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## PUBLIC POLICY AND ECONOMIC GROWTH: Developing Neoclassical Implications

Robert G. King\*

Sergio Rebelo\*\*

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- \* Department of Economics, University of Rochester and the Rochester Center for Economic Research, University of Rochester.
- \*\* Kellogg Graduate School of Management, Northwestern University, Portuguese Catholic University, and the Rochester Center for Economic Research, University of Rochester.

## PUBLIC POLICY AND ECONOMIC GROWTH: Developing Neoclassical Implications

Why do the countries of the world display considerable disparity in long term growth rates? This paper examines the hypothesis that the answer lies in differences in national public policies which affect the incentives that individuals have to accumulate capital in both its physical and human forms. Our analysis shows that these incentive effects can induce large differences in long run growth rates. Since many of the key tax rates are difficult to measure, our procedure is an indirect one. We work within a calibrated, two sector endogenous growth model, which has its origins in the microeconomic literature on human capital formation. We show that national taxation can substantially affect long run growth rates. In particular, for small open economies with substantial capital mobility, national taxation can readily lead to "development traps" (in which countries stagnate or regress) or to "growth miracles" (in which countries shift from little growth to rapid expansion). This influence of taxation on the rate of economic growth has important welfare implications: in basic endogenous growth models, the welfare cost of a 10% increase in the rate of income tax can be 40 times larger than in the basic neoclassical model.

#### I. Introduction<sup>0</sup>

Economists have long suspected that there is a link between national policies and long term rates of economic growth. For example, Schultz [1981] suggests that many public policies contain disincentives for growth because they reduce the rewards to accumulation of a comprehensive concept of capital encompassing human as well as physical capital. In this paper, we show that a basic Schultzian model has the property that modest variations in tax rates are associated with large variations in long run growth rates. Our model follows leads provided by Uzawa [1965], Lucas [1988b], and Rebelo [1987]. In our analysis, changes in public policy can potentially explain periods of secular stagnation or high economic growth. Public policy is particularly powerful in affecting small open economies with freely mobile capital. For these economies, taxes can easily shut down the growth process, leading to "development traps" in which countries stagnate or even regress for lengthy periods.

The specific model that we construct belongs to an important class of endogenous growth models based on work by Uzawa [1965] and retains the following key properties on the basic neoclassical model of Solow [1956], Swan [1963], Cass [1965], and Koopmans [1965]: (i) the existence of a constant asymptotic growth rate; and (ii) competitive and optimal allocations coincide in the absence of public interventions. The crucial attribute of this class of models is that there is a "core" of capital goods which can be produced without the direct or indirect contribution of non-reproducible factors.<sup>1</sup>

In developing our model we begin with the analysis of individual decisions at given prices and then consider the implications of production structure. This path leads us to develop aspects of individual accumulation technology not present in earlier studies that were concerned with aggregate behavior. We then examine the relation between public policy and long term growth, restricting the model's parameters to accord with existing microeconomic and macroeconomic evidence, a methodology that has proven to provide a powerful organizing tool in other areas of research on aggregate behavior.<sup>2</sup>

Our analysis focuses exclusively on taxation of commodity outputs. We have chosen to focus on taxation of this form since we think that a variety of public interventions—including aspects of property rights enforcement and regulation—may be described in this manner, so that our conclusions can potentially be interpreted as bearing on other aspects of governmental activity.

Our investigation of the link between public policy and economic growth is organized as follows. Section II provides a brief overview of the basic neoclassical model and of a very simple endogenous growth model. Both models are calibrated to accord with long run evidence for the U.S. economy and then used to analyze the effects of taxation on real economic activity.

Section III develops our model of growth through human capital accumulation and the incentive effects of public policy on this process. Our analysis proceeds in three stages. Following Rosen [1976] and Heckman [1976], we discuss optimal individual accumulation of human capital and the influence of various taxes on optimal accumulation. To highlight the role of taxes and to conform to prior microeconomic studies, our analysis begins by taking the following key prices to be exogenous: the wage rate per unit of human capital; the price of investing in human capital; and the real interest rate on consumption loans. In the next two stages of model development we add the structure that makes these relative prices endogenous. First, we

study the production of consumption and investment goods, while retaining an exogenous borrowing and lending rate. This provides a framework for discussing a small open economy's accumulation of nontraded human capital. In this section, the nature of the influence of tax policies on the price of investing in human capital is shown to depend on the nature of the technology for producing such investments. Second, we describe a full general equilibrium in which the rate of return adjusts to equate borrowing and lending or, equivalently, savings and investment. The influence of policy on the rate of return is a final factor affecting the growth rate.

Section IV compares the welfare effects of taxation in three economies: the basic neoclassical model, the simple endogenous growth model discussed in section II, and the growth model that we propose in section III. The main conclusion obtained from this comparison is that there are larger welfare costs of taxation in endogenous growth models than in comparable neoclassical models with exogenous technical change. Fundamentally, this is because policy can influence the long run growth rate in endogenous growth models. A concluding section summarizes our results and relates them to ongoing research on the theory of economic growth.

## II. Neoclassical Models of Economic Growth: Old and New

When we think about economic growth, most of us have in the back of our minds some variant of the basic neoclassical model of capital accumulation due to Solow [1956], Swan [1963], Cass [1965], and Koopmans [1965]. In this paper we construct and evaluate a new neoclassical model that alters intertemporal technology in ways which make sustainable growth a feasible outcome when technology is time stationary. Before developing our specific model it is useful to briefly discuss stylized versions of old and new

neoclassical models of economic growth. These two models have the same specification of preferences over consumption ( $C_t$ ), so we begin with these:<sup>3</sup>

$$U = \sum_{t=0}^{\infty} \beta^{t} \frac{1}{1-\sigma} (C_{t}^{1-\sigma} - 1) \quad \text{for } 0 < \sigma < \infty.$$
 (II.1)

With this utility function constant growth in consumption is optimal if the real interest rate is constant over time, which we take as one of the "stylized facts" of economic development (see Kaldor [1961] and Romer [1988a]). As in most of the growth literature we will assume that per capita labor supply is inelastic at N. To simplify the exposition we abstract from population growth.

#### II.1 The Basic Neoclassical Model

In this economy there is a single good that is produced by combining physical capital  $(K_t)$  and labor according to a neoclassical production function  $F(.)^4$ :

$$Y_{t} = F(K_{t}, NX_{t}).$$
 (II.2)

Technological progress occurs at an exogenous rate and its effects on productivity are captured by the variable  $X_t$  which grows at the "gross" rate  $\gamma_X$ ,  $X_t = \gamma_X X_{t-1}$ . We assume that technical progress is labor-augmenting to ensure that steady-state growth is feasible (see Swan [1963] and Phelps [1966]).

The resource constraint on consumption and investment  $(I_t)$ , and the difference equation that describes the accumulation of physical capital complete the specification of the technology.

$$Y_t = C_t + I_t$$
(II.3)

$$\mathbf{K}_{t+1} = \mathbf{I}_{t} + (1-\delta)\mathbf{K}_{t}$$

As usual,  $\delta$  denotes the rate of depreciation which is assumed to be between zero and one.

In this economy there are two modes of economic growth. First, in the steady state, consumption, investment, output and capital all grow at rate  $\gamma_X$ . Second, from a low initial level of the capital stock, the economy may exhibit growth rates exceeding  $\gamma_X$ , during a transition period in which the economy converges to its steady-state growth path. In a companion paper (King and Rebelo [1989]) we argue that the transitional dynamics of the neoclassical model cannot account for much sustained variation in rates of economic growth—either across countries or time periods—without generating counterfactual implications for factor prices or factor shares.

