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INDIVISIBLE LABOR, EXPERIENCE AND INTERTEMPORAL ALLOCATIONS

by

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Abstract

In this paper we show how the introduction of technological restrictions in a general equilibrium model of the labor market can produce dynamic patterns of employment and unemployment which replicate essential features of the empirical evidence. We demonstrate that in equilibrium, the probability of being employed in the future depends on past employment record. Moreover, it will be shown that, when dynamic considerations are taken into account, there is a sense in which, in equilibrium, unemployed individuals are worse off than employed individuals.

I come from a dizzy land where the lottery is the basis of reality.

Jorge Luis Borges, The Lottery in Babylon

Starting with the work of Keynes [1936], there has been a long tradition of interpreting unemployment as the consequence of some sort of nominal rigidity, or stickiness. This paper studies unemployment as the consequence of real rigidities, in particular, a rigidity in the quantity of labor supplied at the individual level.

Following work by Rogerson (1984) we assume that labor is indivisible. In this environment, the social optimum will be such that only a fraction of the labor force actually works. In order to determine who is employed and who is not, we introduce in the consumption set of the individuals a lottery which randomly separates the labor force into the two groups. Besides being a convenient theoretical device which greatly simplifies the computation of the equilibrium, these kinds of lotteries are also optimal in the sense that they dominate, in expected utility terms, any other arrangement in which they are not present. The use of the convexification property of lotteries is not a new idea in the economic literature; Prescott and Townsend [1984a], [1984b], represent a notable previous example of theoretical application of the lottery to economic environments.

Even if individuals are born identical in all regards, through the lottery they become different with respect to the use of their time endowment, i.e. some people use it in production activities (employed), and some use it in leisure (unemployed). This kind of static heterogeneity is also obtained in Rogerson (1985) and Hansen (1985). However, in Hansen's paper, which differently from Rogerson's develops an intertemporal model, this heterogeneity is of no consequences for the evolution of the system over time. Even if individuals are different at the end of each period, they turn out to be identical when entering the next period, i.e. the next lottery. The individual's employment record does not have any effect on the probability of being employed in the future. These two papers also share the unpleasant feature that individuals are always better-off if they are unemployed. In fact, an individual that is selected and receives a job, in addition to not enjoying leisure, is taxed in order to support the jobless through a perfect unemployment insurance scheme.

This paper partially eliminates this shortcoming by incorporating intertemporal gains of working. This is obtained by introducing the concept of experience. By working, an individual's working skills increase. Therefore, in entering future periods, previously employed people differ from the rest of the labor force because of their higher productivity. Experience accumulation affects the probability of getting a job in the future and increases the expected level of earnings. In this way, the static heterogeneity peculiar to the pervious models is extended so as to also hold over time.

In this framework, the perfect unemployment insurance scheme may break down. People with greater experience do not have the incentive to pool together with the others. They will establish their own insurance scheme (lottery) so that they will share a larger consumption bundle than the people that experienced unemployment in the past. Even if total lifetime utility is still higher for the unemployed in this model, there is a sense in which employed people are better-off in the long-run. After the first period, in fact, the expected utility for the experienced people is higher than the expected utility of the unemployed.

We think that this model captures an essential feature of the labor market, i.e. the (ex-post) heterogeneity of the labor force. This framework could produce a great variety of earning profiles which depend on the employment record of the individual, and a full spectrum of earnings within a single period, reflecting the heterogeneous experience of the labor force. Moreover, an individual's employment record not only affects his current level of earnings, but also his future expected level, by influencing the probability of future employment.

It is also important to note the sociological implications of the present model. We start with a society where everybody has the same opportunity set, i.e. all agents are identical. However, because of the existence of imperfections in technology, i.e. the indivisibility of labor and the learning process, the efficient evolution of the society is such that the population gets separated over time into different economic groups.

The rest of the paper is organized as follows. In Section 2 we provide a description of preferences, technology and imperfections which characterize the model economy. In Section 3, we compute the equilibrium allocations under the assumption that perfect pooling over time is enforceable, and Section 4 presents some examples. In Section 5 we show that perfect pooling is not incentive compatible in our dynamic framework, and in Section 6 we characterize the equilibrium which arises when individual incentives are taken into account. In Section 7 we work out some examples for this alternative equilibrium concept. Section 8 compares the allocations generated by the two equilibrium concepts. In addition, it informally describes the equilibria obtainable by extending the time horizon of the economy which is important in order to understand the evolution of the society into heterogeneous groups, and the mobility of individuals across these groups. Section 9 modifies the analysis of Sections 6 and 7 by allowing the individuals to borrow and lend. Section 11 concludes the paper with an economic and a literary view on the relevance of lotteries in the society.

