C_H/P-C_L) \to 1$ as $R \to \infty$, or that $(1+H/1+L)\,(P-C_H/P-C_L) > 1$ for R sufficiently large. Obviously if $C_H \leq C_L$, the inequality is always true. We summarize these remarks in a corollary.

Corollary 4.0. In any stochastic advertising game with repeat purchases and R sufficiently large or $C_H \leq C_L$, there will be no sequential equilibria with $A_H \leq A_L$.

We can also demonstrate these results graphically. In Figure 1 the impossibility of price separation is demonstrated. We can represent the strategies, M(H) and M(L) as points in the price-advertising plane. Any points directly below the strategy M(S) are possible observations which are consistent with the strategy M(S). For example M(S) would be an observation rationalized by zero signal loss, while the projection of M(S) onto the price axis would be rationalized by 100 percent signal loss. However, none of the points below M(S) could be rationalized by the strategy M(T), if M(T) involves a different price. A rational consumer will, therefore, view any observation directly below the strategy M(S) as an indiction that he is facing a monopolist of type S. This, however, would give the monopolist an incentive to deviate from M(S) to a cheaper strategy which involves less advertising. One might then speculate that the only possible equilibria involve those strategies with zero advertising and which lie on the price axis. When the game is *sufficiently quiet* we



The impossibility of advertising with price separation



Potential Price Pooling with Advertising in Equilibrium

The impossibility of price separation

can rule out such price separating equilibria by extending the arguments from the previous section. The only remaining equilibria, shown in Figure 2, involve both types picking the same price but advertising at different levels. When the market R is sufficiently large, the high-quality firm will advertise at the higher level. When the consumer observes a sufficiently high advertising signal he or she can correctly attribute this to the strategy of the high-quality monopolist. When the observed advertising is somewhat less, the consumer must have information about the signal loss function so that Baye's rule can be applied.

All of the previous results characterize a separating equilibrium under the assumption that advertising exists. It would therefore be useful to know how likely it is that advertising will occur in equilibrium. First we recall from Section 3 that if ρ (the prior belief of facing a high quality firm) is not too high, pooling equilibria are not possible. Hence we need only consider price separating equilibria. This leads us naturally to the next result.

Theorem 4.1. **Let t_n be a sequence of random variables with probability density functions $g_n(t)$ such that $g_n(t) > 0$ for all $t \in [0,1]$. Furthermore let $t_n \longrightarrow 0$ in probability, then for t_n sufficiently close to zero (in probability, i.e. n large) there does not exist a price separating equilibria in the stochastic signaling game.**

Proof: From lemma 2.3 we know that the optimal price separating strategy for the high quality firm would be $M(H) = (P_H, 0)$ (this achieves the required separation at minimum cost). [3] Recall that $B(P, A) = H$ if $A \geq \Pi(P, L, H) - \Pi(P_L^L, L, L)$ by the elimination of dominated strategies. We will now consider the strategy $M(H) = (P^T, A^T + \epsilon)$ where (P^T, A^T) is the equilibrium strategy of the high type in the quiet game with repeat business. Recall that $B(P^T, A^T) = H$ (see page 19). Also recall that by the definition of convergence in probability we have that, for every $\epsilon > 0$, $\lim_{n\to\infty} Pr(|t_n - 0| < \epsilon) = 1$. Hence, we can easily see that:

$$E\Pi(P^T, H, B(P, (1 - t_n)(A^T + \epsilon)) - (A^T + \epsilon) \longrightarrow \Pi(P^T, H, H) - (A^T + \epsilon)$$

$$\text{Since} \quad B(P, (1 - t_n)(A^T + \epsilon)) \longrightarrow B(P, A^T + \epsilon) = H \quad \text{in probability.}$$

$$\text{Because} \quad \Pi(P^T, H, H) - (A^T + \epsilon) \geq \Pi(P_H, H, H),$$

$M(H) = (P^T, A^T + \epsilon)$ is prefferred by type H to $(P_H, 0)$ for ϵ sufficiently small (see page 19). This overturns $(P_H, 0)$ as a potential equilibrium. The basic idea is that when the game is not "too noisy" the same factors which overturn a price separation in the quiet game overturn it here as well. This is because as the signal loss converges to zero, the expected profits in the noisy game converge in probability to the deterministic profits in the quiet game. This demonstrates that advertising must occur in equilibrium when the game is sufficiently quiet (i.e. t is close to 0 in probability).

Let $[M_t(H), M_t(L)]$ denote the set of pure strategy equilibria in the game with signal loss t. Let $g(t)$, the pdf of t, be such that $g(t) > 0$ for all $t \in (0, 1)$. Let

[3] The (1+Q) repeat business term does no change the result from Section 2.

$[M_0(H), M_0(L)]$ denote the set of pure strategy equilibria in the quiet game with no signal loss. Then we have:

$$\lim_{t \overset{prob}{\rightarrow} 0} [M_t(H), M_t(L)] = [(P^*, A_H), (P^*, A_L)] \neq [M_0(H), M_0(L)] = [(P^T, A^T), (P_L^L, 0)]$$

Hence the Milgrom and Roberts' game has a discontinuity: no matter how unlikely significant signal loss is, the equilibria under the noisy regime do not converge to the equilibrium in the quiet game. [4]

5. Advertising As Information

In this last section we remove the assumption that advertising is a purely dissipative signal. The rationale is that because this is a model which encompasses the marketing of a new product, it's reasonable to expect that consumers would also find the advertising to be directly informative. In particular, advertising can now inform consumers about the new product's existence, and not just about its quality. The implication is that the level of advertising should have a direct affect on the monopolist's profit function.

In this section I make the simplifying assumption that consumers either view all the firm's advertising or none of it. This enables me to abstract from the possibility that some consumers view only enough advertising to be cognizant about the product's existence, but not enough to be sure of its quality. Under this assumption, informed consumers are "completely informed" consumers. One might imagine this assumption to be approximated if all the firms advertising took place during one television show, for example. Although a few consumers might only see part of the advertising, the vast majority who view the entire show would see it all. We can alternatively interpret this directly informative aspect as an imperfection in the signaling mechanism, since only **some** consumers see all the advertising.

[4] The other possibility is that $[M_t(H), M_t(L)]$ is empty, meaning that there are no (pure strategy) equilibria in the noisy game!

