Collusion, Discounting and Dynamic Games

Dutta, Prajit K.

Working Paper No. 272
April 1991

Collusion, Discounting and Dynamic Games

Prajit K. Dutta

Rochester Center for Economic Research
Working Paper No. 272

Collusion, Discounting and Dynamic Games*

Prajit K. Dutta
Columbia University
and
University of Rochester

Rochester Center for Economic Research
Working Paper No. 272

March 1991

Abstract: It is commonly believed that the more patient the players in a game, the more likely that they can sustain collusive behavior. We show that this intuition, although immediate for purely repeated games, is false for dynamic games. We show that, in general, there is no direct link between discounting and collusion and in fact there are games in which collusion is possible if and only if players are sufficiently impatient. Further, there may be dynamic games arbitrarily "close" to repeated games which exhibit such starkly different behavior. We do however show that any equilibrium outcome that is sustainable by less patient players, is also an equilibrium outcome when players are more patient.

* I have benefitted from discussions with Dilip Abreu, Jeff Banks, Susan Elmes, Christophe Faugare, Jean-Francois Mertens, Changyong Rhee and Raghu Sundaram, although none of these individuals can be implicated in remaining errors. Changyong Rhee also provided very patient and extremely helpful "graphical advice".

1. Introduction

Conventional wisdom holds that the more patient the players in a game, the more likely that they can sustain collusive or first-best outcomes. The intuition is seen most starkly in a repeated game. Deviation from first-best action[1] yields the same one-shot gain, regardless of the discount factor. The cost of deviation is the foregone infinite stream of constant stage-game collusive payoffs and this cost is clearly increasing in the discount factor.[2] The repeated game argument, although compelling and simple, is however critically dependent on the following observation: the first-best action is itself independent of the discount factor, since there is no direct intertemporal linkage across stage games.

In many applications of economic interest a purely repeated framework is unduly restrictive and may not even be approximately correct. The appropriate structure in a number of cases is a dynamic game (also called a Markovian or stochastic game[3]), in which a game environment evolves endogeneously on account of the players' actions and determines which one of many possible stage games gets played in any period.[4] On account of this intertemporal link, the first-best action in a dynamic game will typically depend on the discount factor. Hence, the intuition on collusion possibilities is not as immediate as in the case of repeated games. Yet in much of the literature a belief persists that implicit collusion arises if and only if players are sufficiently patient.

Indeed in many dynamic economic models, as players become more patient the first-best action changes in a manner such that one-shot deviations against it become more profitable. Consider, for example, the neoclassical growth model. It is well known[5] that as the discount factor increases, the corresponding first-best action involves smaller consumption out of any given capital stock. If one considers a strategic version of this model[6], when all other players reduce consumption an optimal deviation clearly yields higher payoffs. Similar considerations apply to pricing problems, investment models, search models etc. Collusion possibilities in a dynamic game are then often shaped by two simultaneous and competing effects: the size of first-best benefits (and these, as in a repeated game, are larger the more patient the players) and the size of benefits to "breaking the cartel" (and these, unlike a repeated game, are also increasing in player patience).

We investigate the sustainability of collusion in a general class of symmetric dynamic games $G(\sigma,\delta)$ where δ is the players' discount factor and $\sigma \varepsilon [0, 1]$ is a parameter that indexes the degree of intertemporal linkage. $\sigma = 0$ will be seen to correspond to a purely repeated game whereas $\sigma = 1$ will be an exhaustible resource problem. Our principal conclusions on discounting and first-best sustainability are the following:

Collusion May Require Impatience (Propositions 5, 7 and 8) For dynamic games, in general there is no direct link between collusion sustainability and discounting, in the following sense: fix any $\sigma^* > 0$ and any closed set of discount factors $D \subset [0,1]$. There is a family of dynamic games $G(\sigma,\delta)$ with the property that whenever $\sigma = \sigma^*$, collusion is sustainable on D and only on this set (Proposition 5). To make the point more dramatic, we present simple sufficient conditions on the one-period reward functions with the following "counter-intuitive" implications: there is $\sigma^* \in (0,1)$ such that for all $\sigma \geq \sigma^*$ either a) collusion is sustainable if and only if players are sufficiently impatient (and this is true if in the repeated game the first-best is a Nash equilibrium and hence collusion is always sustainable there), or b) players can successfully collude if and only if they are neither too impatient nor too patient (and this holds for the alternative case in which in the repeated game the first-best is not a Nash equilibrium and hence collusion is only sustainable by patient players there) (Propositions 7 and 8).

Collusion In Repeated Games Need Not Imply Collusion In Neighboring Dynamic Games (Proposition 9) Is collusive behavior in dynamic games that are "near" repeated games, i.e. for small σ, similar to that in the purely repeated game? We demonstrate that it need not be by showing that for any $\varepsilon > 0$, there is a specification of returns with the following properties: the set of of games (i.e. pairs (σ,δ)) in which collusion is sustainable is of Lebesgue measure smaller than ε but this set always includes the repeated game, i.e. includes the case $\sigma = 0$ and $\delta \in [0,1]$.

Propositions 7-9 immediately yield the following as a corollary: under the overtaking criterion (which we argue in the sequel is the relevant criterion for $\delta = 1$, in the games we study), there is no folk theorem for dynamic games arbitrarily close to repeated games (for which Rubinstein (1979) established just such a result).

A second focus of this paper are the implications of alternative notions of collusiveness. The usual behavioral question, and the one discussed so far, is: can more patient players sustain the best feasible outcome as an equilibrium outcome even if less patient players cannot? An alternative question is: can any outcome that is sustainable in equilibrium by less patient players be similarly sustained by more patient players? This question can be cast in payoff terms by determining whether the highest equilibrium, or second-best, payoffs are increasing in the discount factor. Recall, incidentally, that these notions are equivalent for repeated games. We show:

The Conflict between First and Second-Best Outcomes (Proposition 9) Regardless of σ, if an outcome profile arises in equilibrium for δ^* then it also is an equilibrium outcome for all $\delta > \delta^*$. Increasing player patience does indeed enlarge equilibrium behavioral possibilities, and in this sense conventional wisdom is correct, even if attainment of the best feasible outcome becomes less likely. The highest equilibrium discounted payoffs are increasing in δ, although the second-best discounted average payoffs are not.

Additional results are presented and discussed in the text. The rest of the paper is organized as follows: Section 2 describes the family of games and some preliminary results. The effect of discount factors on collusion possibilities is explored in Section 3. Section 4 compares behavior in repeated and dynamic games, whereas Section 5 presents an illustrative example. Section 6 contrasts first and second best sustainability. Two economic interpretations of the games considered are offered in Section 7 while Section 8 contains concluding comments.

2. Model and Preliminary Results

2.1 The Dynamic Game

A dynamic game $G(\sigma,\delta)$ is defined by a sextuple $<S,A_i,g,r_i,\sigma,\delta;\ i=1,2>$. The player index is i and purely for notational convenience we restrict attention to the two player case. In all statements pertaining to i, j will denote the other player. S is the state space and should be thought of, in the usual way, as the set of possible game environments. A_i is the i-th player's action set and we take $S = A_i = [0,1]$. g is the transition function which associates with the tuple (s_t,a_{1t},a_{2t}), the period -t+1 state s_{t+1}. We consider a transition function which is linear in the state and additively separable in the actions of the two players:

$$g(s_t,\ a_{1t},\ a_{2t}) = \begin{cases} [1 - \sigma\ (a_{1t} + a_{2t})]\ s_t, & \text{if } a_{1t} + a_{2t} \leq 1 \\ 0 & \text{if } a_{1t} + a_{2t} > 1 \end{cases} \tag{2.1}$$

The i-th player's one-period reward is r_i which is dependent on the current state s and action tuple (a_1,a_2). We analyze the following special payoffs which are also linear in the state:

$$r_i(s, a_i, a_j) = \begin{cases} [h\,(a_i + a_j) + f\,(a_i) - f\,(a_j)]\, s, & \text{if } a_i + a_j \leq 1 \\ 0 & \text{if } a_i + a_j > 1 \end{cases} \tag{2.2}$$

In the sequel we make assumptions on the functions h and f. The parameter σ only serves to index the transitions, as described by (2.1) and we take $\sigma \in [0, 1]$. δ is the common discount factor, and of course $\delta \in [0, 1]$. The horizon is infinite and given a time profile of actions and states, the aggregate payoffs[7] are $\sum_{t=0}^{\infty} \delta^t\, r_i(s_t,a_{it},a_{jt})$.

Remark 1 If $\sigma = 0$ we have a purely repeated game.[8] On the other hand, if $\sigma = 1$ we have a pure "cake-eating" or exhaustible resource problem.

Remark 2 The form of the transition function (2.1) and the returns (2.2) can be derived from primitive considerations in specific economic models. Section 7 contains details for exhaustible resource and consumer durables problems.