Section 2: The Economy

There is a continuum of identical agents with names in [0,1]. The economy lasts for two periods and there are two commodities: output and labor. Labor is used in each of the periods to produce output according to the production function f(N), where f(N) has the usual properties: increasing, twice continuously differentiable, concave, and f(0) = 0. The production function is constant over time. It is assumed that output is perishable and thus there is no storage of output over time. Agents have preferences defined over consumption and labor over the two period horizon. Each agent is endowed with one unit of time in each period but it is assumed that labor is indivisible, hence, either the entire unit or none of the unit can be supplied in any given period. Preferences are of the following form:

$$\sum_{t=1}^{2} u(c_t) - mn_t$$

where c_t is consumption in period t and n_t is labor supplied in period t. The function $u(\cdot)$ has the usual properties: increasing, twice continuously differentiable, and strictly concave. The parameter m is assumed to be strictly positive and corresponds to the disutility of work.

At this point the economy possesses no dynamic features: technology and preferences are constant over time and time separable, and there is no storage. The final feature of the economy introduces a dynamic element into the analysis. It is assumed that workers accumulate experience through working which enhances their productivity in future periods. To formalize this notion it is useful to think of labor used to produce output as measured in efficiency units. Thus, a worker with no experience has a time endowment equivalent to one efficiency unit of labor whereas a worker with one period of experience has a time endowment equivalent to (1+s) efficiency units of labor. If in period two there are λ_1 experienced workers and λ_2 inexperienced workers supplying labor to produce output, the output produced will be given by $f((1+s)\lambda_1 + \lambda_2)$. Note however, that the disutility of working does not depend on the quality of the labor supplied. Experience only acts to increase productivity.

One final point is that ownerwhip of the firm is uniformly distributed across the workers, so in equilibrium, profits will be uniformly distributed also.

Section 3: Perfect Pooling

> ... every free man automatically participated in the sacred drawings, which took place in the labyrinths of the god every sixty nights, and which determined his destiny until the next drawing. The consequences were

> incalculable. A fortunate play could bring about his promotion to the council of wise men or the imprisonment of an enemy (public or private) or finding, in the peaceful darkness of his room the woman who begins to excite him and whom he never expected to see again.
>
> Jorge Luis Borges, The Lottery in Babylon

The economy described in the previous section is similar in many respects to the one period indivisible labor framework studied by Rogerson [1985]. There it was shown that introducing lotteries over employment into the consumption sets of workers could both increase welfare and facilitate computation of equilibrium. The same result holds in this case, although the presence of experience accumulation necessitates that this claim be proven.

Without going through the details of defining a standard Arrow-Debreu equilibrium for this economy, it should be clear that there are four possible groups of individuals in equilibrium. Those who work in both periods; those who work in the first period but not in the second; those who work in the second but not in the first; and, those who don't work in either period. Call these groups one through four, respectively. Within each group all individuals will have the same consumption profile. Let $(c_1^i, n_1^i, c_2^i, n_2^i)$ be the allocation for group i members. Since all agents are identical, in equilibrium they must all obtain the same utility. Hence

$$\sum_{t=1}^{2} u(c_t^i) - mn_t^i = \sum_{t=1}^{2} u(c_t^j) - mn_t^j$$

for all $i,j = 1,\dots,4$. Call this utility level $\bar{U}$. Let λ_i be the fraction of agents in each of the four groups. It is now possible to construct a lottery which assigns agents to these groups randomly so that the sizes of the groups are unchanged, but where consumption is constant across groups. Define

$$c^*_t = \sum_{i=1}^{4} \lambda_i c^i_t \qquad t = 1,2.$$

Then if each worker is assigned to group i with probability λ_i and guaranteed a consumption profile of (c^*_1, c^*_2) the expected utility obtained by each worker is

$$EU = \sum_{i=1}^{4} \lambda_i [u(c^*_1)+u(c^*_2) - mn^i_1 - mn^i_2].$$

By definition of c^*_1 and c^*_2 and concavity of $u(\cdot)$:

$$EU \geq \sum_{i=1}^{4} \lambda_i [u(c^i_1) + u(c^i_2) - mn^i_1 - mn^i_2] = \sum_{i=1}^{4} \lambda_i \bar{U} = \bar{U}$$

where the inequality is strict as long as $c^*_t \neq c^i_t$ for all i and t. Since aggregate labor supply is unchanged by introducing the lottery this allocation is feasible and thus the result is proven.