This differs from Milgrom and Roberts [1986] where **all** consumers are perfectly informed about advertising expenditures.

I assume that the percentage of consumers who view all the advertising is positively correlated to expenditures on advertising (at least within a given range). I view the probability of being informed by advertising as independent of one's marginal valuation of a satisfactory product (i.e., their address). Uninformed consumers will not purchase any of the good in question. In other words, only informed consumers participate in the signaling game. I define by $f(A)$ the fraction of consumers who will view all of the advertising. Under these assumptions the relevant profit function facing each type of monopolist is:

$$\Pi'(P,Q,B) = f(A)\Pi(P,Q,B) - A \quad , \tag{5.0}$$

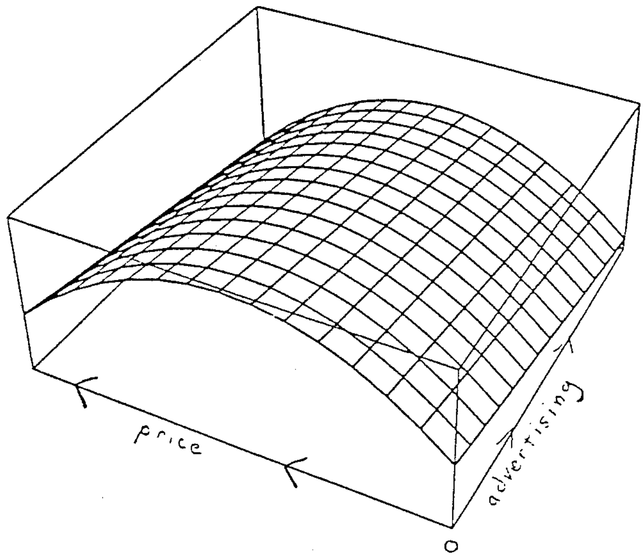
where $f'(A) \geq 0$ for small A and $f(0) = 0$, $f(A) \leq 1$. $\Pi(P,Q,B)$ is the profit function from the previous section. Hence the marginal value of advertising for a firm which sets price P and is believed to be of quality B is: $f'(A)\Pi(P,Q,B) - 1$. This implies that under complete-information, and $f \in C^2$, that the optimal level of advertising satisfies: $f'(A) = [\Pi(P_Q^Q, Q, Q)]^{-1}$.

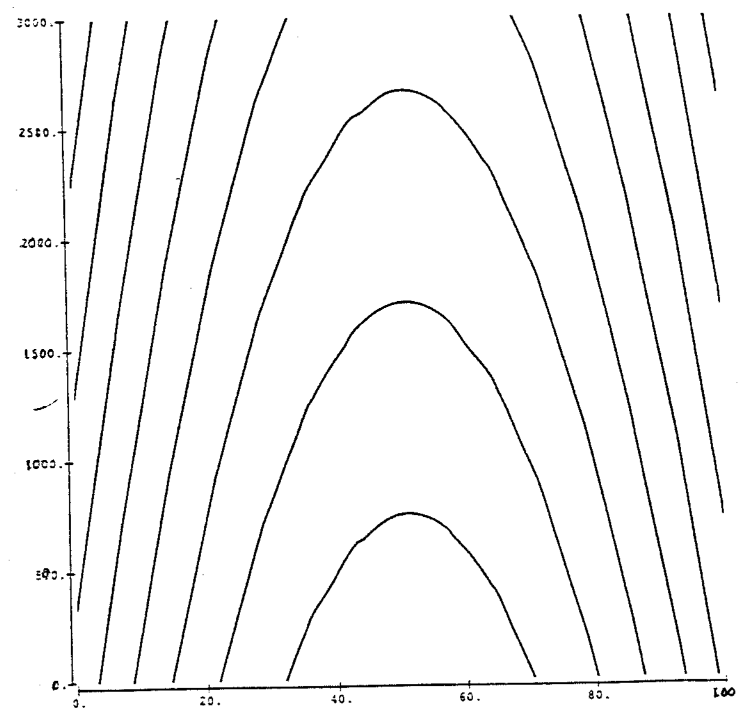
Note that under "fixed beliefs" the pricing and advertising decisions are made independently of one another and hence P_B^Q is still optimal. (To see this just differentiate the new profit function with respect to P and set equal to zero.)

Since there exists a "limited" number of agents to inform it's reasonable to assume that: $f'(A) \to 0$ as $A \to \infty$, $f'(A) \to \infty$ as $A \to 0$, $f(A) \uparrow 1$ as $A \to \infty$ and finally $f''(A) < 0$ (indicating that the some consumers are very difficult to reach).

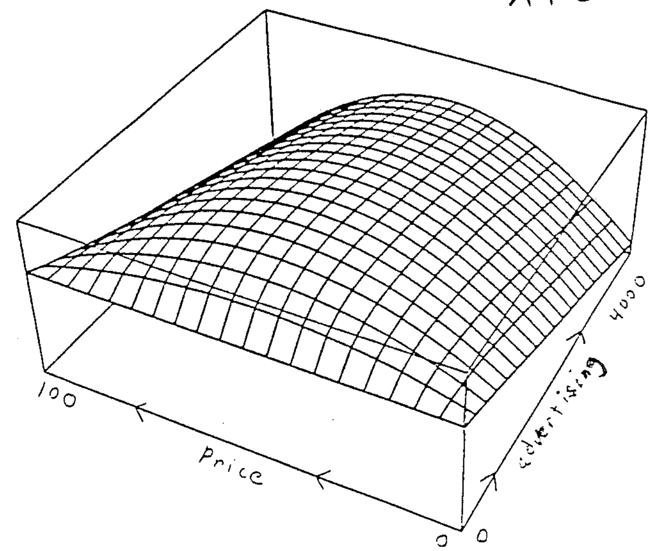
We can gain some insight into the model by comparing the new complete-information profit function (5.0) to the profit function (3.0) from Section 3. When advertising is purely a dissipative signal, the maximum profits will occur on the price axis where advertising is zero (see diagram-next page). As advertising increases, the profit surface slopes back in a linear fashion. This is because we are holding beliefs fixed and advertising is an unnecessary expenditure which serves only to reduce profits. The isoprofit curves have the standard concave shape.