Remark 3 The formulations (2.1) and (2.2) have been chosen to maximize expositional ease. Results similar to those that follow can be derived for more general continuous, non-additively separable functions. We defer to the end of Section 3 the relevant discussion. It will be clear then that the only crucial property that we require are that the functions be linear in the state. We do that to factor out the effect of the state variable in everything that follows. It will be seen that first-best and deviation payoffs are linear in the state under this specification. Consequently, collusion prospects will be exactly the same from all states. In turn this avoids a usual problem with dynamic games that particular idiosyncracies of a few states drive anomalous behavior from those states but not others.

We consider games of complete information. A strategy π_i is simply a sequence $\pi_{io}, \pi_{i1}, \ldots \pi_{it}, \ldots$ where π_{it} selects an action a_{it} as a function of game history $h_t = (s_0, a_{10}, a_{20}, \ldots s_{t-1}, a_{1t-1}, a_{2t-1}, s_t)$. Any strategy pair (π_1,π_2) determines an outcome $(s_{t+1}, a_{1t+1}, a_{2t+1}, \ldots)$ after every history. In particular, after each history starts a subgame.

Nash and perfect equilibrium are defined in the usual way. Let $\phi(s;\sigma,\delta)$ denote the set of perfect equilibrium payoffs for $G(\sigma,\delta)$ when the state at period zero is s. The following proposition is immediate

Proposition 1 *Suppose that $h(1) + f(0) - f(1) \leq 0 \leq h(1) + f(1) - f(0)$. Then,*

i) $a_1 = a_2 \equiv 1$ is a perfect equilibrium. Hence, $(0,0) \in \phi(s;\sigma,\delta)$ for all s, σ, δ.

ii) 0 is the smallest equilibrium payoff.

2.2 First-Best Outcomes

Much of the subsequent analysis will be concerned with sustainability of the symmetric first-best strategy. The one-shot first-best payoffs are $1/2\ [r(s,a_1,a_2) + r_2(s,a_2,a_1)] = h(a_1 + a_2)s$. So, the first-best optimization problem is

$$\max_{a_{1t},\ a_{2t}} \sum_{t=0}^{\infty} \delta^t\ h\ (a_{1t} + a_{2t})\ s_t \quad \text{s.t. } (2.1) \tag{2.3}$$

The following assumption will be maintained:

(A1) i) h is a strictly concave, increasing[9], twice continuously differentiable function

ii) $h(0) < 0$ and $\lim_{a \uparrow 1} \left[\frac{h(a)}{h'(a)} - a\right] = \infty$

Remark $h(a) = a - 1/2\ a^2 - b,\ b \in (0,1/2)$ satisfies (A1).

Let $V(s;\sigma,\delta)$ denote the symmetric first-best payoffs in $G(\sigma,\delta)$. Clearly the associated Bellman equation is

$$V(s_t;\ \sigma,\ \delta) = \max_{0 \leq a_{1t}+a_{2t} \leq 1} \left\{h\ (a_{1t}+a_{2t})\ s_t + \delta V([1-\sigma(a_{1t}+a_{2t})]s_t;\sigma,\delta)\right\} \tag{2.5}$$

Proposition 2i) *The first-best strategy is to take a constant action, denoted $a(\sigma,\delta)$, regardless of history. Of course $a_1 = a_2 = \frac{a(\sigma,\ \delta)}{2}$. Moreover, whenever $\sigma > 0$ and $\delta > 0$, $a(\sigma,\ \delta)$ is determined by the solution to*

$$\frac{1-\delta}{\delta} = \left[\frac{h(a)}{h'(a)} - a\right]\sigma \tag{2.6}$$

$a(\sigma,\delta) = 1$ when either $\sigma = 0$ or $\delta = 0$. Further, $a(\sigma,\delta)$ is continuous, and is in fact a continuously differentiable function on $(0,1]^2$, which monotonically declines in its arguments with $a(1,1) > 0$.

ii) The value function is given by

$$V(s;\sigma,\delta) = \begin{cases} \dfrac{h(a(\sigma,\delta))}{1-\delta(1-\sigma a(\sigma,\delta))}\, s & \sigma,\delta \neq 0,1 \\ \infty & \sigma,\delta = 0,1 \end{cases} \tag{2.7}$$

Finally, $V(s;.,.)$ is continuous (continuously differentiable on $(0,1]^2$) and monotonically increasing (respectively decreasing) in δ (respectively σ), for fixed s.

Proof: Substituting (2.7) as a candidate value function into (2.5) yields (2.6) by way of first-order conditions (which characterize the maximum in (2.5), given the concavity of h). From (2.6) it is clear that $h(a(\sigma,\delta)) \geq 0$, for all σ,δ. Given (A1i), $\frac{h(a)}{h'(a)} - a$ is increasing in a whenever $h(a) \geq 0$. From this observation and (2.6), the monotonicity properties of $a(\sigma,\delta)$ follow immediately. That the optimal actions are continuously differentiable on $(0,1]^2$ can be deduced from (2.6). When either $\sigma\downarrow 0$ or $\delta\downarrow 0$ (or both), (A1ii) and (2.6) imply that $a(\sigma,\delta) \uparrow 1$. Hence, it follows that the function is continuous at the boundary. The assumption, $h(0) < 0$, yields the conclusion that $a(\sigma,1) > 0$. The monotonicity properties of the value function is immediate from the fact that on the relevant set of actions, one-period rewards are non-negative. The continuous differentiability of V follows from (2.6) and (2.7). Continuity at $\sigma,\delta = 0,1$ follows by taking limits, while using the above properties of optimal actions, for $\sigma,\delta = 0,1$.

■

In the sequel it will be convenient to write $V(s;\sigma,\delta) \equiv v(\sigma,\delta)s$. Since the value function is linear in s, we can restrict further attention solely to the constant $v(\sigma,\delta)$.

2.3 Optimal Deviation

Suppose a player were to deviate from the collusive strategy. Suppose further that deviation triggers a move to the worst equilibrium in the next period (from Abreu (1988) it is known that it suffices to only look at this threat; indeed the sufficiency of checking for one-shot deviations holds for all δ, provided $\sigma > 0$, and of course for $\delta < 1$ in the repeated game). The optimal deviation is then determined by

$$\max_{a_i} \left[h(a_i + \frac{a(\sigma, \delta)}{2}) + f(a_i) - f(\frac{a(\sigma, \delta)}{2}) \right] s \qquad (2.8)$$

Denote the maximized returns $M(s;\sigma,\delta)$. We make the following assumption on f:

(A2) f is an increasing, continuously differentiable function.

Given (A1 i) and (A2) it immediately follows that:

Proposition 3i) *The optimal deviation is* $1 - \frac{a(\sigma, \delta)}{2}$.

ii) *Further,* $M(s;\sigma,\delta) = \left[h(1) + f(1 - \frac{a(\sigma, \delta)}{2}) - f(\frac{a(\sigma, \delta)}{2}) \right] s.$

(2.9)

Finally, $M(s;.,.)$ is continuous (continuously differentiable on $(0,1]^2$)and increasing in both σ and δ.

It will be convenient to write $M(s;\sigma,\delta) = \mu(\sigma,\delta)s$ in the sequel and from hereon we only look at $\mu(\sigma,\delta)$.

3. Collusion and Impatience

In this section we investigate the connection between collusion and the players' discount factor. To begin with, we explore the question for a fixed degree of intertemporal linkage (for convenience we take $\sigma = 1$). We then use those results to establish some possibilities for all games with sufficient linkage (for all $\sigma > \sigma^*$ where $\sigma^* \in [0,1)$). Under (A1) the first-best is a Nash equilibrium in the stage game (and hence collusion is trivially sustainable in the repeated game). We end this section with a generalization of (A1) which removes that feature and consequently yields further implications for dynamic games.