The above discussion suggests that attention be focused on the following planner's problem:

$$\text{(P-1)} \qquad \underset{\lambda_i, c^t_{ij}}{\text{Max}} \ \lambda[u(c^1_w)-m + \lambda_w(u(c^2_{ww})-m) + (1-\lambda_w)u(c^2_{wN})]$$

$$+ (1-\lambda)(u(c^1_N) + \lambda_N(u(c^2_{NW})-m) + (1-\lambda_N)u(c^2_{NN}))$$

$$\text{s.t. } c^t_{ij} \geq 0, \qquad 0 \leq \lambda_i \leq 1$$

$$\lambda c^1_w + (1-\lambda)c^1_N \leq f(\lambda)$$

$$\lambda(\lambda_w c^2_{ww} + (1-\lambda_w)c^2_{wN}) + (1-\lambda)(\lambda_N c^2_{Nw} + (1-\lambda_N)c^2_{NN})$$

$$\leq f(\lambda\lambda_w(1+s) + (1-\lambda)\lambda_N).$$

The notation used above is as follows. The variable λ is the fraction of agents who work in period one, or analogously, the probability that an individual will work in period one. The variables λ_w and λ_N are the probability of employment in period two conditional on whether or not the individual worked (w) or didn't work (N) in period one. The c's represent the consumptions. The superscript denotes the time period and the subscripts refer to the individual's profile of employment experience. Note that the term multiplied by λ in the objective function is the lifetime expected utility conditional on working in period one, whereas the term multipled by $(1-\lambda)$ is the lifetime expected utility conditional on not working in period one. So the objective function is simply the expected utility of each agent. The constraints simply state that all the probabilities are between zero and one and that total consumption in each period must not exceed total production. Problem (P-1) appears rather unmanageable in its present form. However, it is straightforward to show that any solution to problem (P-1) will have $c^1_w = c^1_N$ and $c^2_{wN} = c^2_{wN} = c^2_{Nw} = c^2_{NN}$. This allows us to write problem (P-1) in the following form:

(P-2)
$$\max_{\lambda_i, c_i} \; u(c_1) + u(c_2) - \lambda m - \lambda\lambda_w m - (1-\lambda)\lambda_N m$$

$$\text{s.t.} \quad c_i \geq 0 \qquad o \leq \lambda_i \leq 1$$

$$c_1 \leq f(\lambda)$$

$$c_2 \leq f(\lambda\lambda_w(1+s) + (1-\lambda)\lambda_N)$$

This problem has a unique solution which we will denote by $\lambda^*, \lambda^*_w, \lambda^*_N, c^*_1, c^*_2$. The next proposition characterizes this solution.

<u>Proposition 1</u>: The solution to problem (P-2) satisfies the following:

(i) $\lambda^* > 0, \quad \lambda^*_w > 0, \; \lambda^*_N = 0$

(ii) $\lambda^* < (1+s)\lambda^*\lambda^*_w$

(iii) $c^*_2 > c^*_1$

(iv) $f'(\lambda^*) < (1+s)f'((1+s)\lambda^*\lambda^*_w)$ if $\lambda^* < 1$.

The interpretation of this proposition is fairly straightforward. Condition (i) says that there are always some workers who work in both periods ($\lambda^*_w > 0$) and there are no workers who work in the second period after not working in the first period ($\lambda^*_N = 0$). Because of experience, labor supply in efficiency units is always higher in the second period, $\lambda^* < (1+s)\lambda^*\lambda^*_w$, and, hence, consumption is greater in period two than in period one. Condition (iv) says that if workers were paid their marginal product, then wages would be higher in period two than in period one. More will be said about this condition later in the paper.

<u>Proof of Proposition 1:</u>

At this point it is not clear which of the constraints on the λ's will be binding. It is possible that some of the λ's are equal to one and that some of them are equal to zero. Because of the multiple kinds of corner solutions that may be involved here it will be easiest to proceed in stages. First we show that the results hold if the constraints requiring the λ's to be less than or equal to one are not binding. After this has been established it is straightforward to show that the results hold even if this is not the case.