## Calibrating the Model

To study the effects of taxation in the neoclassical model it is useful to calibrate it with parameters that accord with the U.S long term experience. We will use the parameters of the baseline economy studied in our companion paper (King and Rebelo [1989]) which contains a detailed discussion of the model's calibration. Each time period is taken to represent a year and the discount factor  $\beta$  is chosen so that the after-tax steady-state interest rate is 3.2%.<sup>5</sup> Momentary utility is taken to be logarithmic ( $\sigma$ =1). The production function is assumed to be Cobb-Douglas,  $F(K,N)=AK^{1-\alpha}N^{\alpha}$ . We normalize A=1, which is a choice of units for measuring output, and select a conventional value for labor's share ( $\alpha$ =2/3). Finally, we set the depreciation rate equal to 10% ( $\delta$ =.10), the growth rate of technical progress to 2% ( $\gamma_{\rm X}$  = 1.02), and the fraction of time devoted to work to 20% (N=.20).

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(II.4)

#### The Effects of Taxation

Throughout the paper, the main tax experiment that we consider is an unanticipated increase in the output/income tax rate—applied equally to all sectoral activities—from 20% to 30%.<sup>6</sup> Roughly, this difference represents a change from the average Japanese tax rate during 1965-1975 to the average U.S. tax rate over that period. (Atkinson and Stiglitz [1980, Figure 1-2], provide this measure of the average tax rate—tax revenues as a share of gross domestic product—for a number of countries).<sup>7</sup>

To isolate the effects of taxation from those of government expenditures we assume that the tax revenue is used to finance lump sum transfer payments. Within the basic neoclassical model, as is well known, an increase in the income tax rate occasions a shift in the level of the steady-state path—but not in its slope—and sets in motion transitional dynamics. For our calibrated economy, the steady-state effects of an increase in the rate of income tax from 20% to 30% are a 18.2% drop in the capital stock and a 3.6% drop in consumption. The dynamics that characterize the transition between the two steady states are depicted in Figure 1, where the dashed line represents the old steady-state path. The qualitative features of these dynamics are familiar: during the transition period the initial level of consumption rises in response to the tax increase so the economy "works off" the capital stock through lower net investment and temporarily high levels of consumption.

#### II.2 A Simple Neoclassical Model of Endogenous Growth

For organizing our thinking about economic growth it is useful to consider the simplest endogenous growth model. In this model all factors of

production are reproducible and their quantity is summarized by the composite capital good  $K^*$ . The production technology is given by

$$Y_t = A K_t^*, \qquad (II.2)$$

where A>0 is the time invariant productivity parameter. The resource and accumulation constraints are

$$Y_{t} = C_{t} + I_{t}^{*},$$
 (II.3)

$$K_{t+1}^* - K_t^* = I_t^* - \delta K_t^*.$$
 (II.4)

It is easy to show that the common growth rate of consumption, investment, output and capital in this economy is:

$$\gamma = \left[\beta R(\tau)\right]^{(1/\sigma)}.$$
(II.5)

In this expression we have defined  $R(\tau) = [(1-\tau)A+1-\delta]$  as the gross private rate of return to the composite capital good. Thus, in an application of standard Fisherian principles, the growth rate depends on the gap between the rate of interest and the rate of time preference, with the strength of this relation depending on the intertemporal elasticity of substitution  $(1/\sigma)$ .

## Calibrating the Model

We parameterize this economy to make it comparable to the basic neoclassical model just discussed. We choose  $\sigma=1$ ,  $\delta=.10$  and determine the values of  $\beta$  and A such that the economy grows at 2% a year and has an annual after-tax real interest rate of 3.2% when the rate of income tax is 20%.<sup>8</sup>

#### Effects of Taxation

As is clear from expression (II.5), an unanticipated increase in the tax rate  $\tau$  produces an immediate shift in the level and slope of the growth path—there are no transitional dynamics. Higher taxes work to lower the rate of return,  $R(\tau)$ , and thus to lower the reward to accumulation. A rise in the tax rate thus lowers the long run growth rate, which is a general characteristic of endogenous growth models stressed by Rebelo [1987]. The effects of increasing the rate of income tax from 20% to 30% on consumption and capital are represented in Figure 2. The economy's rate of growth falls from 2% to 0.37% (in Table 4 this is reported as a decline of 1.63% in the growth rate). The reduction in investment associated with this slowdown makes the initial level of consumption rise by 36%.

We think that this model is a useful starting point for consideration of the effects of policy on long term growth. However, in a strict interpretation of K<sup>\*</sup>, it delivers endogenous long run growth only by effectively ignoring labor input, which is the sole force inducing diminishing returns to capital in the basic technology of Solow [1956] and Swan [1963]. (Alternatively if K<sup>\*</sup> is viewed as a composite of physical and human capital then the assumption is that these are produced according to an identical technology.) In our analysis below, we will follow Lucas [1988b] in permitting labor input to be reproducible, i.e., permitting human capital accumulation. Then, we can reintroduce the smooth substitution between factor inputs that was a key motivation for Solow's [1956] specification of production technology.

## III. Economic Growth through Human Capital Accumulation

Our interest is in models of endogenous growth that accord with the major facts of economic development. One stylized fact is that national growth rates do not display trends in the absence of major policy interventions. Another is that there is little evidence of long run trends in real interest rates. Following Solow [1970], we interpret these observations as evidence that steady-state models are a reasonable first approximation to reality, and we focus on economies in which the real interest rate is constant along steady-state paths. We therefore require that the production of both physical and human capital goods is governed by constant returns to scale technologies so that there are feasible steady-state growth paths. We also continue to utilize the preferences described in (II.1) since these lead individuals to choose a constant growth of consumption when faced with a constant interest rate.

The model economy that we construct is thus of the class studied previously by Uzawa [1965], Lucas [1988b] and Rebelo [1987] in that it highlights the societal allocation of resources between current consumption and comprehensive accumulation (physical and human capital) under constant returns to scale.<sup>9</sup> However, our model is different in three respects from these earlier studies. First, in contrast to Uzawa and Lucas, we allow for a commodity input into the production of new human capital, which seems empirically reasonable given our broad interpretation of this process. In this regard, we are also motivated by the analysis of Rebelo [1987], which indicated that the Uzawa-Lucas specification restricts certain tax policies to have negligible effect on the steady-state growth rate. Second, since we want to understand (i) the decentralization of accumulation decisions and (ii) growth in an open economy with traded physical capital and nontraded

human capital, we require that the rate of human capital investment be subject to diminishing point-in-time returns as in Rosen [1976]. Third, our model is designed to permit a quantitative evaluation of the effects of policy on economic growth.

#### III. 1 The Core Elements

To study the accumulation of physical and human capital, we use a two sector endogenous growth model. As in the neoclassical model of the previous section, there is a single consumption/physical investment good. This good is produced in sector 1 according to a constant returns-to-scale production technology with physical and human capital as its inputs. Hence, one technical constraint for the economy is

$$C_t + I_t = Y_{1t} = F_1(K_{1t}, N_{1t}H_t).$$
 (III.1)

As previously,  $C_t$  and  $I_t$  denote consumption and physical investment. Output of this commodity is  $Y_{1t}$  and physical capital and labor (human capital) inputs into this sector are denoted respectively by  $K_{1t}$  and  $N_{1t}^{H}_{t}$ .