Consider the exhaustible resource case. Momentarily suppress references to σ and write $a(\delta)$, $\nu(\delta)$ and $\mu(\delta)$. Note, from (2.6) and (2.7) that $a(\delta)$ and hence $\nu(\delta)$ are completely independent of f. Now pick any function μ satisfying $\nu(0) = \mu(0)$, $\mu(\delta) > \nu(\delta)$ iff $\delta > \hat{\delta}$ and $\mu'(\hat{\delta}) > \nu'(\hat{\delta})$. Keeping in mind the fact that $a(\delta)$ is monotone, it is easy to see that there is a continuously differentiable, increasing function f (in fact there are uncountably many) such that $\mu(\delta) = h(1) + f(1-\frac{a(\delta)}{2}) - f(\frac{a(\delta)}{2})$. So such an f satisfies:

(A3) $f(1-\frac{a(\delta)}{2}) - f(\frac{a(\delta)}{2}) \lesseqgtr \nu(\delta) - h(1)$, as $\delta \lesseqgtr \hat{\delta}$, where $\hat{\delta} \in (0, 1)$

Section 5 contains an example which satisfies (A3). Since collusion is sustainable if and only if $\nu(\delta) \geq \mu(\delta)$ (and then it is sustainable after all histories) we have

Proposition 4 *Under (A1)-(A2), and the additional assumption (A3), in the dynamic games $G(1,\delta)$, collusion is sustainable if and only if $\delta \leq \hat{\delta}$.*

In fact the same logic yields: given any $\mu(\cdot)$ which is an increasing, continuously differentiable function, satisfying $\mu(0) = \nu(0)$, there is an associated increasing, continuous f_μ such that it satisfies (2.9). It is not difficult to see, and we show this formally in the appendix, that given any arbitrary closed set $D \subset [0,1]$, there is an increasing, continuous function $\mu(\cdot)$ such that $\nu(\delta) \geq \mu(\delta)$, if and only if $\delta \in D \cup \{0\}$. Furthermore, all these arguments repeat for any $\sigma > 0$. It then follows that we have:

Proposition 5 **For any closed set $D \subset [0,1]$ and $\sigma > 0$, there exists payoffs f, and hence a family of games $G(\sigma,\delta)$, satisfying (A1)-(A2) such that collusion is sustainable if and only if $\delta \in D \cup \{0\}$.**

Proposition 5 is the general impossibility result. Proposition 4 we will further develop to analyze properties of dynamic games, which hold over a range of σ. Note, incidentally, that had we analyzed dynamic games of the renewable resource type, i.e. games in which some states can be maintained indefinitely, then for δ close to one, collusion must appear. This must be so since losing an infinite stream of steady-state consumption is an "infinite" punishment. However, the relationship between collusion and discounting is unlikely to be monotone even in that case. We would conjecture that in the renewable resource framework, there are formulations such that collusion is sustainable either at very low or at very high discount factors but not in between.

We turn now to a simple characterization of the collusion set. This set is defined as $C \equiv \{(\sigma,\delta) \in [0,1]^2: \nu(\sigma,\delta) \geq \mu(\sigma,\delta)\}$. In the light of Proposition 5 above, the reader might expect that the collusion set could have a very arbitrary structure. Although that might well be true, we now present a result which implies that there is a reasonably simple way to describe this set.

<u>Proposition 6</u> *Fix δ and suppose that (A1)-(A2) hold. If collusion is not susainable at $\bar{\sigma}$, then it cannot be sustained at any $\sigma > \bar{\sigma}$. Equivalently, for all δ there is $\sigma(\delta) \in [0,1]$, such that collusion is sustainable iff $\sigma \leq \sigma(\delta)$. Further, $\sigma(\delta)$ is a continuous function.*

Proof: The first part of the result follows immediately from Propositions 2ii) and 3ii). The continuity of the boundary $\sigma(\delta)$ follows from the identical property of the first-best and deviation payoffs. ■

So the collusion set has the following structure: $C = \{(\sigma,\delta): \sigma \leq \sigma(\delta)\}$. Propositions 4 and 6 can be used to establish the second main result of this section:

<u>Proposition 7</u> **Suppose that (A1)-(A3) hold. Then, there is $\sigma^* \varepsilon [0,1)$ such that for any $\sigma > \sigma^*$, collusion is sustainable if and only if players are sufficiently impatient, i.e. there is $\delta(\sigma)$ with $(\sigma,\delta) \in C$ iff $\delta \leq \delta(\sigma)$.**

Proof: Note that from Proposition 6 it is known that $v(\sigma,\delta) \geq \mu(\sigma,\delta)$, for all σ, $\delta \leq \hat{\delta}$. In the light of this, a contradiction to the proposition implies the existence of sequences $\{(\sigma_n,\delta_n,\delta_n'\}$, $\sigma_n \uparrow 1$, $\delta_n < \delta_n'$ and $v(\sigma_n,\delta_n) = \mu(\sigma_n,\delta_n)$, $v(\sigma_n,\delta_n') = \mu(\sigma_n,\delta_n')$. From (A3) it is immediate that $\delta_n \to 1$ and $\delta_n' \to 1$.

Since $\mu_2(1,\hat{\delta}) > v_2(1,\hat{\delta})$, there is a neighborhood $N(1,\hat{\delta})$ and $\varepsilon > 0$ s.t. $\mu_2(\sigma,\delta) > \mu_2(1,\hat{\delta}) - \varepsilon > v_2(1,\hat{\delta}) + \varepsilon > v_2(\sigma,\delta)$, for all $\sigma,\delta \in N(1,\hat{\delta})$. For large n, σ_n,δ_n and σ_n,δ_n' are in $N(1,\hat{\delta})$. But then,

$$\begin{aligned} v(\sigma_n,\delta_n') - v(\sigma_n,\delta_n) &< [v_2(1,\hat{\delta}) + \varepsilon]\ (\delta_n'-\delta_n) \\ &< [\mu_2(1,\hat{\delta}) - \varepsilon]\ (\delta_n'-\delta_n) \\ &< \mu(\sigma_n,\delta_n') - \mu(\sigma_n,\delta_n) \end{aligned}$$

The last inequality yields the desired contradiction.■

<u>Remark</u> Section 5 contains an example of an explicitly computable collusion set.

In the model discussed thus far collusion is sustainable in the purely repeated game for all discount factors. This followed from the simplifying assumption that h is strictly increasing on [0,1] as a conseqence of which $a(0,\delta) \equiv 1$ for all δ. Hence, the first-best is a Nash equilibrium in the stage game. We now dispense with this assumption. Suppose we replace (A1) with:

(A1'i) h is concave, twice continuously differentiable and increasing on (0,k) where k $\in$ (0,1) and constant thereafter.

ii) $h(0) < 0$ and $\lim_{a \uparrow k} \left[\frac{h(a)}{h'(a)} - a \right] = \infty$.

It is easy to see that $a(0,\delta) = k$ and further that this is not a Nash equilibrium in the one-shot game. Hence, in the purely repeated game we have the standard feature that collusion is sustainable if and only if players are sufficiently patient. From Proposition 6 it then follows that collusion is not sustainable for impatient players no matter what the level of intertemporal linkage is. We give conditions now under which the added dynamic game effect, that more patient cohorts are more profitable to deviate against, implies that players deviate if and only they are <u>either</u> very impatient or very patient.

We need an appropriate modification of (A3). As before, consider first the exhaustible resource case and denote the first-best and deviation payoffs $\nu(\delta)$ and $\mu(\delta)$ respectively. A consequence of (A1') is that $\nu(0) < \mu(0)$ no matter what specification of f we choose. Fix $0 < \tilde{\delta} < \hat{\delta} < 1$. Propositions 2 and 3 still hold. So choose f s.t.:

(A3') $$f(1- \frac{a(\delta)}{2}) - f(\frac{a(\delta)}{2}) < \nu(\delta) - h(1), \text{ iff } \delta \in (\tilde{\delta},\hat{\delta})$$

<u>Proposition 8</u> **Suppose (A1'),(A2) and (A3'). Then there is $\sigma^* \in [0,1)$, s.t. for all $\sigma > \sigma^*$, collusion is sustainable if players are neither very impatient nor very patient; i.e. there exist $0 < \delta_1(\sigma) < \delta_2(\sigma) < 1$, with collusion sustainable iff $\delta \in [\delta_1(\sigma),\delta_2(\sigma)]$. The purely repeated game exhibits collusion only if players are sufficiently patient.**

Proof: From (A3') the posited behavior is immediate for $\sigma = 1$. Proposition 6 and a modified mimic of the proof of Proposition 7 (applying arguments both to the left of $\tilde{\delta}$ as well as to the right of $\hat{\delta}$), then yields the current proposition. ■

A brief discussion of possible generalizations of the transition and reward functions are in order. As they stand, the formulations (2.1)-(2.2) have three convenient features: linearity in the state, additive separability in actions and a discontinuity at $a_1+a_2 = 1$. As explained above, state linearity is a desired characteristic since it renders collusion prospects identical from all states. The following modification of (2.1) and (2.2) shows that the discontinuity is inessential to the argument. Take $g(s,a_1,a_2) = [1 - \sigma]s$, whenever $a_1+a_2 > 1$ (and defined as in (2.1) for $a_1+a_2 \leq 1$). Let $r_i(s,a_i,a_j) = [h(a_i+a_j)+ f(a_i)-$

$f(a_j)$]s, for all a_i,a_j. Evidently, (under (A1) and (A2)), the transition and reward functions are continuous. Add to (A1) the following: h decreases on (1,2], h(2)=0 and $h(a+1) + f(a) - f(1) \leq 0 \leq h(a+1) + f(1) - f(a)$. It is immediate that Propositions 1 and 2 are completely unchanged (indeed the first-best actions remain exactly the same). The optimal deviation need not be $1 - a(\sigma,\delta)/2$, but is still determined by the trivial one-period optimization exercise, (2.8). With added complexity in this last step, we can now find f to further satisfy (A3).