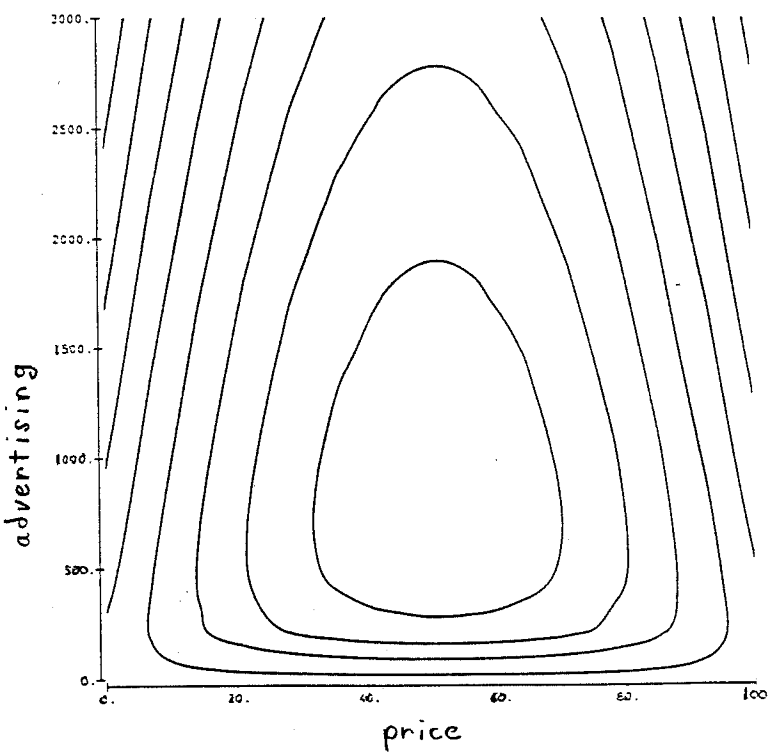
$f(A) = 1$

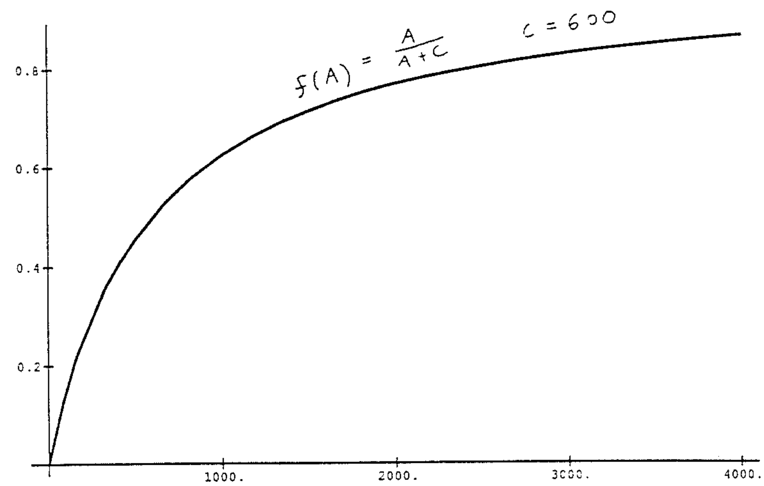




$f(A) = \frac{A}{A+c}$







A Reasonable Advertising Functions

When we multiply the profit function by $f(A)$ this changes. The surface in the upper-left-hand corner indicates profits as a function of both price and advertising. This surface dips down in the front when advertising is near zero because $f(A) \to 0$ as $A \to 0$. Hence, the complete-information profits are maximized at some positive level of advertising. As A increases beyond this level, advertising will increase costs more than revenues and is undesirable. Since $f(A) \to 1$ as A becomes large, when advertising is very high, the two profit functions are indistinguishable. By looking at the contour map of the profit function (lower-left-hand corner) it is clear that we no longer have isoprofit curves, we instead have isoprofit correspondences. In other words, for a given price, advertising may be less than optimal either because it is too high or because it is too low.

With this behind us we can now easily generalize Proposition 1 and Proposition 2 of Milgrom and Roberts.

Theorem 5.0. Suppose the following conditions hold:

(S1) $f(A_H^H)\Pi(P_H^H, L, H) - A_H^H > f(A_L^L)\Pi(P_L^L, L, L) - A_L^L$

(S2) There exists $(P_H, A_H) > 0$ such that:

$f(A_H)\Pi(P_H, H, H) - A_H > f(A_L^H)\Pi(P_L^H, H, L) - A_L^H$

$f(A_H)\Pi(P_H, L, H) - A_H \leq f(A_L^L)\Pi(P_L^L, L, L) - A_L^L$

Then $M(H) = (P_H, A_H)$ and $M(L) = (P_L^L, A_L^L)$ is a sequential equilibrium. Furthermore, if dominated strategies are eliminated from the game and f is continuously differentiable and $f'(A) > 0$ for all A, then $(P_H, A_H) \neq (P_H^H, A_H^H)$ satisfy:

$$(C1) \qquad f(A_H)\Pi(P_H, L, H) - A_H = f(A_L^L)\Pi(P_L^L, L, L) - A_L^L$$

$$(C2) \qquad \frac{\partial\Pi/\partial P(P_H, H, H)}{\partial\Pi/\partial P(P_H, L, H)} = \frac{f'(A_H)\Pi(P_H, H, H) - 1}{f'(A_H)\Pi(P_H, L, H) - 1}$$

where $f'(A_H^H) = [\Pi(P_H^H, H, H)]^{-1}$ $f'(A_B^Q) = [\Pi(P_B^Q, Q, B)]^{-1}$

Condition (S1) says that if the high quality firm were to play his complete information strategy then the low quality firm would have an incentive to mimic this strategy. Hence the complete information strategies could not be part of a separating equilibrium. This means that any separating equilibrium will involve signaling. Condition (S2) says that there exists a strategy $M(H) = (P_H, A_H)$ which is too costly for the low quality firm to mimic. In other words, even if type L could trick everyone into believing he was type H, he would still prefer the payoff from his complete information strategy. Condition (S2) also says says that type H prefers $M(H) = (P_H, A_H)$ to the best possible profits under a low quality perception. This implies that by setting the out-of-equilibrium beliefs to L, the equilibrium $[M(H) = (P_H, A_H), M(L) = (P_L^L, A_L^L)]$ can be sustained. The equilibrium beliefs are defined in the obvious manner: $B(P_H, A_H) = H$ and $B(P' \neq P, A' \neq A) = L$

If we eliminate dominated strategies from the game, then (P_H, A_H) will achieve the required separation at minimum cost and so:

$$(P_H, A_H) = \quad \text{ArgMax} \quad [f(A)\Pi(P, H, H) - A] \quad \text{subject to}$$

$$f(A_H)\Pi(P_H, L, H) - A_H \leq f(A_L^L)\Pi(P_L^L, L, L) - A_L^L$$

Since deviations from the complete information strategy are costly, the constraint will hold with equality, this gives us (C1). Setting up the appropriate Lagrangian and combining FOC conditions yields (C2).