The human capital investment good, which we call  $I_{Ht}$ , is produced in the second sector with another production technology that is constant returns to scale in the two inputs, i.e.,

$$I_{Ht} = Y_{2t} = F_2(K_{2t}, N_{2t}H_t).$$
(III.2)

The physical capital goods are taken to obey standard neoclassical accumulation equations, i.e.,

$$K_{j,t+1} - K_{j,t} = I_{jt} - \delta_{Kj} K_{jt}, \qquad (III.3)$$

where  $\delta_{Kj}$  is the depreciation rate in sector j. Aggregate physical capital investment is then the sum of the sectoral investments, i.e.,  $I_t = I_{1t} + I_{2t}$ .

Our specification of the evolution of human capital embodies diminishing point-in-time capacity to grow, as in Rosen [1976],

$$H_{t+1} - H_t = \Theta(I_{Ht}/H_t)H_t - \delta_H H_t, \qquad (III.4)$$

with  $D\Theta > 0$  and  $D^2\Theta < 0.^{10}$  This specification of "adjustment costs" permits steady-state growth if  $I_{Ht}$  and  $H_t$  grow at the same rate. Further, combined with (III.2), our setup is consistent with the view that growth in human capital combines labor and other inputs according to a production function as in Heckman [1976]. We assume that both physical and human capital investment are irreversible.

Finally, the sectoral allocations of labor must sum to the available stock, N.

$$N_{1t} + N_{2t} \leq N$$
 (III.5)

Since human capital is embodied in workers' time, this allocation also determines the sectoral allocation of human capital.

With this specification of intertemporal technology, our model has a range of feasible balanced growth equilibria in which consumption, physical investment, sectoral outputs, and capital stocks all grow at the same rate; this rate, which we denote by  $\gamma_{\rm H}$ , is the human capital growth rate.

#### Calibrating the Model

Our objective is to explore the quantitative effects of tax policies on rates of economic growth. For this purpose, we need to specify aspects of the investment technologies (parameters of the  $\Theta$  function,  $\delta_{\rm H}$ ,  $\delta_{\rm K}$ );

production technologies (parameters of the functions  $F_1$  and  $F_2$ ); and tax structure. Our parameter selections are reported in Table 1.

Throughout our analysis, we concentrate on the case where the production functions  $F_1$  and  $F_2$  are Cobb-Douglas, with  $F_i = A_i K_i^{(1-\alpha_i)} (N_i H)^{\alpha_i}$ . We uniformly assume that the share of labor in sector 1  $(\alpha_1)$  is 2/3, so that our results are compatible with those for the neoclassical model discussed above. Further, as in section II, we normalize the constant term in the sector 1 production function to unity  $(A_1 = 1)$  which, as earlier, represents a choice of units. In our quantitative analysis, we assume that there is a uniform depreciation rate on physical capital in its two uses,  $\delta_{K1} = \delta_{K2} = \delta_K = .10$ .

The parameter choices for sector 1 are well within the range of selections studied in other settings, such as public finance, quantitative growth theory, and business cycle analysis. However, in specifying the human capital production process, there is less guidance from prior aggregative studies. We consequently start with a benchmark view that human capital production is not much different from the production of output, so that  $\alpha_2 = \alpha_1 = 2/3$  and  $\delta_H = \delta_K = .10$ . This set of parameter choices has the convenient implication that there are no transitional dynamics in response to tax changes if these are uniform across the sectors. Rather, both capital goods—K and H—simply grow forever at the new steady-state rate, as in the simple model of section II. This implication is independent of the choice of the  $\Theta$  function; it holds generally if the two sectoral production functions are the same.

There has been little research since that of Rosen [1976] on the estimation of the parameters of the  $\Theta$  function, which is a primary determinant of the rate of human capital accumulation. We employ a parameterization of the human capital accumulation technology which implies

that: (i) human capital declines at the depreciation rate if there is no investment expenditure ( $\Theta(0)=0$ ); and (ii) there are locally no "adjustment costs" at zero gross investment so that  $D\Theta(I_H/H) = 1$  at  $I_{Ht} = 0$ . The specific function we use is

$$\Theta(I_{\rm H}/{\rm H}) = [I_{\rm H}/{\rm H} + \theta^{1-\theta}]^{\theta} - \theta^{1-\theta}. \qquad (III.6)$$

Our benchmark assumption is that the parameter heta takes on the value .5.

To conduct quantitative experiments within our general equilibrium model, it is also necessary to specify aspects of preferences— $\beta$  and  $\sigma$ —since these influence the equilibrium interest and growth rates. Our procedure is as follows: we choose a value of the intertemporal elasticity of substitution in consumption  $(1/\sigma)$  and a baseline value for the after-tax interest rate  $R(\tau)$  and growth rate  $\gamma_{\rm H}$ . Then, we can compute two "unknowns" in the model, the utility discount factor  $\beta$  and the productivity parameter for sector 2 output,  $A_2$ . The former is pinned down by the Fisherian link between interest and consumption growth,  $\beta(\gamma_{\rm H})^{-\sigma}R(\tau)=1$ ; the latter is determined by the required growth rate given efficient factor input proportions. Fixing these parameters, we then explore how steady-state interest rates and growth rates vary as the tax structure is altered.<sup>11</sup> Our benchmark preference case is to assume that the intertemporal elasticity of substitution is unity  $(1/\sigma = 1)$ . Finally, as in section II, we use the after-tax steady-state real interest rate of 3.2% and growth rate 2%.

Sensitivity Analysis: Given our uncertainty about the values of the human capital production technology, we looked at the implications of alternative parameter values, suggested by prior theory or measurement, which would act to reduce the growth implications relative to our benchmark. First, the Uzawa [1965]-Lucas [1988b] specification is that only labor is used to produce the human capital investment good  $(\alpha_2=1)$ . This parameterization has the very special property that taxation of sector 1 output has no effect on the economy's steady-state growth rate. Consequently, we study the implications of increasing our choice of  $\alpha_2$  from our benchmark of .67 to a level of .95. Second, in the applied labor economics literature, there is a variety of evidence on the magnitude of the depreciation of human capital: Mincer's [1974] estimate of  $\delta_{\rm H} = .012$  for individuals is the lowest one that we found; Haley [1976] reports estimates in the range of 3 to 4%. In our sensitivity analysis, we reduce  $\delta_{\rm H}$  to Mincer's value. Third, reasoning that increasing "adjustment costs" would mitigate the sensitivity of growth to economic policy, we reduce  $\theta$  to .25. Fourth, we explored the implications of a value of  $\sigma$  that implies a smaller degree of intertemporal substitution ( $\sigma=5$ ). Table 1 provides a summary of the parameterizations that we consider.

#### Taxation of the Two Sectors

Throughout the analysis, we consider taxation of sectoral outputs at rates  $\tau_j$ , with the proceeds rebated lump sum. As in the basic neoclassical model of section II, with constant returns-to-scale technologies this is equivalent to taxing the incomes from all factor services allocated to sector j at the uniform rate  $\tau_j$ .

## III. 2 Individual Human Capital Accumulation

We start by studying the individual's decision problem when the following prices are taken as given (i) the wage rate, (ii) the price of investing in human capital, and (iii) the interest rate. These are assumed constant over time, as they will be in a steady state. The introduction of diminishing returns to point-in-time production is necessary for the individual's human

capital investment demand to be well defined. Otherwise, when the rates of return to the accumulation of physical and human capital are identical (as they are in equilibrium) the individual allocations across these two activities are indeterminate.