Finally, to generalize the additive separability feature, we could take $g(s,a_1,a_2) = \sigma\gamma(a_1,a_2)s$ and $r_i(s,a_1,a_2) = \rho(a_1,a_2)s$, where γ and ρ are continuous functions with appropriate monotonicity and boundary properties. Again, increasing patience would engender an increase in both first-best payoffs $v(\delta)$ and (if $a(\delta)$ declines in δ) deviation payoffs, $\mu(\delta)$. Analytical complexity would arise from the fact that first-best actions, optimal deviations and the associated payoffs would have to be determined simultaneously. Additive separability allowed us to treat them sequentially and demonstrate from elementary considerations that μ might rise faster than v. We do believe, however, that Propositions 4-8 can be proved within this class of transitions and reward functions as well.

4. Repeated and Dynamic Games: A Comparative Analysis

Two questions are simultaneously addressed in this section: i) can one get an estimate of how large might the set of games be whose behavioral implications differ from repeated games? ii) must games "near" repeated games atleast have similar properties? For this section, we revert to assumption (A1) (together with (A2) of course). Hence, the repeated game is characterized by the fact that collusion is sustainable at all δ. Consider the following assumption:

(A4i) There is $H > 0$ such that $h'(a) \leq H(1-a)$, $a \in [0,1)$

(A4ii) There is $F > 0$ such that $f(1-(a/2)) - f(a/2) \geq F(1-a)$, $a \in [0,1]$

Remark $h(a) = a - 1/2\, a^2 - b$ and $f(a) = a$, satisfy (A4).

Proposition 9 **Suppose that f and h satisfy (A1, A2 and A4) . Pick any $\varepsilon > 0$. Then, there is a family of games $G(\sigma,\delta;\varepsilon)$ such that $\lambda(C) \leq \varepsilon$, where λ is the Lebesgue measure on the parameter space and C is the collusion set. Moreover, $(\sigma,\delta) \in C$, whenever $\sigma = 0$. In other words, the set of dynamic games for which collusion is sustainable can be made arbitrarily small, although this set always includes the repeated game.**

Proof: Consider a family of games $G_n(\sigma,\delta)$ whose one-period rewards are determined by $r_i = h + n[f(a_i)-f(a_j)]$, where n is some positive integar. Note that the first-best actions are independent of n. The boundary between collusion and non-collusion, the function $\sigma(\delta)$, comes from

$$\frac{h(a(\sigma,\delta)}{1- \delta(1 - \sigma a(\sigma,\delta)} = h(1) + n[f(1- \frac{a(\sigma,\delta)}{2}) - f(\frac{a(\sigma,\delta)}{2})] \tag{4.1}$$

It is straightforward to check that substituting the first-order condition (2.6) into (4.1) yields (for $\sigma,\delta > 0$)

$$\frac{h'(a(\sigma,\delta))}{\sigma\ \delta} = h(1) + n[f(1- \frac{a(\sigma,\delta)}{2}) - f(\frac{a(\sigma,\delta)}{2})] \tag{4.2}$$

Substituting (A4) into (4.2) yields

$$[1-a(\sigma,\delta)][\ \frac{H}{\sigma\delta} - nF] - h(1) \geq 0 \tag{4.3}$$

From (4.3) it is immediate that $\sigma(\delta)\delta < c/n$, for some constant $c > 0$. Hence, choosing n large enough makes the collusion region arbitrarily small. Of course, $a(0,\delta) \equiv 1$ and so collusion is sustainable in the repeated game for all discount factors.■

Remark A similar argument would work even if collusion is not always sustainable in the repeated game. In the n-th variation, modify f in such a way that its value is unchanged at the first-best action, k/2. Clearly, the collusion prospects in the repeated game are then independent of n. By an appropriate choice of f at other points in the domain, which are the dynamic games' first-best, estimates as (4.3) can be established.

Propositions 7-9 have some simple implications for folk theorem analysis in $G(\sigma,\delta)$. Consider any $\sigma > 0$. If the payoffs in the undiscounted game are calculated according to long-run average, then the only feasible payoff is clearly zero. This payoff is also an equilibrium payoff and hence, in a trivial sense, the undiscounted folk theorem holds. However, if undiscounted payoffs are evaluated according to the overtaking criterion, arguably the more relevant criterion in exhaustible-resource type problems, non-sustainability of the first-best at $\delta = 1$ immediately implies that there is there is no undiscounted folk theorem in the overtaking criterion (unlike the case of repeated games for which Rubinstein (1979) establishes just such a result). Moreover, Proposition 8 says that dynamic games for which the folk theorem breaks down, exist in arbitrarily small neighborhoods of repeated games. Of course, the folk theorem implications do rely critically on the exhaustible resource nature of the problem, as a consequence of which infinite punishments cannot be imposed near $\delta = 1$.

5. An Illustrative Example

In this section we illustrate Propositions 7-9 by way of computing the collusion set in an example. There will be a central example whose data will be varied in the course of this section in order to bring out different possibilities. Consider :

$$h(a) = a - 1/2\, a^2 - 1/4 \tag{5.1}$$

It is straightforward to check that (5.1) satisfies assumption (A1). Write $\Psi(\sigma,\delta) \equiv (1-\delta)/\sigma\delta$. The first-order characterization (2.6) yields, for $\sigma,\delta \neq 0$

$$a(\sigma,\delta) = - \Psi(\sigma,\delta) + \left[\Psi(\sigma,\delta)^2 + 2\, \Psi(\sigma,\delta) + 1/2 \right]^{1/2} \tag{5.2}$$

Whenever σ or δ equals zero, $a(\sigma,\delta) = 1$. The arguments leading upto (4.2) give:

$$v(\sigma,\delta) = \frac{1 - a(\sigma,\delta)}{\sigma\delta}, \quad \sigma,\delta \neq 0 \tag{5.3}$$

At $\sigma = 0$ or $\delta = 0$, $v(\sigma,\delta) = h(1)/[\, 1 - \delta(1 - \sigma)]$. Write I(B) for the indicator function on a set B. Suppose now that we specify the following functional form for f:

$$f(x) = [25(2x-1)^2 - 1/4]\ I(x > 11/20) \tag{5.4}$$

It is easy to check that f satisfies assumption (A2). The optimal deviation is $1\text{-}a(\sigma,\delta)/2$. Substituting this into (5.4) yields (writing $B=\{(\sigma,\delta):\ 1\text{-}a(\sigma,\delta)/2 \geq 11/20\}$):

$$\mu(\sigma,\delta) = [25(1-a(\sigma,\delta))^2]\ I(B) + h(1)\ I(B^c) \tag{5.5}$$

The boundary of the collusion region, $\sigma(\delta)$ can now be solved from $v(\sigma,\delta) = \mu(\sigma,\delta)$.[10] Some tedious algebra, after substituting (5.2) into (5.3) and (5.5) yields[11]:

$$\sigma(\delta) = 0.08\, \delta^{-1} + [\, 0.1632\, \delta^{-2} - 0.16\, \delta^{-1}\,]^{1/2} \tag{5.6}$$

The implied collusion set for this example, an illustration of Propositions 4 and 7, is pictured in Figure 1. We now give variations of this example to illustrate the further conclusions of Propositions 8 and 9. For any $k \in [0,1]$, consider the following variant of the one-period reward h:

$$h_k(a) = h(a)\ I(a \leq k) + h(k)\ I(a > k) \tag{5.7}$$

We now have a function that that satisfies (A1').[12] By standard arguments it can be shown that the first-best action and payoffs associated with these returns is given by (writing $E = \{(\sigma,\delta): \Psi(\sigma,\delta) \leq [h(k)/h'(k)] - k\}$):

$$a_k = a(\sigma,\delta)\ I(E) \quad + k\ I(E^c) \tag{5.8}$$

$$v_k = v(\sigma,\delta)\ I(E) \quad + h(k)/[1-\delta(1-\sigma k)]\ I(E^c) \tag{5.9}$$

On the other hand,[13]

$$\mu_k = 25\ [1 - a_k(\sigma,\delta)]^2 \tag{5.10}$$

The equation $v_k = \mu_k$ is solved in two parts. On E, the solution corresponds to (5.6). On E^c, we solve: $h(k)/[1-\delta(1-\sigma k)] = 25[1-k]^2$, and that solution is given by:

$$\Sigma(\delta) = \frac{51k - 25\cdot 5k^2 - 25\cdot 25}{25k(1 - k^2)}\ \delta^{-1} + \frac{1}{k} \tag{5.11}$$

The collusion set can be described as:$C = \{(\sigma,\delta)\in [0,1]^2: \sigma \leq \min\ [\sigma(\delta),\Sigma(\delta)]\}$. Figure 2 illustrates Proposition 8 for k = 0.9. The conventional effect determines the north-west collusion region whereas the dynamic game effect determines the north-east region of deviation. Figures 3a) and 3b) report the results for k = 0.895 and k = 0.85 respectively (as the conventional effect becomes stronger, k decreases, the size of non-collusion on this count increases).