Intuitively, (C2) describes the efficient signaling frontier, while (C1) describes the minimum amount of signaling required. The fact that $\partial\Pi/\partial P(P_H^H, H, H) = 0$ and $f'(A_H^H)\Pi(P_H^H, H, H) = 1$ implies that initially, the cost of signaling via either channel is zero. Hence, both signaling channels will always be employed and the first-order condition is relevant.

In theorem 5.0 the new first order condition (C2) encompasses new incentives or disincentives for advertising as compared to price signaling. The numerator on the right, for example, represents the net cost of a dollar spent on advertising. Whenever advertising increases profits, this will be less than one dollar. If the ratio on the right-hand side is less than one, then the high quality type has a relative advantage in advertising and can achieve a dollar's worth of separation for a fraction of a dollar. Whenever the right side is less than the left side, a dollar's worth of separation is also cheaper with advertising as opposed to price changes (and vice versa). More specifically, when $\Pi(P_H^H, H, H) > \Pi(P_H^H, L, H)$ and $f'(A) > 0$, the high-quality firm will have this advantage in advertising separation. Furthermore, since $f'(A_H^H) = [\Pi(P_H^H, H, H)]^{-1}$ the first dollar's worth of advertising separation is virtually free. This implies that part of the separation will always be done by advertising. This contrasts with the Milgrom and Roberts one-period model where only price signaling may exist.

Our Inada assumptions about $f(A)$ imply that as advertising increases, its relative advantage as a signaling tool must diminish. This indicates that at some point price signaling must also play a role. For example, consider the case where $C_H > C_L$ and $\Pi(P_H^H, H, H) > \Pi(P_H^H, L, H)$. The high-quality type starts out at (P_H^H, A_H^H) and wants to move along the signaling frontier the minimum distance necessary to separate. This is because signaling is costly as it represents movement away form the best possible profits: $f(A_H^H)\Pi(P_H^H, H, H) - A_H^H$. Initially the high type has an advantage in price markups. After it increases the price above P_H^H the left side of (C2) switches from zero to some positive number less than one. Since a price increase above P_H^H reduces $\Pi(P, H, H)$ less than it does $\Pi(P, L, H)$, type H still has an advertising advantage. The right-hand side of (C2) varies continuously between 0 and 1 as A varies between A_H^H and infinity (under Inada conditions given above). This implies that there exists a corresponding advertising level on the efficient signaling frontier which satisfies (C2). Movement along the frontier continues until (C1) is satisfied. Since $f(A) \rightarrow 1$ as $A \rightarrow \infty$ this will clearly be satisfied for large enough A. Hence, in this example, price and advertising are both correlated with quality. Although price and advertising decreases below (P_H^H, A_H^H) will also satisfy (C2), these points represent the most inefficient signaling possible. Speaking informally, (C2) does not distinguish between the cheapest and most costly signaling direction.

If, on the other hand, a reduced marginal cost more than compensates for a loss in repeat business we may instead have: $\Pi(P_H^H, L, H) > \Pi(P_H^H, H, H)$. In this case the high-quality firm has an advantage in "disadvertising."

As with price reductions from P_H^H when $C_H < C_L$ the first dollar is virtually free and an initial reduction in advertising is always desired. As advertising falls, however, the concavity assumption implies that f' increases. This increases the cost of additional cuts in advertising and will eventually cause price signaling to be more cost effective. This contrasts sharply with the Milgrom and Roberts model where only increases in advertising can be used to signal quality. That result hinged on the assumption that zero advertising was the complete information optimum. Since negative advertising has no meaning, " the only direction to go was up." By making positive advertising a necessity as is a positive price, we now open up the possibility for downward signaling. While the relationship between C_H and C_L determines the direction of price signaling, the relationship between $(1+H)(P_H^H - C_H)$ and $(1+L)(P_H^H - C_L)$ determines the direction of advertising. When the former is smaller, the high-quality firm will choose to **reduce** advertising. When $\Pi(P_H^H, L, H) > \Pi P_H^H, H, H)$, the low-quality firm is in some sense more profitable. Hence a dollar cut in advertising reduces profits for type L by more than for type H.

Notice that $(1+H)$ is the reflection of repeat business in out model. It could therefore be argued that in a multiperiod model where repeat business is more important, downward advertising is unlikely. On the other hand, in a market without repeat business, downward advertising would be the norm, provided that marginal cost was proportional to quality. It must be pointed out that although $\Pi(P_H^H, L, H) > \Pi(P_H^H, H, H)$ implies signaling by "disadvertising", it need not imply that $A_H < A_L^L$ but only that $A_H < A_H^H$. Hence advertising and quality may still be positively correlated. The next theorem demonstrates the existence of a separating equilibrium with a negative correlation.

Theorem 5.1. If $C_H > C_L$ then there exists an R and a corresponding separating equilibrium where advertising and quality are inversely correlated.

Proof: From lemma 2.1 we know that for small R, $\Pi(P_L^L, L, L) > \Pi(P_H^H, L, H)$. For larger R, however, $\Pi(P_L^L, L, L) < \Pi(P_H^H, L, H)$. Also $\Pi(P_H^H, H, H)$ is always less than $\Pi(P_H^H, L, H)$ provided that $C_H > C_L$. Since $\Pi(P, Q, B)$ is continuously increasing with respect to R, this implies that there exists an R such that:

$$\Pi(P_H^H, H, H) < \Pi(P_L^L, L, L) < \Pi(P_H^H, L, H)$$

This last line indicates that natural separation will not occur and that advertising separation will be in a downward direction. Since $A_H^H < A_L^L$ clearly $A_H < A_L^L$. Hence, quality and advertising are inversely correlated in a signaling equilibrium.