The individual maximization problem involves choosing sequences of consumption  $\{C_t\}_0^{\omega}$  and human capital investments  $\{I_{Ht}\}_0^{\omega}$  so as to maximize lifetime utility (II.1), subject to an intertemporal budget constraint,

$$\sum_{t=0}^{\infty} [R(\tau)]^{-t} C_{t} \leq B_{o} + \sum_{t=0}^{\infty} [R(\tau)]^{-t} [wNH_{t} - pI_{Ht}],$$

where  $R(\tau)$  is the market discount factor ( $R = 1+r(1-\tau)$ , with  $r(1-\tau)$  being the after-tax real interest rate) and  $B_0$  is the level of initial financial assets. The optimal human capital program is also constrained by the evolution equation for human capital,  $[H_{t+1}-H_t]/H_t = \Theta(I_{Ht}/H_t) - \delta_H$ .

There is a separation of consumption from production decisions in this model, so that preferences do not influence the rate of human capital formation. Consumption growth occurs at the familiar rate  $(C_{t+1}/C_t) = [R(\tau)\beta]^{1/\sigma}.$ 

To examine the determination of the optimal growth rate, it is convenient to work with the inverse of the adjustment technology, which states the inputs required to yield a given flow of human capital outputs. Call this function  $\Psi$ , so that  $I_{Ht}/H_t = \Psi[(H_{t+1} - (1-\delta_H)H_t)/H_t]$ . Substituting this expression into the lifetime budget constraint and maximizing with respect to the human capital stocks, we are led to the following efficiency condition:

$$\mathbf{wN} = (\mathbf{R}(\tau) - \gamma_{\mathrm{H}})\mathbf{p}\mathbf{D}\Psi[\gamma_{\mathrm{H}} - 1 + \delta_{\mathrm{H}}] + \mathbf{p}\Psi[\gamma_{\mathrm{H}} - 1 + \delta_{\mathrm{H}}]. \qquad (III.7)$$

Implicitly, this efficiency condition determines a function for the optimal growth rate of human capital in the presence of "adjustment costs,"

$$\gamma_{\rm H} = \gamma[{\rm w/p, R}(\tau), \delta_{\rm H}]. \qquad (\text{III.8})$$

The growth rate depends positively on the wage rate w and negatively on the price of investing in human capital p.<sup>12</sup> Since investment in human capital—like other investments—depends negatively on the interest rate  $R(\tau)$  and on the depreciation rate of the capital good  $\delta_{\rm H}$ , the growth rate also depends negatively on these factors. In addition, it depends implicitly on the parameters of the  $\Theta$  function as in other models with adjustment costs to investment.

This formulation provides a convenient basis for discussing some theoretical results from the labor economics literature-which generally views the investment process for human capital as untaxed-and to preview some of our results in the general equilibrium models studied below. If we subject labor income to a tax at rate  $\tau_{_{\overline{w}}}$ , then the after-tax wage falls to  $(1- au_w)$ w so that one would expect a slowdown in human capital growth from this (These implications for human capital accumulation translate channel. directly into implications for the growth of individual income, wNH<sub>t</sub>). However, there may be a countervailing effect from the influence of taxation on the cost of investing in human capital. If labor is the only input into human capital investment and there is no direct taxation of the human capital activity, as in Rosen [1976], then p is simply proportional to  $(1-\gamma_{y})$ w, so that there is a full offset on the relative price w/p and, thus, no effect on the growth rate. In the alternative specification studied by Heckman [1976], labor is only one of the factors employed in producing new human capital, so that there is a smaller countervailing move in p from  $au_{_{_{f W}}}$  and, thus, there

continues to be a reduction in the growth rate induced by the tax increase even if the human capital investment sector is not taxed.

## III. 3 Prices Facing & Small Open Economy

For the purpose of studying a small open economy, we need to explore the implications of the production structure for the prices w and p. Thus in this section we proceed part way to a full dynamic general equilibrium, but we retain the assumption that the interest rate is exogenous. We make the conventional assumptions that there is international borrowing and lending; trade in capital and consumption goods; and international immobility of labor/human capital. We also assume that all countries follow the "worldwide tax system" according to which agents pay taxes in their home country on capital income from foreign investments but receive a tax credit for any taxes paid abroad on this income.<sup>13</sup>

## The Price of Investment in Human Capital

For a small economy facing a given world interest rate, the cost of producing a marginal unit of the human capital investment good is independent of the level of investment, since both of the production functions are constant returns to scale. In discussing the implication of analogous results for specialization in the production of traded goods in a world economy, Baxter [1988] notes that they fundamentally derive from the "nonsubstitution theorem" of Samuelson [1961] and Mirrlees [1969]. In our setting, since human capital is not traded, a small open economy will not specialize if it grows, i.e., it will generally produce both final product and human capital investment. But it still faces a price of investing in human capital that is determined solely by domestic technology and taxes along with the world interest rate; this price is not influenced by the domestic economy's choice between production of final output and human capital investment.

Following Baxter's [1988] line of argument, the capital intensity in sector 1 is pinned down by the cost of capital under international capital mobility,  $K_{1t}/(N_{1t}H_t) = \kappa_1[(R(\tau)-1+\delta_{K1})/(1-\tau_1)]$ . Hence, the real wage rate w is also determined by these variables, as  $w = (1-\tau_1)D_2F_1(\kappa_1,1)$ . Finally, with all of the input costs determined, the price of the investment good is given by

$$p = \{ (R(\tau) - 1 + \delta_{K2}) v_{K} + v_{N} \} / (1 - \tau_{2}), \qquad (III.9)$$

where the "unit factor demands"  $v_{\rm K}$  and  $v_{\rm N}$  are functions of the relative factor price for sector 2, w/[R( $\tau$ )-1+ $\delta_{\rm K2}$ ].

## Implications of Taxation

With these solutions in hand, we can return to the effects of the analysis of taxation of sector 1 output—i.e., of a rise in  $\tau_1$ —discussed at the end of the previous section. First, if there is only labor in the sector 2 production function ( $\alpha_2 = 1$ ), so that  $v_{\rm K} = 0$ , then  $p=w/(1-\tau_2)$  and w/p = $(1-\tau_2)$ . Hence, in our Cobb-Douglas setup,  $\alpha_2 = 1$  will be associated with no effect of sector 1 taxation on economic growth. However, sector 2 taxation will have a negative influence on growth.

As stressed by Heckman [1976], with  $v_{\rm N}^{<1}$ , a sector 1 tax increase will reduce w/p and, hence, reduce the growth rate of human capital. (The same logic also implies that this relative price change will generally occur when other produced inputs are used in the creation of new human capital; it can even occur if these are flow inputs rather than capital ( $\delta_{\rm K2}$ =1).) To some extent, there will also be factor substitution induced by an increase in  $\tau_1$ ; that is, the input ratio  $\kappa_1$  will rise and an increase in the marginal product of labor will occur. This factor substitution effect on the marginal product of labor will partly mitigate the direct effect of taxation.

These general equilibrium adjustments are complex and, for this reason, we resort to simulations of parametric economies in studying the effects of taxation in small open economies. To maintain the link to individual consumer choice, we also discuss the adjustment in the prices w and p so that one can see how the equilibrium outcomes are decentralized.

Table 2 reports the results of some basic experiments with our small open economy setup, which holds fixed the rate of return  $R(\tau)$ . Prior to the fiscal change, we assume that the economy has a growth rate of 2% and the initial configuration of tax rates is  $\tau_1 = \tau_2 = .20$ . Further, we assume that the economy does not trade with the rest of the world at these tax rates. We then explore the implications of increasing  $\tau_1$  to .3 and of increasing both  $\tau_1$  and  $\tau_2$  to .3.

In our benchmark case, we assume the parameter values discussed in section III.1 above govern the economy. We also consider two other sets of parameter values. The first is a "high  $\alpha_2$ " case with  $\alpha_2$  = .95, which is designed to illustrate the effects discussed earlier in this section. The second is to bring in greater investment adjustment costs; we consider  $\theta$ =.25.