Finally, we turn to an illustration of Proposition 9. We revert to k = 1, so h is as defined in (5.1). We vary the payoff f as follows (writing $F=\{x:x>1/2+1/(20\sqrt{n})\}$:

$$f_n\ (x) = [n25(2x-1)^2 - 1/4]\ I(F) \tag{5.12}$$

The first-best actions (5.2) and associated payoffs (5.3) are completely unchanged. The deviation payoffs are now (writing $G = \{(\sigma,\delta): 1-a(\sigma,\delta)/2 \geq 1/2+1/(20\sqrt{n})\}$:

$$\mu_n = 25n[1-a(\sigma,\delta)]^2\ I(G) \quad + \quad h(1)\ I(G^c) \tag{5.13}$$

From (5.3) and (5.13) it follows that the collusion boundary is:

$$\sigma_n = 0.08(n\delta)^{-1} + [(0.0032\ n^{-2} + 0.16\ n^{-1})\delta^{-2} - 0.16\ n^{-1}\ \delta^{-1}]^{1/2} \tag{5.14}$$

Decreasing (increasing) n makes collusion more (less) sustainable. We illustrate Proposition 9 in Figures 4a) and 4b) which correspond to n = 0.4 and n = 400 respectively. Of course, collusion is sustainable in the repeated game for all δ.

6. Monotonicity of the Set of Equilibrium Outcomes

Thus far,we have identified collusion with first-best sustainability. This is a conceptually and computationally simple notion (the symmetric first-best in a symmetric game is both easy to calculate and a natural focal point); but it can be supplemented in one way. In games in which the first-best is not sustainable, we can still talk about the extent of collusiveness by way of the second-best, or best equilibrium outcome. In this section we show that greater patience is in fact more conducive to collusion in this alternative sense. In particular, we show that any outcome which arises in equilibrium for a discount factor δ, also arise for any higher discount factor. We also investigate the closely related questions of the monotonicity of the second-best discounted (and second-best discounted average) payoffs.

The analysis in this section is actually carried out for a wider class of dynamic games than just $G(\sigma,\delta)$. Indeed consider any dynamic game $G(\delta)$ which satisfies:

(A5) i) The one-period payoffs, r_i are bounded.

ii) The worst equilibrium payoffs are zero, for all initial states and discount factors.

Recall that $\phi(s;\delta)$ denotes the set of equilibrium payoffs for initial state s and discount factor δ. Define the second-best discounted and discounted average payoffs (respectively) as:

$$W_i(s;\delta) = \text{Sup } \{w_i: (w_i,w_j) \ \varepsilon \ \phi(s;\delta)\} \tag{6.1}$$

$$\bar{W}_i(s;\delta) = (1 - \delta) \ W_i(s;\delta) \tag{6.2}$$

Proposition 9 **Suppose (A5) holds. Consider dynamic games $G(\delta_1)$ and $G(\delta_2)$, $\delta_1 < \delta_2 < 1$. Consider an equilibrium strategy pair π_i, i=1,2 for δ_1 and initial state s with associated outcome ω. Then, there is an equilibrium strategy pair for the game $G(\delta_2)$, from initial state s, with the same outcome ω. Further, $W_i(s;\delta_1) \leq W_i(s;\delta_2)$.**

Proof: Some additional notation would be useful at this point. We shall investigate the following strategy for δ_2: play π_i till either player defects. Then switch to the worst equilibrium. Let h_t denote a partial history along the equilibrium path ω and let the associated continuation payoffs be denoted $P_i(h_t;\delta)$. Denote the t+τ - th period one-shot payoffs $r_{it+\tau}(h_t)$ and hence

$$P_i(h_t;\delta) = \sum_{\tau=0}^{\infty} \delta^\tau r_{it+\tau}(h_t) \tag{6.3}$$

For the rest of the proof, we will simplify notation by suppressing the player index i and history h_t. Further, we normalize the time index and set t = 0. (6.3) is hence written as $P(\delta) = \sum_{\tau=0}^{\infty} \delta^\tau r_\tau$. Finally, let $M_i(h_t;\delta)$ be the payoffs to optimal deviation when the continuation strategies are to go to the worst equilibria: (and from hereon, this will be referred to as $M(\delta)$).

Lemma 9.1 *Continuation payoffs are increasing in the discount factor: $P(\delta_2) \geq P(\delta_1)$.*

Proof: Fix $\varepsilon > 0$ and suppose that T satisfies $\delta_2^T \left[\frac{r^*}{1-\delta_1} - \frac{r_*}{1-\delta_2}\right] < \varepsilon$, where $r^* \equiv \sup r_i$ and $r_* \equiv \inf r_i$ (and these are finite, given (A5i)). Note that

$$r_{T-1} + \delta_2 P(h_T;\delta_1) \geq r_{T-1} + \delta_1 P(h_T;\delta_1) \geq 0 \tag{6.4}$$

Since $P(h_T;\delta_1)$ is non-negative (by (A5ii)), the first inequality in (6.4) holds. The second follows since the nested expression in (6.1) is precisely $P(h_{T-1};\delta_1)$. (6.4) yields

$$r_{T-2} + \delta_2 r_{T-1} + \delta_2^2 P(h_T;\delta_1) \geq r_{T-2} + \delta_1 r_{T-1} + \delta_1^2 P(h_T;\delta_1) \tag{6.5}$$

Iterating T times we get

$$\sum_{\tau=0}^{T-1} \delta_2^\tau r_\tau + \delta_2^T P(h_T;\delta_1) \geq P(\delta_1) \tag{6.6}$$

However,

$$P(\delta_2) = \sum_{\tau=0}^{T-1} \delta_2^\tau r_\tau + \delta_2^T P(h_T;\delta_1) + \delta_2^T [P(h_T;\delta_2) - P(h_T;\delta_1)]$$

$$> P(\delta_1) - \varepsilon \tag{6.7}$$

(6.7) follows from the properties of T and (6.6). Since (6.7) holds for all $\varepsilon > 0$, the lemma is proved. ■

The deviation payoffs $M(\delta)$ are independent of the discount factor, by (A5ii). The proposition is immediate. ■

Remark An immediate corollary is that for fixed σ, the set of equilibrium outcomes and second-best discounted payoffs are monotonically increasing in the discount factor, in the games $G(\sigma,\delta)$ of Sections 2-5. Note however, that second-best discounted average payoffs in those games need not be monotonically increasing in δ. To see this note that $W(s;\sigma,1)$, which is just the second-best long-run average returns, is zero whenever $\sigma > 0$. On the other hand, second-best discounted average returns are clearly strictly positive, for $\delta < 1$, whenever the first-best is sustainable.

7. Two Economic Models

In this section we describe two economic models in which the transition function (2.1) and the immediate rewards (2.2) arise from primitives. The discussion has the limited purpose of suggesting that our specifications are not wholly unrealistic. In particular, we do not intend to present a complete model for the economic phenomena. For simplicity we only discuss the case $\sigma = 1$.

Exhaustible Resource Suppose two firms operate in a market with a perfectly elastic demand curve at price p. The firms independently (and simultaneously) determine the fraction (a_i) of the jointly owned available resource (s) to be extracted. The cost of extraction is a variable cost which depend on ($a_1 + a_2$) and s; say $c(a_1+a_2)^2 s + bs$. Then,

$$\begin{aligned} \text{Profits} &= pa_i s - c(a_i+a_j)^2 s - bs \\ &= [\frac{p}{2}(a_i+a_j) - c(a_i+a_j)^2 - b]s + [\frac{p}{2} a_i - \frac{p}{2} a_j]s \end{aligned} \tag{7.1}$$

Taking $h(a_i+a_j) \equiv \frac{p}{2}(a_i+a_j) - c(a_i+a_j)^2 - b$ and $f(a_i) \equiv \frac{p}{2} a_i$, we have (2.2). Of course extraction of $a_i s$ leaves $(1 - a_1 - a_2)s$ next period.

Consumer Durables This formulation is very similar to the exhaustible resource problem. Consider two firms selling a consumer durable to a fixed market of size x; a_i is the fraction of the market that is serviced. Assuming that all consumers have an identical reservation price and that costs of production are interdependent, we can recover a formulation like (7.1).

8. Conclusions

It is widely believed that tacit collusion is more likely when economic agents are patient. We showed in this paper that this intuition is somewhat specific to purely repeated games. When the game environment is endogeneous, as in dynamic games, the collusive action itself depends on the discount factor. Consequently, deviation possibilities are different at each discount factor and deviation payoffs are often higher if all players are more patient. Of course a patient player is also hurt more by furture punishments (as in the case of repeated games). Hence, an increase in player patience has to compare these two competing effects. There is no general identifiable link between collusion and discounting; in fact the outcome may well be that more patience is detrimental to the sustainability of collusion. Moreover, such contrary behavior is possible in dynamic games close to the purely repeated one. We showed that equilibrium behavior, however, is monotone in the discount factor. An implication of our results is that folk theorems may not hold for games that are close to repeated games (although this last conclusion is predicated on the exhaustible resource nature of the dynamic games we studied[14]).