This result hinges on the small size of the market and on $C_H > C_L$. As the market R grows larger $(P_H^H - C_H)/(P_H^H - C_L) \to 1$. This implies that for large enough R, $(1 + H) > (1 + L)$ will force $\Pi(P_H^H, H, H) > \Pi(P_H^H, L, H)$. When this happens, the high-quality firm will signal its quality via increased advertising. In other words, the large market size eventually mitigates any marginal cost advantage of the low-quality firm. This coupled with more repeat purchases will cause the high-quality firm to be more profitable and so advertising will be positively correlated with quality. When $C_H < C_L$ the high-quality firm has both a *marginal cost advantage* and also a *repeat business advantage* and so is unambiguously better off than the low type. Hence, regardless of market size, any advertising signals will be in the "upward direction."

It is also possible for advertising to be negatively correlated with quality in a natural separation. For example, when C_H is sufficiently higher than C_L, P_H^H will be sufficiently high that: $\Pi(P_H^H, L, H) < \Pi(P_L^L, L, L)$. In this situation, the low-quality firm has no incentive to mimic the high-quality type. The low type has greater profits from the increases business which results form a lower price. The end result is that both types naturally separate. When this happens:

$$f'(A_H^H) = [\Pi(P_H^H, H, H)]^{-1} \quad \text{and} \quad f'(A_L^L) = [\Pi(P_L^L, L, L)]^{-1}$$

$f' > 0$ and $f'' < 0$ implies that $A_L^L > A_H^H$. Hence we have an example with the inverse correlation but without the signaling. Simply put, the greater profit potential of the low-quality type encourages it to advertise more. Notice that in the limit as $R \to \infty$:

$$\lim_{R\uparrow\infty} \Pi(P_L^L, L, L) \approx R^2L/4 \;<\; (H/L)R^2L/4 \approx \lim_{R\uparrow\infty} \Pi(P_H^H, L, H)$$

This indicates that for large markets, natural separation with advertising inversely correlated to quality is impossible.

To summarize, if $C_L < C_H$ the low quality firm will be more profitable when R, the market size is sufficiently small. When its cost advantage is very significant both types will naturally separate at their complete information strategies with $A_L^L > A_H^H$. As R increases, the repeat business "disadvantage" begins to counteract the marginal cost advantage. When this advantage is less pronounced, type L may still have an incentive to mimic type H, provided that:

$$\Pi(P_H^H, H, H) < \Pi(P_L^L, L, L) < \Pi(P_H^H, L, H) \quad .$$

To deter this, the high-quality type would reduce advertising below A_H^H and increase it price above P_H^H. Hence $A_H < A_H^H < A_L^L$ in equilibrium. As R increases still further, eventually:

$$\Pi(P_H^H, L, H) > \Pi(P_H^H, H, H) > \Pi(P_L^L, L, L) \quad .$$

In this situation type H will reduce advertising below A_H^H, but not necessarily below A_L^L. Finally as R increases even more $\Pi(P_H^H, H, H) > \Pi(P_H^H, L, H)$ and only advertising increases are used to signal quality. Recall that the model presented has only two periods. If the model were extended to many periods, this might better counterbalance any marginal cost advantage of type L, and thereby reduce the liklihood of an inverse correlation between advertising and quality. Notice that in none of these examples are consumers fooled by the inverse relationship between quality and advertising. The implication is that in equilibrium consumers understand market characteristics enough to infer what the relationship between the two should be.

If marginal cost in inversely related to quality, $C_H < C_L$, then the high quality firm is unambiguously better off. This created a greater incentive for type H to generate business through informative advertising. When this incentive is sufficiently high, the two firms naturally separate. If this incentive is somewhat less, then the high type must signal its quality by advertising above A_H^H and by pricing below P_H^H. I now summarize the preceding remarks in a theorem:

Theorem 5.2. **In an informative advertising game with repeat purchases and R sufficiently large or $C_H \leq C_L$ there will be no equilibria with $A_H \leq A_L$.**

We are now ready to consider an example which will illustrate the basic points of this section. We consider the model where $f(A) = A/(A+c)$. Equations (C1) and (C2) will then reduce to:

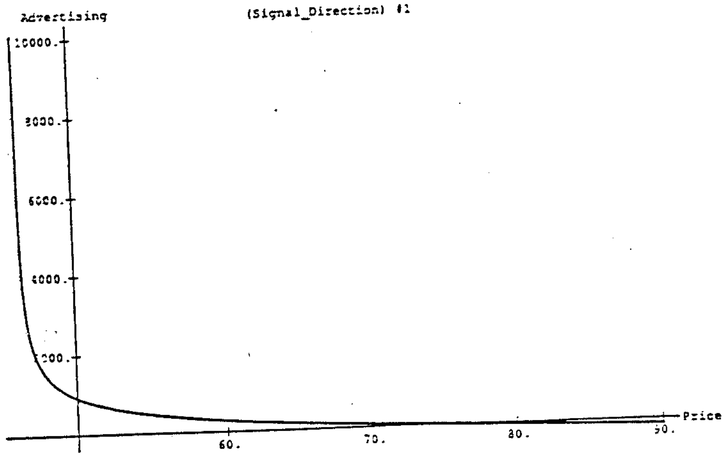
$$\frac{(1+H)(R-2P_H/H+C_H/H)}{(1+L)(R-2P_H/H+C_L)} = \frac{(A_H+c)^2 - c(1+H)(R-P_H/H)(P_H-C_H)}{(A_H+c)^2 - c(1+L)(R-P_H/H)(P_H-C_L)}$$