In the benchmark case with  $\alpha_1 = \alpha_2 = 2/3$ , the 10% sector 1 tax increase—from  $\tau_1 = .30$ —lowers the growth rate by over eight percentage points. (Even a one percent increase in the sector 1 tax rate lowers growth by two percentage points). This slowdown is induced by a decline in w/p, the gross return to human capital investment from 14.92 to 13.97 percent per year. Taxing sector 2 at the same higher rate ( $\tau_1 = \tau_2 = .30$ ) leads to a complete shutdown of human capital investment ( $\gamma_{\rm H} = (1 - \delta_{\rm H})$ ).

With a higher labor share,  $\alpha_2$ =.95, the effects of taxation of sector 1 are reduced, but still important: the increase from  $\tau_1$ =.20 to  $\tau_1$ =.30 reduces the growth rate by 2.84%, so that—starting from a 2% growth rate—the economy would display negative growth. As discussed previously, with lower capital input, the relative price w/p is less sensitive to taxation in sector 1. Taxing both sectors at the same higher rate ( $\tau_1 = \tau_2 = .30$ ) continues to lead to a complete shutdown of the growth process.

With more sharply increasing costs of growth ( $\theta$  smaller), there is a smaller magnitude impact: growth falls by five percent when the tax change is concentrated on sector 1 alone; previously, in the benchmark model, it fell by over eight percent.

## III. 4 The Closed Economy General Equilibrium

In the closed economy general equilibrium, the rate of return exerts a stabilizing influence relative to the prior analysis. In particular, the increases in the tax rate that lead to lower growth bring about a decline in the real rate of return, which raises the amount of human capital investment undertaken at a given relative price w/p. Therefore, the effects that we report in this section are necessarily smaller than those in the prior section.<sup>14</sup>

Results on the effects of taxation on the growth rate are provided in Table 3: in the benchmark economy, the basic tax experiment of raising the sector 1 tax rate from  $\tau_1$ =.20 to  $\tau_1$ =.30 leads to a one half percent decrease in the growth rate; raising both tax rates by ten percent leads to a cut in the growth rate of 1.5%. This number is broadly in line with the growth

effects that we found in the simple endogenous growth model of section 2 (which implied a growth rate decline of 1.6%), but it is slightly attenuated due to the presence of "investment adjustment costs" to human capital formation in the current setup.

The balance of the table reports a battery of sensitivity experiments abstracting from the transitional dynamics induced by the tax increase and focusing on steady-state effects. First, when we consider the higher labor's share value,  $\alpha_2^{=.95}$ , we find that the effects of taxing only sector 1 is sharply limited: instead of a .5% cut in the growth rate there is a .1% reduction. This sensitivity analysis suggests that it is important to obtain good estimates of the relative importance of taxed factors and untaxed factors in the human capital production process. Second, when we consider the Mincer [1974] value of depreciation  $\delta^{}_{
m H}$ =.012, the effects of a general tax increase are attenuated, falling from 1.5% to .67%. At the aggregate level, the depreciation rate on human capital involves the training of new population members; the retraining of agents reallocated across jobs; and the continuing development of population members staying on the same job. Our sensitivity analysis indicates that it would be valuable to obtain better measurements of the depreciation of human capital associated with these activities. Third, there is a major influence of the intertemporal elasticity of substitution  $(1/\sigma)$  on the rate of economic growth. Our value of  $\sigma$ =5 is only about halfway from our benchmark of  $\sigma$ =1 to the largest values found by Hall [1988], but it is nevertheless sufficient to substantially mitigate the effect of taxation on economic growth. Fourth, we experimented with values of the investment technology parameter, reducing it to heta=.25. As in the small open economy, the growth rate declines relative to the benchmark

experiment, but this influence is smaller than that of the other sensitivity experiments.

From this battery of results, we conclude that taxation may affect the growth rate in a quantitatively important way, but that the magnitude of this influence depends, not surprisingly, on the production and tax structure.

## IV. Welfare Implications of Taxation

In the models of sections II and III we examined the effects on real activity of an increase in the income tax rate from 20% to 30%. In this section we evaluate the predictions of our models for the welfare cost of this tax increase. Our objective is to illustrate the general principle that there are larger welfare effects in endogenous growth models than in the basic neoclassical model. As in the previous sections we assume that the tax proceeds are rebated in a lump-sum fashion in order to isolate the substitution effects of taxation.

## Method of policy analysis

The method that we employ is based on Lucas [1988a] and works as follows.<sup>15</sup> Denote by  $\{C_t\}_{t=0}^{\omega}$  the consumption path associated with the steady state of an economy with a 20% tax rate, and let  $\{C_t'\}_{t=0}^{\omega}$  denote the path that results after an unanticipated increase in the tax rate from 20% to 30%. The welfare loss associated with this tax increase is the number  $\phi$  such that

$$U(\{C_{t}(1-\phi)\}_{t=0}^{\infty}) = U(\{C_{t}'\}_{t=0}^{\infty}).$$
 (IV.1)

Since  $C_t$  grows at a constant rate,  $\phi$  is determined so that consumers are indifferent between (i) an increase in the tax rate to 30% and (ii) a

situation in which the tax rate remains at 20% but their consumption level is reduced by  $100 \times \phi$ % in every period.

## IV. 1 The Basic Neoclassical Model

In response to an increase in the income tax rate from 20% to 30%, our parameterized version of the neoclassical model predicts that steady-state capital falls by 18.2% and consumption declines by 3.6%. As Judd [1987] and Jorgenson and Yun [1988] have stressed, it is inappropriate to evaluate tax policies solely on the basis of these long run effects. In fact the welfare cost of taxation  $\phi$  would be independent of preferences in the neoclassical growth model if only steady-state comparisons were utilized. However, the decline in steady-state consumption gives us an upper bound to the welfare cost associated with the tax increase. Since consumption is higher along the transition path than it is in the new steady state, the welfare cost is lower than the cost associated with an immediate, permanent 3.6% drop in consumption.

The first line of Table 4 shows that, taking into account the entire transition path, the welfare cost of raising the income tax rate from 20% to 30% in the benchmark model is 1.6%. That is, the tax rate increase is equivalent (in utility terms) to an immediate 1.6% downward shift in the steady-state consumption path.

We studied the sensitivity of this welfare cost to intertemporal elasticities of substitution different from the benchmark value of one. When the intertemporal elasticity of substitution in consumption is increased (i.e.  $\sigma$  is reduced), the initial jump in consumption illustrated in Figure 1 is magnified and the welfare cost increased. For example, when  $\sigma = 1/2$ , the welfare cost is 1.71%. Symmetrically, when  $\sigma$  is increased, the welfare cost falls: when  $\sigma = 2$ , the welfare cost is 1.46%. The intuition behind these results is familiar from discussions of the welfare cost of taxation in partial equilibrium (e.g., Musgrave and Musgrave [1980], page 310)): increasing the willingness of agents to substitute across goods increases their response to the tax distortion and leads to a higher excess burden of taxation.

## IV. 2 The Simple Endogenous Growth Model

In the simple endogenous growth model of section II there are no transitional dynamics. A permanent change in the tax rate implies an immediate shift in the level of the consumption path and a permanent change in the rate of growth of consumption. For our benchmark case of unitary elasticity of substitution, increasing  $\tau$  from 20% to 30% results in a 1.63% reduction in the growth rate. As sketched in Figure 2, the reduction in rates of growth induced by taxation is accompanied by an increase in the initial level of consumption. This effect on consumption is typically large—with  $\sigma = 1$  there is a 36.2% increase in initial consumption. Yet, with lower growth due to taxation, welfare unambiguously declines in this economy. Table 4 shows that this economy predicts dramatically higher values for the welfare cost of taxation than the neoclassical model. Fundamentally, this difference reflects the fact that the long run growth rate is affected in the linear technology economy but not in the neoclassical model.