There were two principal restrictions placed on the family of dynamic games we studied. The worst equilibrium payoffs after all histories, and for all discount factors, was taken to be zero. Secondly, only games whose data are linear in the state variable were analysed. The first restriction considerably facilitated the study of optimal deviations. The second ensured that all statements regarding collusion held over the entire state space. We believe that both restrictions, though analytically important, are inessential to the general conclusions of the paper. Indeed, many of the conclusions would hold if the worst equilibrium payoffs were known to decrease in the discount factor (which is the case in repeated games). A third restriction, that of symmetry, is easily seen to be inessential.

Footnotes

1 In all of what follows we consider symmetric games. So the first (and second) best are shorthand for the symmetric first (and second) best.

2 In this discussion, and indeed throughout the paper, we normalize the payoffs to the worst equilibrium to zero, and this is the continuation after a deviation. The conclusion, that in repeated games collusion prospects improve with the discount factor, is true even without this simplification. By Abreu, Pearce and Stachetti (1990), it is known that the worst equilibrium discounted average payoff is decreasing in the discount factor. Hence, the costs to deviation (the first-best less the worst equilibrium payoffs) are increasing in player patience.

3 Introduced first by Shapley (1953), they have been widely studied since; see Fudenberg and Tirole (1991) for a further discussion and references.

4 Recent applications of dynamic games to economic problems include Stokey (1990) (who analyzes public policy issues), Maskin and Tirole (1988) (who study dynamic price competition in oligopoly) and Benhabib and Radner (1988) (who model oligopoly extraction of a common property resource). Other applications are discussed in Fudenberg and Tirole (1991).

5 For instance, see Becker (1985).

6 For instance, see Dutta and Sundaram (1989).

7 The payoffs are well-defined for all $(\sigma, \delta) \varepsilon [0, 1]^2$, except possibly for the case $\sigma = 0$, $\delta = 1$. We ignore this case momentarily.

8 Strictly speaking, on account of the discontinuity of g, we have a repeated game with endogeneous termination (if $a_1 + a_2 > 1$). Qualitatively similar conclusions follow in a model where the transition function is continuous (and hence, $\sigma = 0$ is indeed a purely repeated game) - see the discussion at the end of Section 3.

9 We use the term increasing as a synonym for what is also called non-decreasing (and distinguish this from strictly increasing) - the definitions being obvious are not repeated here.

10 Note that $v(\sigma,\delta) > h(1)$ (since a=1 in the first period followed by a>1 in a subsequent period is a feasible, and strictly inoptimal policy). Hence, $\sigma(\delta)$ can be simply solved from $1\text{-}a(\sigma,\delta)/\sigma\delta = 25(1\text{-}a(\sigma,\delta))^2$.

11 (5.6) is the positive root that arises from solving a quadratic equation. That this is the relevant root follows from the fact that, in this example, $\sigma(\delta)$ has to be downward sloping. In turn, this is immediate given that we have $25\sigma(\delta)\delta[1\text{-}a(\sigma(\delta),\delta)]=1$ (recall that $a(\sigma,\delta)$ monotonically declines in its arguments).

12 Strictly speaking (A1') is not satisfied since the left limit, at k, of h(a)/h'(a) - a is finite. The reader is invited to check that with first-best defined as in (5.8), Proposition 7 goes through.

13 Notice that whenever $k \leq 0.9$, $1 - a(\sigma,\delta)/2 \geq 0.55$, for all σ,δ and hence $\mu(\sigma,\delta) = 25[1-a(\sigma,\delta)]^2$ everywhere.

14 Indeed Dutta (1990) shows that for wide classes of stochastic games , a folk theorem similar to that of Fudenberg and Maskin (1986) for repeated games, obtains.

Appendix

In this appendix we prove

Proposition A: Fix $\sigma^* > 0$ and a closed set $D \subset [0,1]$. Under (A1), there is an increasing continuously differentiable function μ such that $v(\sigma^*,\delta) \geq \mu(\sigma^*,\delta)$ iff $\delta \in D \cup \{0\}$.

Proof: For convenience, we suppress σ^* wherever possible. Note first that there is $\varphi>0$ such that $v(\delta) - v(\delta') > \varphi(\delta-\delta')$, whenever $\delta>\delta'$. To see this notice that $a(\delta')\equiv a'$ is a feasible action for δ. Hence,

$$v(\delta) - v(\delta') \geq \frac{h(a')}{1-\delta(1-\sigma^*a')} - \frac{h(a')}{1-\delta'(1-\sigma^*a')}$$

$$= \frac{h(a')(1-\sigma^*a')}{[1-\delta(1-\sigma^*a')][1-\delta'(1-\sigma^*a')]} (\delta - \delta')$$

$$\equiv \varphi (\delta - \delta') \tag{A.1}$$

Since $h(a(\delta))(1-\sigma^*a(\delta)) \geq h(a(1))(1-\sigma^*)$, it follows that $\varphi > 0$. Now define

$$\tilde{\mu}(\delta) = v(\delta) + \varphi\rho(\delta,D) \tag{A.2}$$

where $\rho(\delta,D) = \min|\delta - d|$, $d\in D$. Clearly, the function so defined equals v on D and is strictly greater otherwise. Since $|\rho(\delta,D) - \rho(\delta',D)| \leq |\delta - \delta'|$, it is an increasing function. As the sum of two continuous functions, it is clearly continuous but possibly not continuously differentiable. But then we could take any continuously differentiable approximation of ρ, say $\hat{\rho}$, satisfying $\hat{\rho}' \leq 1$, $\hat{\rho} \leq \rho$ and $\hat{\rho} > 0$ if $\rho > 0$. The construction of (A.2) then yields the desired conclusion. ■

References

Abreu, D., 1988, On the Theory of Infinitely Repeated Games with Discounting, Econometrica 56, 383-396.

Abreu, D., D. Pearce and E. Stachetti, 1990, Toward a Theory of Discounted Repeated Games with Imperfect Monitoring, Econometrica 58, 1041 - 1063.

Becker, R., 1985, Comparative Dynamics in Aggregate Models of Optimal Capital Accumulation, Quarterly Journal of Economics 100, 1235 - 1256.

Benhabib, J. and R. Radner, 1988, Joint Exploitation of a Productive Asset: A Game-Theoretic Approach, mimeo, New York University.

Dutta, P.K. and R. Sundaram, 1989, The Tragedy of the Commons? A Complete Characterization of Stationary Equilibria in Dynamic Resource Games, mimeo, University of Rochester.

Dutta, P.K., 1990, A Folk Theorem for Stochastic Games, mimeo, Columbia University.

Fudenberg, D. and E. Maskin, 1986, The Folk Theorem in Repeated Games with Discounting or Incomplete Information, Econometrica 54, 533-554.

Fudenberg, D. and J. Tirole, 1991, A Course in Game Theory, MIT Press (forthcoming).

Maskin, E. and J. Tirole, 1988, A Theory of Dynamic Oligopoly II: Price Competition, Kinked Demand Curves and Edgeworth Cycles, Econometrica 56.

Rubinstein, A., 1979, Equilibrium in Supergames with the Overtaking Criterion, Journal of Economic Theory, 31, 227 - 250.

Shapley, L., 1953, Stochastic Games, Proceedings of the National Academy of Sciences.

Stokey, N., 1990, Credible Public Policy, mimeo, University of Chicago.