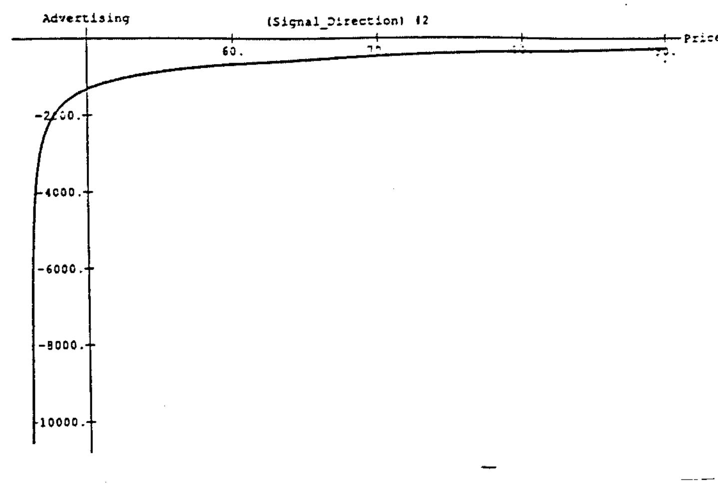
$$\frac{A_H}{A_H+c}(1+L)(R-P_H/H)(P-C_L)-A_H = \frac{A_L^L}{A_L^L+c}(1+L)(R-P_L^L)(P_L^L-C_L)-A_L^L$$

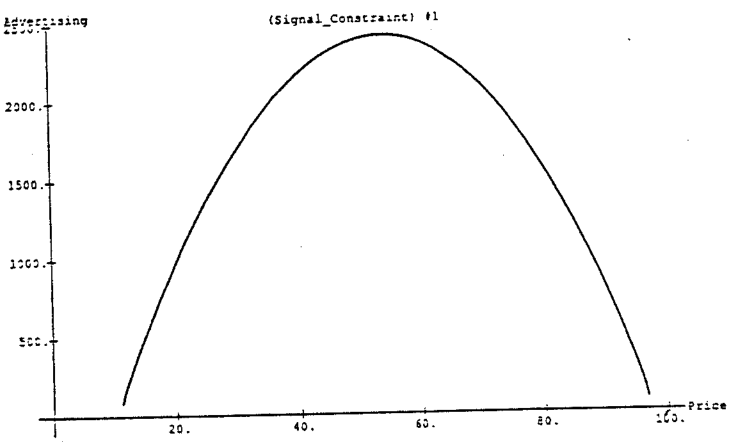
In order to be able to graph the solution we consider a model with specific parameters: R=100; c=200; H=1; L=0.25; $C_H = 2$; $C_L = 8$. We can solve (C1) and (C2) to derive advertising as a function of price. Since both equations are of degree two, we will get two solutions for each equation. On the top of the next page the two solutions to (C1) are labeled (Signal Direction #1) and (Signal Direction #2). The second solution yields only negative advertising and so can be discarded. The two solutions to (C2) are found in the middle of the page. The first one is concave and the second one is convex. These two curves correspond to the concave and convex regions of the isoprofit correspondence. If the high- quality firms picks a point above the curve (Signal Constraint #1) then it will be too costly for type L to mimic. Similarly, if type H picks a point below the curve (Signal Constraint #2) it will also be too costly to mimic. The first curve is what Milgrom and Roberts refer to as the A(p) curve. The convex curve, on the other hand, represents the new "disadvertising" possibilites.

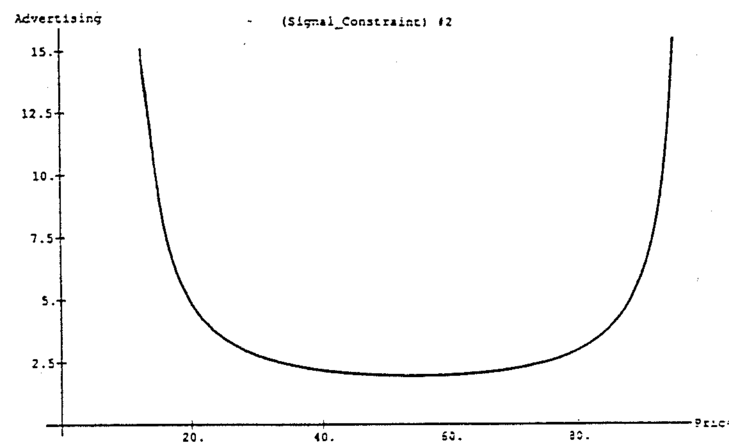
The intersections between the solutions to (C1) and (C2) represent potential equilibria. These are graphed below. In the first solution the high quality firms sets $M(H) = (P_H = 47,\ A_H = 2355)$. This yields a profit of \$2039. In the second solution $M(H) = (P_H = 75.5,\ A_H = 2.56)$. This yields a profit of \$ 43. Clearly the first solution must be the real equilibrium. This should be of no surprise since $C_H < C_L$, implying that the high-quality firm is unambiguously better off. The other solution would only make sense if $(1+H)(P_H^H - C_H) < (1+L)(P_H^H - C_L)$, indicating that the low quality firm was more profitable. In equilibrium the low quality firm will follow his complete-information strategy: $M(L) = (P_L^L = 16.5,\ A_L^L = 68.75)$.

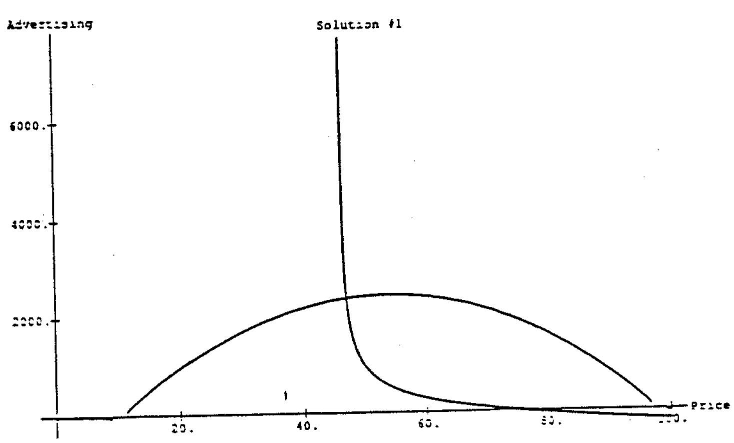
We can graph the two solutions to (C1) and (C2) as points of tangency between isoprofit curves. The equilibrium strategy occurs where type H's lowest isoprofit curve touches the A(p) curve of type L. If type H picks a strategy above the A(p) curve then he is spending more than necessary on advertising and not maximizing profits. If type H picks anything below the A(p) curve than his signal of quality is not credible since type L will be willing to mimic that strategy if it would convince consumers that he was a high quality firm. Profits for type H increase as we pick lower and lower isoprofit curves, since increasing advertising above A_H^H is costly. Hence, the equilibrium can only occur at a point of tangency. (See the Figure— Up Solution).