However, one aspect of our calibration procedure contributed to the extraordinarily high welfare cost predicted by the model. By requiring that the discount rate  $\beta$  be such that the economy chooses to grow at 2% when the income tax rate is 20% we endowed this artificial economy with extremely patient agents (the value of  $\beta$  we adopted was .9884). The life-time utility

of these agents was severely affected when the tax rate increase reduced the returns to private accumulation of capital and hence the rate of growth.

If we calibrate the model by requiring that without taxes ( $\tau$ =0) the economy grows at 2%, the welfare effect is 16.3%, which is much lower than the number we reported in Table 4 but still significantly greater than the 1.6% welfare cost for the neoclassical model. In this case the  $\beta$  adopted is .9576 which coincides with the discount factor used in our benchmark parameterization of the neoclassical model.

As in the neoclassical model, the welfare cost of taxation depends on the extent of intertemporal substitution in consumption. When  $\sigma = 2$ , for example, the effect on the growth rate is .82% and the welfare cost is 63%. When  $\sigma = 1/2$ , the effect on the growth rate is 3.2% and the welfare cost is 72%.

## IV. 3 The Two-Sector Endogenous Growth Model

When we consider tax increases that are uniformly levied on both sectors 1 and 2, the two-sector endogenous growth model has positive implications that are broadly the same as the simple model of endogenous economic growth. Only the existence of "adjustment costs" alters these implications, yielding slightly smaller growth effects.

In terms of welfare effects of uniform sectoral taxation, the final line of entries in Table 4 makes clear that the welfare effects are also very close to those we found for the simple model. The cost of a 10% increase in taxation is around 60% of consumption. As in the simple endogenous growth model, the growth and welfare effects are increased (decreased) if individuals are more (less) willing to substitute over time. For  $\sigma = 1/2$ ,

the growth rate falls by 2.97% and the welfare cost measure is 76%. For  $\sigma =$  2, the growth rate falls by .77% and the welfare cost is 58%.

Other tax experiments—such as taxing only sector 1 or considering different labor share in the two sectors—generally imply transitional dynamics which requires that we explicitly solve for complete equilibrium paths as in the basic neoclassical model. We plan to pursue these experiments in our future research.

#### V. Conclusions

In this paper we proposed a model of economic growth in which a comprehensive measure of "technical progress" is made endogenous along the lines suggested by Uzawa [1965], Lucas [1988b] and Rebelo [1987]. By interpreting this comprehensive measure as social investment in "human capital," our analysis provides a potentially valuable formalization of the ideas of Schultz [1961, 1981] on economic development. Using this interpretation, we build explicit microfoundations for a two sector model of endogenous economic growth. When we calibrate our model with parameter values that accord with the U.S. long run experience, we reach three major conclusions, as follows.

First, we find that public policies can exert a quantitatively large influence on the average growth rates of economies operating in isolation. Policies can display these effects because they influence private incentives for accumulation of physical and human capital as in Schultz [1981]. Further, these incentive effects of taxation are reinforced in open economies that have access to international capital markets. In both open and closed economies, relatively small changes in tax rates can lead countries to stagnate or even regress for lengthy periods, if these policies eliminate

incentives for growth. Our explanation of "no-growth steady states" contrasts with that offered by Becker, Murphy and Tamura [1988] and Azariadis and Drazen [1988]. In those analyses, aspects of the technology give rise to multiple steady states so that economies with different initial conditions may converge to steady states with different rates of growth even in the absence of cross-country heterogeneity in public policy.

Second, the effects of taxation depend importantly on aspects of the production technology for new human capital, about which there is presently insufficient information. In part, this reflects the fact that our human capital good is a composite of many different activities and that we have not taken a sufficiently precise stand on its essential content. On the other hand, there has been little research in labor economics since that of Rosen [1976] and Heckman [1976] on the parameters of individual technologies for investment in human capital. Our research indicates that macroeconomic policy analysis would be aided by additional microeconomic measurement.

Third, since policies have the potential to influence the growth rate in models with endogenous long run growth, there is generally a much larger quantitative influence of policies on welfare than in the neoclassical model where the growth rate is governed by the exogenous rate of technical progress. Some experiments comparing neoclassical and endogenous growth models suggest that this difference can be quantitatively important.

In summary, with the results of the present paper, we find new promise for the hypotheses of Schultz [1981] that incentive effects of policy can influence economic activity—taxation can readily lead to development traps and growth miracles. Models of endogenous economic growth thus provide new analytical paths for studying old problems in the economics of development.

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#### Footnotes

<sup>o</sup>This paper is a substantially revised version of one prepared for the conference on "The Problem of Development" held at the State University of New York at Buffalo in May 1988. The analysis of the two sector model in our work, particularly in its open economy versions, draws heavily on some recent research by Marianne Baxter [1988]. We also thank her for pointing out the relationship between our model building activity and the arguments of T.W. Schultz [1981]. Finally, we have benefited from comments by Stanley Fischer and Arnold Harberger—who discussed the paper at the Buffalo conference—as well as by Kenneth Judd, who discussed the paper at the Summer Econometric Society meetings. Support from the National Science Foundation is gratefully acknowledged.

<sup>1</sup>This class of models is very large, including structures with many capital stocks in the growth "core" and with nonreproducible factors outside the growth "core" (Rebelo [1987]); or with steady states that are only asymptotically obtained (Jones and Manuelli [1990]).

<sup>2</sup>Lucas [1980] provides cogent arguments for combining aggregate and microeconomic evidence to restrict dynamic macroeconomic models of business fluctuations. Other applications of this strategy include Mehra and Prescott's [1985] work on asset pricing and recent work on real business cycles, as surveyed by King, Plosser and Rebelo [1988a,b].

<sup>3</sup>As is conventional with constant elasticity specifications, we assume that  $\sigma=1$  corresponds to logarithmic momentary utility.

<sup>4</sup>A neoclassical production function is constant returns to scale, concave, twice continuously differentiable, satisfies the Inada conditions, and specifies that each production factor is essential in production. <sup>5</sup>With a twenty percent tax rate on final output, this number is consistent with the 6.5% figure used in King and Rebelo [1989]. Let the before-tax marginal product of capital be MPK and the after tax marginal product be  $(1-\tau)$  MPK. The rate of return to capital investment is then  $(1-\tau)$  MPK -  $\delta$ . If we take the before-tax rate of return, MPK -  $\delta$ , to be .065, then using  $\delta$ =.10 and  $\tau$ =.2, we arrive at an after-tax rate of return of .8(.165) - .10 = .032.

<sup>6</sup>The equivalence between output and input taxation under constant returns to scale is discussed by Break [1974] and McLure [1975]; Atkinson and Stiglitz [1980] provide a convenient summary.

<sup>7</sup>The average tax rate is clearly a crude proxy for the income tax rate in our model but we view it as a natural starting point given that it is very hard to map the complex tax systems that most countries adopt into stylized descriptions that can be used in calibration exercises such as ours. For general discussions of these difficulties and suggestions for improvements on our proxy see Braun [1989], McGratten [1989] and Wynne [1988].

<sup>8</sup>These two requirements imply values of A and  $\beta$  as follows. First, the value of A is .1650, since the before-tax interest rate is 6.5% and the depreciation rate is 10%. The value of  $\beta$  is then given by the equation  $\beta R(\tau) \gamma^{-\sigma} = 1$ , where the after-tax interest rate,  $R(\tau)$ , is 3.2%.