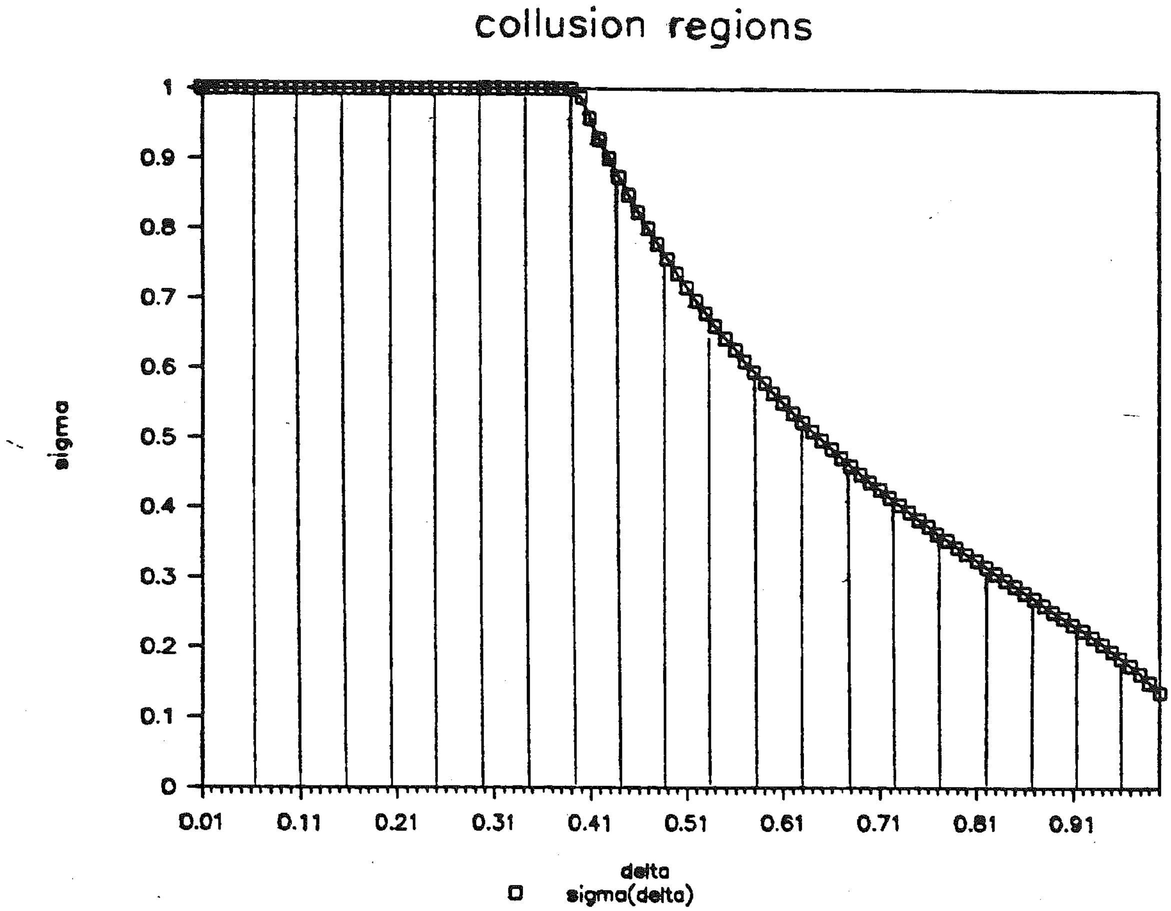

Figure 1

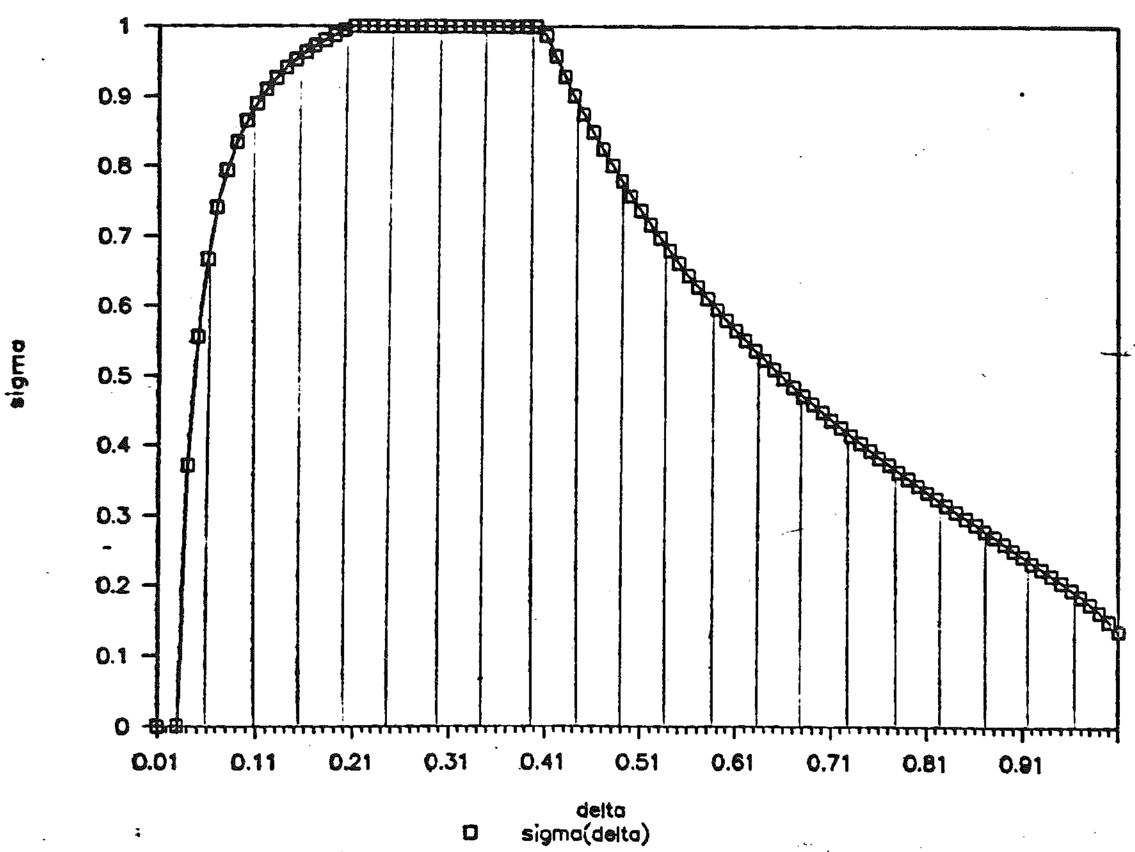

Figure 2: k = 0.9

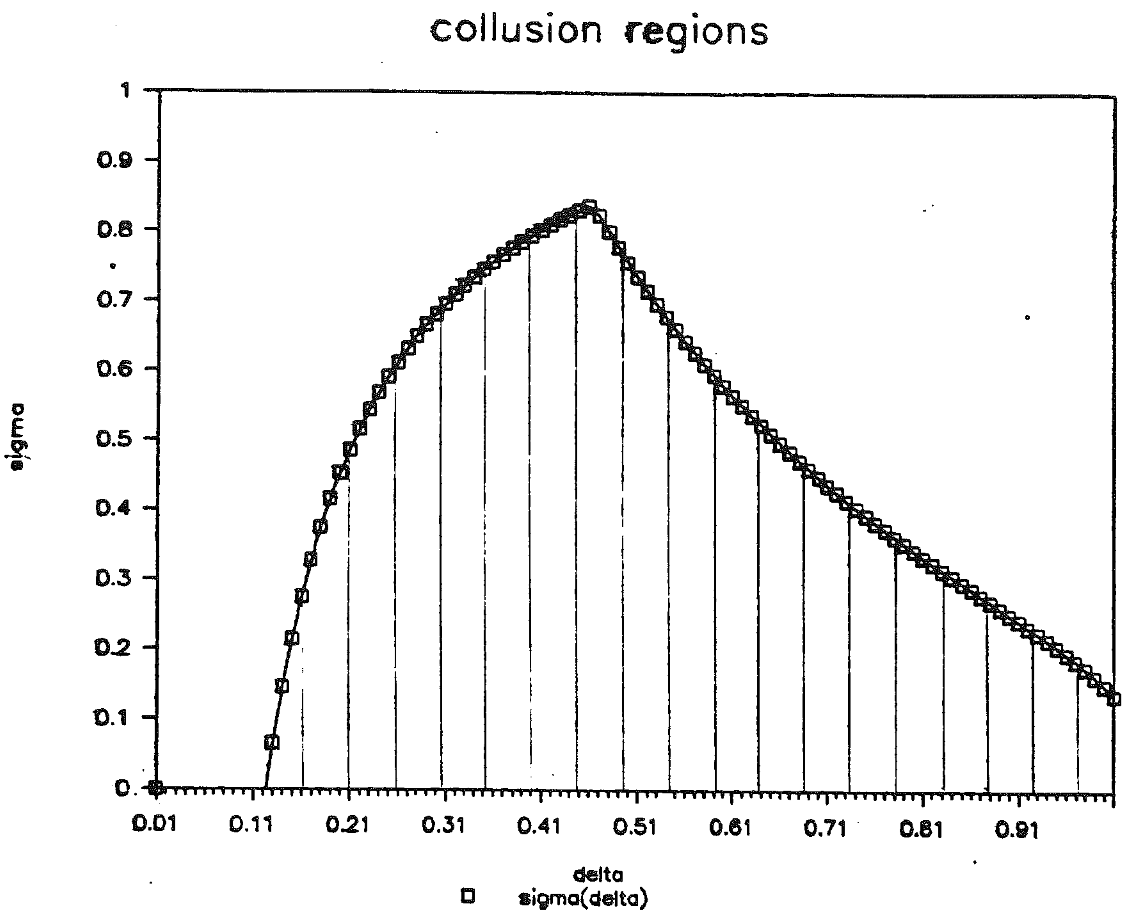

Figure 3a: k = 0.895