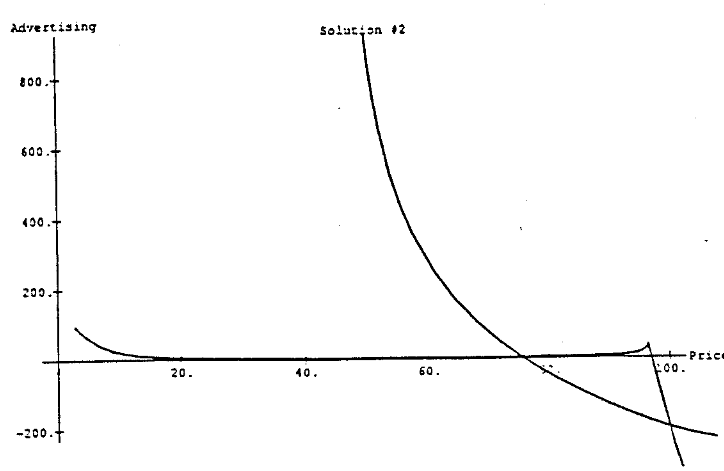

Advertising
(Signal_Direction) #1
10000.
2000.
60.
70.
80.
90.
Price

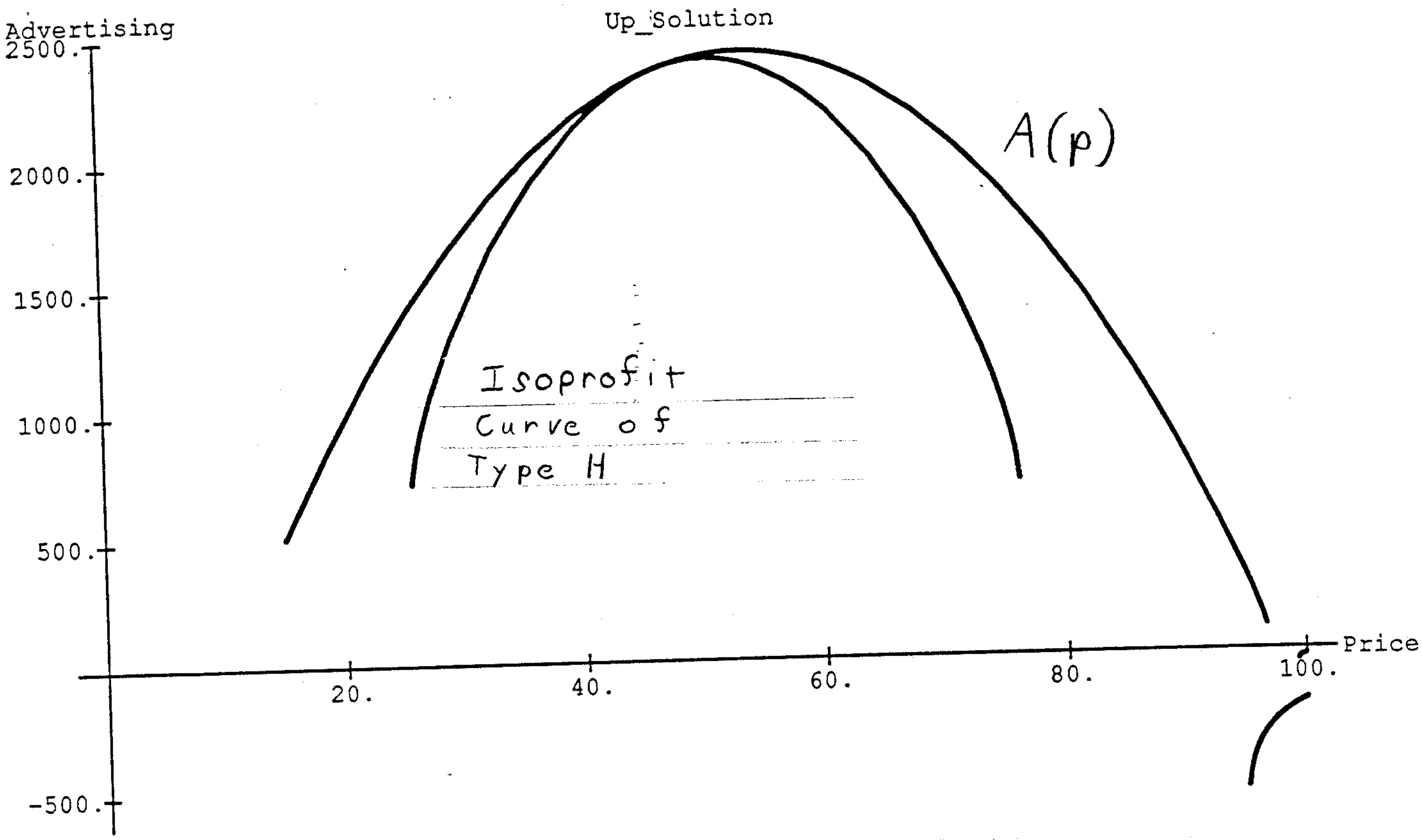

Advertising
(Signal_Direction) #2
60.
Price
-4000.
-6000.
-8000.
-10000.