<sup>9</sup>An important assumption in our approach is that the changes in productivity summarized by the evolution of the composite human capital good are embodied in the representative worker. See Romer [1986, 1988b] for analyses that do not rely on this embodiment assumption.

<sup>10</sup>By Dg(x), we mean the derivative of the function g with respect to x; correspondingly,  $D^2g(x)$  denotes the second derivative.

 $^{11}$ The details of this procedure are reported in Appendix A .

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<sup>12</sup>It also depends positively on the number of hours, N, which is exogenous in our model. In economies with variable labor supply, therefore, policy may affect human capital accumulation via the supply of labor, a channel not considered here.

<sup>13</sup>The U.S., Japan and the U.K. follow this tax system. An alternative tax convention is the "territorial system" which exempts from taxes all capital income earned abroad. See Swenson [1989] for a detailed discussion.

<sup>14</sup>To compute the closed economy general equilibrium, essentially, we add the requirement that  $\beta(\gamma_{\rm H})^{-\sigma}$  R = 1 to the preceding analysis. See appendix A.

<sup>15</sup>This method is closely related to that used by Hamilton in his [1987] study of the effects of taxation on risk taking. Our measure of the welfare effects of taxation would not be appropriate if we were addressing normative questions such as the design of an optimal tax system, since we do not impose that the tax revenue must be the same in the two regimes compared. If this restriction were imposed, our welfare measure would coincide with the compensating variation used by Hamilton [1987]. We thank Ken Judd for making us aware of Hamilton's work.

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TABLE 1: Parameters for Tax Experiments in Two Sector Endogenous Growth Model				
Model Component:	Parameter Values Studied:			
Sector 1: Consumption/Physical Investment				
(III.1) $F_1 = A_1 K_1^{(1-\alpha_1)} (N_1 H)^{\alpha_1}$ :	$\mathbf{A}_1 = 1 \qquad \alpha_1 = 2/3$			
Sector 2: Human Capital Investment				
(III.2) $F_2 = A_2 K_2^{(1-\alpha_2)} (N_2 H)^{\alpha_2}$ :	$A_2 = *  \alpha_2 = \{2/3, .95\}$			
Evolution of Physical Capital Stocks				
(III.3) $K_{j,t+1} - K_{j,t} = I_{jt} - \delta_{Kj} K_{jt}$ :	$\delta_{\mathrm{K1}} = \delta_{\mathrm{K2}} = .1$			
Evolution of Human Capital Stocks				
(III.4) $H_{t+1}-H_t = \Theta(I_{Ht}/H_t;\theta) H_t - \delta_H H_t$ :	$\theta = \{.5, .25\}  \delta_{\text{H}} = \{.10, .012\}$			
Preferences				
$U = \sum_{t=0}^{\infty} \beta^{t} u(c_{t}; \sigma):$	$\sigma = \{1, 10\}$ $\beta = *$			
Initial Rates				
After-Tax Real Interest Rate	$R(\tau) = 1.032$			
Real Growth Rate	$\gamma_{\rm H}$ = 1.02			
Sectoral Tax Rates	$\tau_1 = .20$ $\tau_2 = .20$			

When multiple parameter values are given, e.g.,  $\alpha_2 = \{2/3, .95\}$ , the initial

value is the benchmark and the subsequent one is used in sensitivity analysis. Parameters indicated by an asterisk (\*) are determined so that the closed economy general equilibrium produces the initial values of interest and growth rates at initial tax rates.

The functions u(c) and  $\Theta(I_H/H)$  are given in text equations (II.1) and (III.6) respectively. The relevant parameters follow the principal argument, i.e.,  $u(c;\sigma)$  and  $\Theta(I_H/H;\theta)$ .

Parameter Choices:	Tax Rate Increases %		Growth Rate Decrease %
	$\tau_1$	$ au_2$	
Benchmark	10 10	0 10	8.6 12.0*
Higher $\alpha_2$	10	0	2.8
nigher u2	10	10	12.0*
Lower $ heta$	10 10	0 10	5.0 9.5

Those model parameters not changed in the experiments take the benchmark values listed in Table 1. An asterisk (\*) indicates that the economy contracts at the maximum rate  $(\gamma_{\rm H} = 1 - \delta_{\rm H})$ .

#### TABLE 2: Tax Effects on Growth Rate in Two Sector Endogenous Growth Model OPEN ECONOMY VERSION

Parameter Choices:	Tax Rate Increases %		Growth Rate Decrease %
	$\tau_1$	$\tau_2$	
Benchmark	10	10	1.52
Benchmark	10	0	.52
Higher $\alpha_0$	10	0	.11
Higher $\sigma$	10	10	.31
Lower $\delta_{\rm H}$	10	10	.67
Lower $\theta$	10	10	1.38

### TABLE 3: Tax Effects on Growth Rate in Two Sector Endogenous Growth Model CLOSED ECONOMY VERSION

Table 1.

Welfare Effects of Tax Increase in Three Dynamic Models						
	Initial Consumption	Growth Rate Decrease	Welfare Cost			
	Increase %	%	۲.			
Basic Neoclassical Model	6.6	0	1.6			
Simple Endogenous Growth Model	36.2	1.63	65.4			
Two-Sector Endogenous Growth Model	34.5	1.53	62.6			

The results reported here are for the benchmark versions of each model; parameter choices are discussed in section II of the text for the first two models and in Table 1 for the third model.

#### TABLE 4



Figure 1

Effects of Taxation in the Neoclassical Model

time (years)

time (years)



Effects of Taxation in the Basic Endogenous Growth Model



#### Appendix A

# Calibration of Two Sector Endogenous Growth Model

This appendix discusses the formal structure of the two-sector endogenous growth model outlined in the main text, as well as our procedures for calibrating its steady state and exploring policy implications.

The representative agent in this economy solves the dynamic optimization problem

$$\max \sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the accumulation constraints

$$K_{t+1} = (1-\delta_K)K_t + I_{Kt}$$

 $\mathbf{H}_{t+1} = \Theta(\mathbf{I}_{Ht}/\mathbf{H}_{t})\mathbf{H}_{t} + (1-\delta_{H})\mathbf{H}_{t}$ 

the resource constraints

$$C_t + I_{Kt} \leq \Omega_1 F_1(K_{1t}, M_{1t}) + T_{1t}$$

$$I_{Ht} \leq \Omega_2 F_2(K_{2t}, M_{2t}) + T_{2t}$$

and the factor allocation constraints

 $M_{1t} + M_{2t} \leq NH_{t}$ 

$$K_{1t} + K_{2t} \leq K_t$$

To analyze equilibrium behavior, we form the Lagrangian

$$\begin{split} \mathfrak{L} &= \sum_{t=0}^{\infty} \beta^{t} u(C_{t}) \\ &+ \sum_{t=0}^{\infty} \tilde{\Lambda}_{1t} [\Omega_{1}F_{1t} + T_{1t} + (1-\delta_{K})K_{t} - K_{t+1} - C_{t}] \\ &+ \sum_{t=0}^{\infty} \tilde{\Lambda}_{2t} [\Theta(\frac{\Omega_{2}F_{2t} + T_{2t}}{H_{t}})H_{t} + (1-\delta_{H})H_{t} - H_{t+1}] \\ &+ \sum_{t=0}^{\infty} \tilde{Q}_{Kt} [K_{t} - K_{1t} - K_{2t}] \\ &+ \sum_{t=0}^{\infty} \tilde{Q}_{Mt} [NH_{t} - M_{1t} - M_{2t}] \end{split}$$