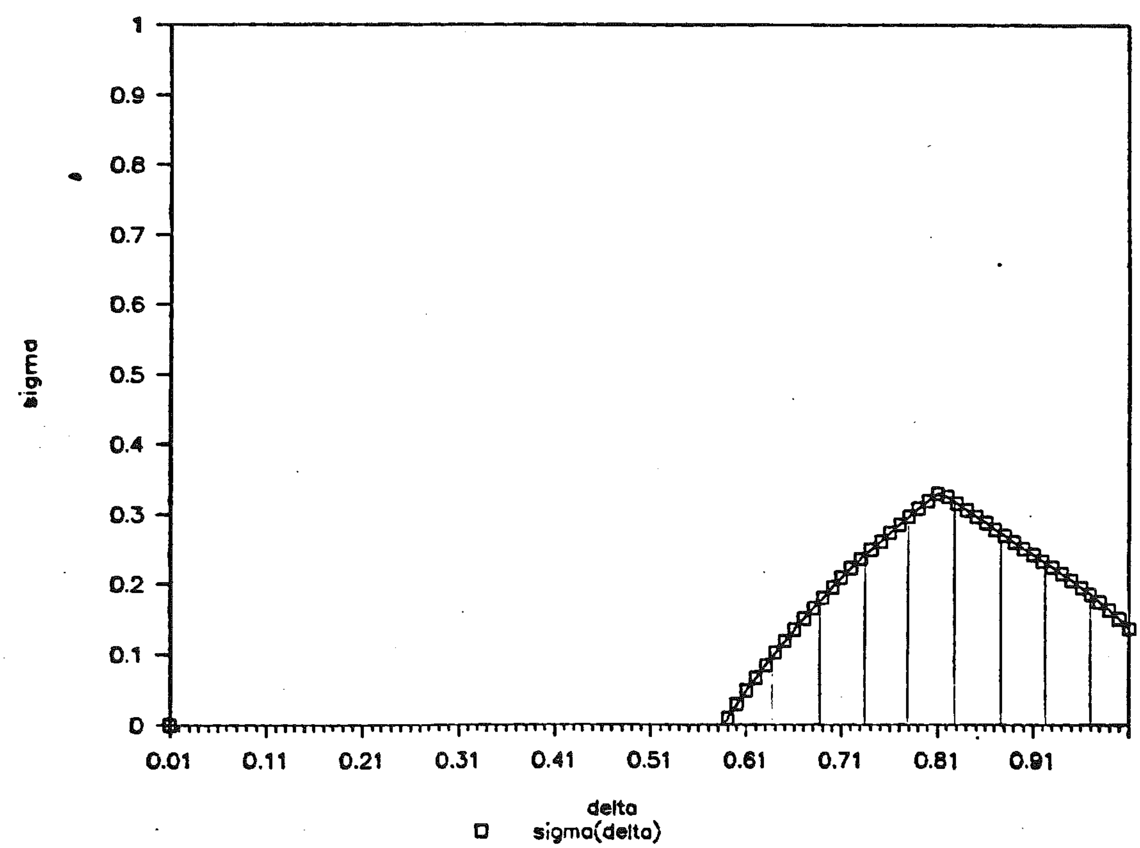

Figure 3b: k = 0.85

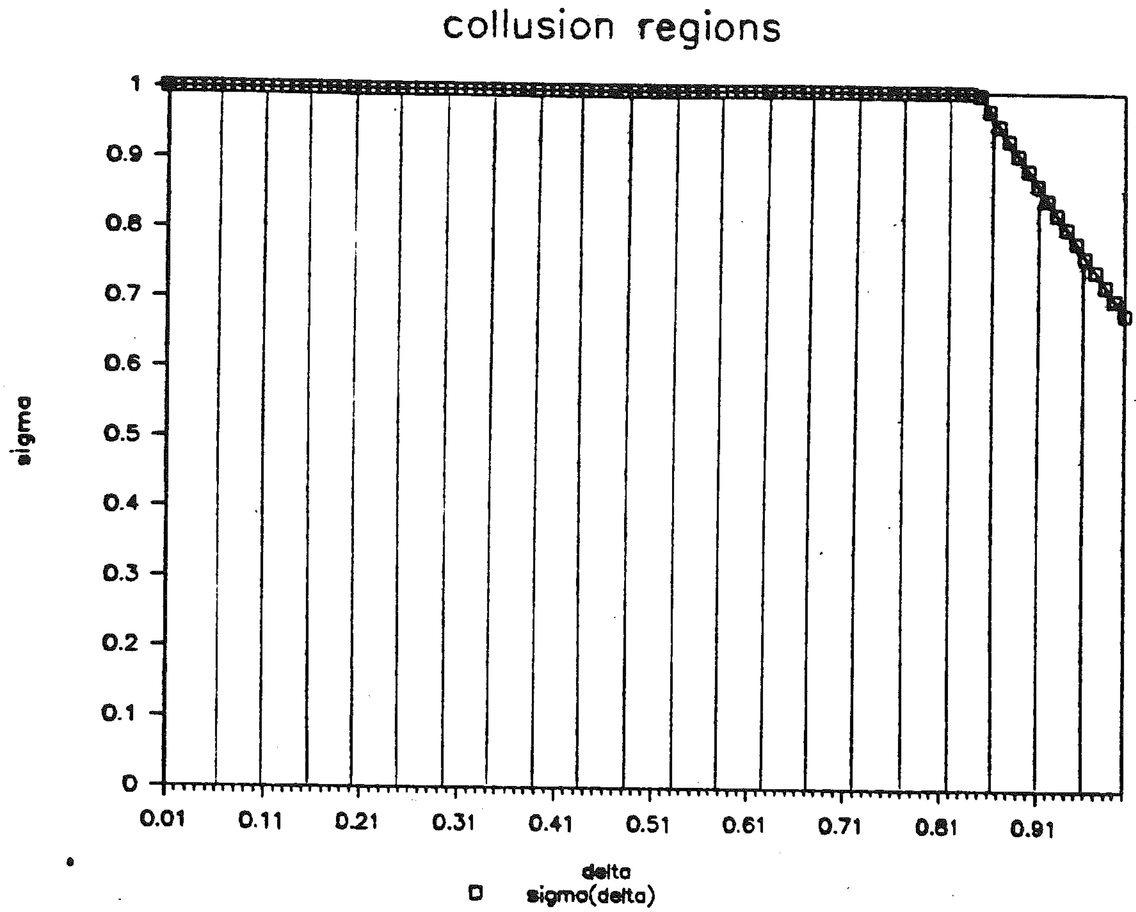

Figure 4a: n = 0.4

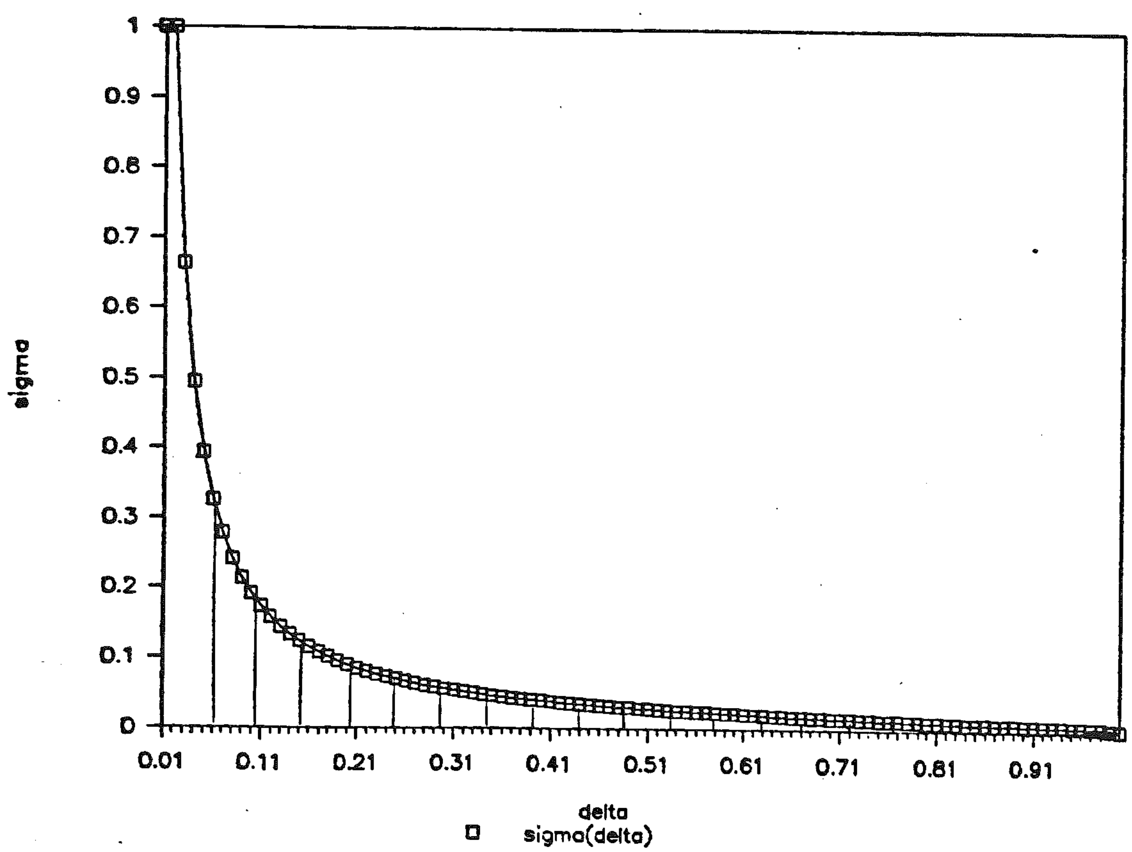

Figure 4b: n = 400