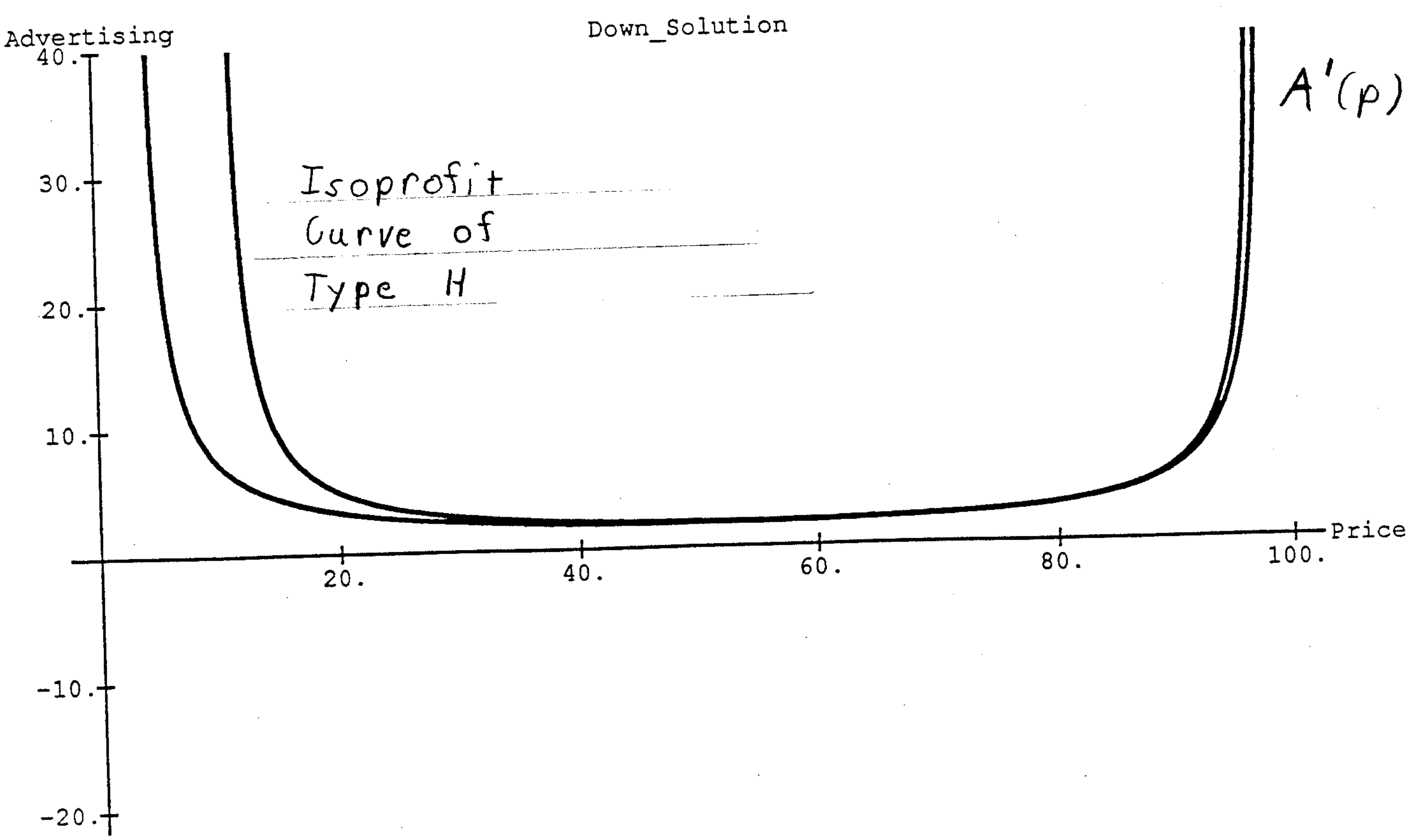

Advertising
(Signal_Constraint) #1
2000.
1500.
500.
20.
40.
60.
80.
Price


Advertising
(Signal_Constraint) #2
15.
12.5
10.
7.5
5.
2.5
20.
40.
60.


Advertising
Solution #1
6000.
20.
40.
60.
Price


Advertising
Solution #2
800.
600.
400.
200.
-200.
20.
40.
60.
Price


Up_Solution
Advertising
2500.
2000.
1500.
1000.
500.
-500.
20.
40.
60.
80.
100.
Price
A(p)
Isoprofit
Curve of
Type H


Down_Solution
Advertising
40.
30.
20.
10.
-10.
-20.
20.
40.
60.
80.
100.
Price
A'(p)
Isoprofit
Curve of
Type H

In the figure labeled (Down Solution) we are looking at strategies which involve less advertising than A_H^H. Here, type H would prefer to move in the upward direction, but to keep his signal credible he must keep his strategy on or below the $A'(p)$ curve. Hence, in equilibrium, type H picks the highest isoprofit curve which still touches the $A'(p)$ curve. The is obviously a point of tangency.

6. Conclusions

In conclusion, the paper has answered three fundamental questions about advertising: (1) When will price and quality be correlated? (2) When will advertising and quality be correlated? (3) Why might low-quality firms advertise? The main result of Section 4 is that price will not be correlated to quality if advertising is imperfectly observed. This differs from Milgrom and Roberts [1986] where advertising exists even though price alone is a sufficient statistic of quality. In my model a necessay condition for the emergence of advertising is that quality can not be inferred from the price. Hence only price pooling is consistent with advertising. It was also shown that if advertising costs can be observed with only a small amount of signal loss, then the price alone will never be used to signal quality. This rules out all pure strategy equilibria except those with price pooling and an advertising signal which is correlated to quality. It is still unknown to the author whether such equilibria actually exist. In either case, however, this result should cause us to reconsider the conclusions reached by Milgrom and Roberts.

In addition to this result, Section 4 gave us a possible explanation for why low quality firms might advertise. If consumers can only view a perturbed advertising signal, then any observed level of advertising $A \in [0, A_H]$ is potentially "in-equilibrium." This gives the low-quality type an incentive to masquerade as a higher type through advertising. Since there's no upper bound on the signal loss, no amount of advertising by type H can assure it of separation. Instead, the high-quality firm can only increase the probability of a successful separation. This competition between the two types must involve a price which is uncorrelated to quality. If type L were to deviate from the pricing strategy of type H, this would reveal its true quality and render all of its advertising useless. Similarly, if type L were to fail to advertise this would cause the advertising of type H to be excessive, since then a much smaller amount of advertising would achieve the required separation at reduced cost.

Section 5 provided an alternative explanation for low quality advertising, namely its directly informative component. Since the model deals only with new products, its reasonable to expect that people can be informed about the product's existence through advertising. In this section we saw that the high quality firm advertises both because of the direct effect on demand and also to signal its quality. One interesting conclusion is that advertising would always be employed as a signal of quality. This differs from Milgrom and Roberts where it is possible to have only price signaling. Taken together the results of Section 4 and Section 5 suggest that advertising is a more important signal of quality than price. In other words, when we add imperfections into the signaling process the equilibrium will always involve advertising signals, it may or may not involve price signals. This depends on the nature of the imperfection.

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