The efficiency conditions take the following forms. For consumption, we have the familiar requirement that

$$\beta^{t} Du(C_{t}) = \tilde{\Lambda}_{1t}.$$

For the cross-sectoral allocations of factor stocks, we have the four conditions

$$\tilde{\Lambda}_{1t}\Omega_1 D_1 F_1(K_{1t}, M_{1t}) = \tilde{Q}_{Kt}$$

$$\begin{split} \tilde{\Lambda}_{1t} \Omega_1 D_2 F_1(K_{1t}, M_{1t}) &= \tilde{q}_{MT} \\ \tilde{\Lambda}_{2t} D\Theta_t \Omega_2 D_1 F_2(K_{2t}, M_{2t}) &= \tilde{q}_{Kt} \\ \tilde{\Lambda}_{2t} D\Theta_t \Omega_2 D_2 F_2(K_{2t}, M_{2t}) &= \tilde{q}_{Mt} \end{split}$$

For the efficient evolution of capital stocks, we have the two shadow price requirements

$$\tilde{\Lambda}_{1t} = \tilde{\Lambda}_{1,t+1}(1-\delta_{K}) + Q_{K,t+1}$$

$$\tilde{\Lambda}_{2t} = \tilde{\Lambda}_{2,T+1}(1-\delta_{H}) + Q_{1t,T+1}N + \Lambda_{2,T+1}[\Theta_{t+1} - D\Theta_{t+1}(\frac{\Omega_{2}F_{2,t+1} + T_{2,t+1}}{H_{t+1}})]$$

and the transversality conditions

$$\lim_{t \to \infty} \tilde{\Lambda}_{1t} K_{t+1}$$
$$\lim_{t \to \infty} \tilde{\Lambda}_{2t} H_{t+1}$$

Finally, we have the four resource constraints

$$K_{t+1} = \Omega_1 F_{1t} + T_{1t} + (1 - \delta_K) K_t - C_t$$

$$H_{t+1} = \Theta\left(\frac{\Omega_2^{F_2t} + T_2t}{H_t}\right)H_t + (1-\delta_H)H_t$$

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$$K_t = K_{1t} + K_{2t}$$

$$\mathbf{NH}_{t} = \mathbf{M}_{1t} + \mathbf{M}_{2t}$$

## Steady State Requirements

Consolidating the preceding conditions, we find that the steady state is described by

(SS1) 
$$\beta(\gamma_{\rm H})^{-\sigma} = 1/(1+r)$$
.

(SS2) 
$$(1+r) = (1-\delta_{K}) + \Omega_{1}D_{1}F_{1}(K_{1}/M_{1}, 1)$$

(SS3) 
$$(1+r) = [(1-\delta_{\rm H}) + N D\Theta \Omega_2 D_2 F_2 + \Theta - (F_2/H) D\Theta]$$

(SS4) 
$$\frac{D_1F_1(K_1/M_1, 1)}{D_2F_1(K_1/M_1, 1)} = \frac{D_1F_2(K_2/M_2, 1)}{D_2F_2(K_2/M_2, 1)}$$

$$(SS5) \qquad 1 = \frac{M_1}{NH} + \frac{M_2}{NH}$$

(SS6) 
$$\frac{K}{NH} = \left(\frac{K_1}{M_1}\right) \left(\frac{M_1}{NH}\right) + \left(\frac{K_2}{M_2}\right) \left(\frac{M_2}{NH}\right)$$

(SS7) 
$$\gamma_{\rm H} = \Theta(I_{\rm E}/{\rm H}) + 1 - \delta_{\rm H}$$

(SS8) 
$$I_E/H = F_2(\frac{K_2}{M_2}, 1) \frac{M_2}{H}$$

(SS9) 
$$(\frac{Y}{NH}) = F_1(K_1/M_1, 1)(\frac{M_1}{NH})$$

(SS10) 
$$\frac{C}{NH} = \frac{Y}{NH} - (\gamma_H + 1 - \delta_K) \frac{K}{NH}$$

This system is 10 equations in the 10 unknowns  $\gamma_{\rm H}$ , (1+r), (K<sub>1</sub>/M<sub>1</sub>), (K<sub>2</sub>/M<sub>2</sub>), (M<sub>1</sub>/NH), (K/NH), (I<sub>E</sub>/H), (Y/NH) and (C/NH).

#### **Calibration**

For the purpose of determining the parameters of the steady state to match observed average growth and real interest rates, we proceed as follows. First, we postulate CES forms for the  $F_1$  and  $F_2$  functions, so that  $F_1$ 

$$F_{i}(K_{i}, M_{i}; A_{i}, \alpha_{i}, \rho_{i}) = A_{i}[(1-\alpha_{i})K_{i}^{-\rho_{i}} + \alpha_{i}M_{i}^{-\rho_{i}}]^{-1/\rho_{i}}$$

Then, we compute the steady state according to the following algorithm:

Step 1: Given  $A_1 = 1$ ,  $\alpha_1$ , and  $\rho_1$ , invert  $(1+r) = D_1 F_1(K_1/M_1, 1) + 1 - \delta_K$  to obtain steady state  $K_1/M_1$  ratio using (SS2).

Step 2: Given  $\alpha_1$ ,  $\alpha_2$ ,  $\rho_1$  and  $\rho_2$  calculate  $K_2/M_2$  from requirement that marginal rates of transformation are equated in the two sectors, using (SS4).

Step 3: Given the parameters of  $\Theta$  function—the coefficients developed in Appendix A (i.e.,  $\theta$  and A<sub>3</sub>)—compute (I<sub>E</sub>/H) consistent with specified  $\gamma_{\rm H}$  using (SS7). Step 4: Given the results of the preceding steps, (SS3) permits—with specification of  $\Omega_2$ —solution for the parameter  $A_2$ 

$$\mathbf{A}_{2} = \frac{\mathbf{r} + \delta_{\mathrm{H}} - [\Theta(\mathbf{I}_{\mathrm{E}}/\mathrm{H}) - \mathrm{D}\Theta(\mathbf{I}_{\mathrm{E}}/\mathrm{H})(\mathbf{I}_{\mathrm{E}}/\mathrm{H})]}{\mathrm{N}\mathrm{D}\Theta(\mathbf{I}_{\mathrm{E}}/\mathrm{H})\Omega_{2}[\mathrm{D}_{2}\mathrm{F}_{2}(\mathrm{K}_{2}/\mathrm{M}_{2}, 1)/\mathrm{A}_{2}]}$$

Step 5: Use (SS8) to compute the fraction of time in efficiency units allocated to investment in human capital, given previously determined  $(I_E/H)$  and  $(K_2/M_2)$  with specified N.

$$(M_2/NH) = I_E/[F_2(\frac{K_2}{M_2}, 1)]$$

Step 6: Compute  $M_1/NH = 1-(M_2/NH)$ , using (SS5).

Step 7: Compute  $\frac{Y}{NH} = (\frac{Y}{M1})(\frac{M_1}{NH}) = F_2[(\frac{K_1}{M_1}), 1; A_1, \alpha_1, \rho_1] \bullet (\frac{M_1}{NH})$ , using (SS8) and the results above.

Step 8: Compute  $\frac{K}{NH} = (\frac{K_1}{M_1} \frac{M_1}{NH} + \frac{K_2}{M_2} \frac{M_2}{NH})$ , using (SS6) and the results above.

Step 9: Compute  $\frac{C}{NH} = \frac{Y}{NH} - \gamma_H \frac{K}{NH} + (1 - \delta_K) \frac{K}{NH}$ , using (SS9) and the results above.

Step 10: Compute  $\beta = \gamma_{\rm H}^{\sigma}/(1+r)$  using (SS1) and the results above.