Assuming that the λ's are all less than one the first order conditions for problem (P-1) are:

(3.1) $u'(f(N_1))f'(N_1) + u'(f(N_2))f'(N_2)(\lambda_w(1+s)-\lambda_N) \leqq m(1+\lambda_w-\lambda_N)$

with equality if $\lambda > 0$.

(3.2) $u'(f(N_2))f'(N_2)\lambda(1+s) \leqq \lambda m$, with equality if $\lambda_w > 0$

(3.3) $u'(f(N_2))f'(N_2)(1-\lambda) \leqq (1-\lambda)m$, with equality if $\lambda_N > 0$,

where $N_1 = \lambda$ = first period employment

$N_2 = \lambda\lambda_w(1+s)+(1-\lambda)\lambda_N$ = second period employment in efficiency units.

To see that $\lambda^* > 0$ it is enough to note that as $\lambda \to 0$ the left hand side of (3.1) approaches infinity because $u'(0) = \infty$ and $f'(0) > 0$. By a similar argument, $N_2^* > 0$, which implies it must be the case that $\lambda_w^* + \lambda_N^* > 0$. Inspection of equations (3.1) and (3.2) shows that if $0 < \lambda < 1$ then it is not possible for both $\lambda_w^* > 0$ and $\lambda_N^* > 0$ to be true as long as $s > 0$. Also, if (3.3) is satisfied with equality then it is clear that (3.2) cannot be satisfied at all. Hence it must be that $\lambda_N^* = 0$ and $\lambda_w^* > 0$. This proves (i). Given that $\lambda_N^* = 0$ equation (3.1) can be written as

$$u'(f(N_1))f'(N_1) + u'(f(N_2))f'(N_2)\lambda_w(1+s) = m(1+\lambda_w)$$

Since $\lambda_w^* > 0$, (3.2) implies that

(3.4) $u'(f(N_2))f'(N_2)(1+s) = m$

Substituting this into the above expression gives:

(3.5) $u'(f(N_1))f'(N_1) = m$

Comparison of (3.4) and (3.5) implies that $N_2 > N_1$ if $s > 0$. This proves (ii). Since technology is constant across time this also proves (iii).

To prove (iv) note that (3.4) and (3.5) together imply that

$$u'(f(N_1))f'(N_1) = u'(f(N_2))f'(N_2)(1+s)$$

Since $N_2 > N_1$ it follows that $u'(f(N_1)) > u'(f(N_2))$. But then the above equation implies that $f'(N_1) < f'(N_2)(1+s)$.

We now show that none of the results are altered if we allow for the possibility that some of the λ's could equal the maximum value of one. The argument that $\lambda^* > 0$ and $\lambda_N^* + \lambda_w^* > 0$ is clearly unaffected.

To prove that $\lambda_N^* = 0$ still holds, first note that if $\lambda^* = 1$ then the value of λ_N is irrelevant so there is no problem in choosing $\lambda_N^* = 0$. Now consider the case where $\lambda^* < 1$, $\lambda_w^* = 1$, and the constraint on λ_w is binding. Then (3.2) becomes

$$u'(f(N_2))f'(N_2)(1+s) > m$$

and (3.1) holds with equality. Substituting the above expression into (3.1) gives:

$$u'(f(N_1))f'(N_1) + \lambda_w m - u'(f(N_2))f'(N_2)\lambda_N < m(1+\lambda_w-\lambda_N)$$

or $$u'(f(N_1))f'(N_1) - u'(f(N_2))f'(N_2)\lambda_N < m(1-\lambda_N).$$

If $\lambda_N > 0$ then (3.3) holds with equality and substitution into the above expression gives:

$$u'(f(N_1))f'(N_1) < m.$$

But $N_2 > N_1$ (since $\lambda_w^* = 1$), hence:

$$u'(f(N_1))f'(N_1) > u'(f(N_2))f'(N_2) = m.$$

This is a contradiction.

We now show that $N_1^* < N_2^*$ still holds. Clearly if $\lambda_w^* = 1$ then $N_1^* < N_2^*$. So assume that $\lambda^* = 1$, $\lambda_w^* < 1$, and $N_1^* \geq N_2^*$. Equation (3.1) will have the left hand side strictly greater than the right hand side and equation (3.2) will hold with equality. Substituting (3.2) into (3.1) and recalling that $\lambda_N^* = 0$ gives

$$u'(f(N_1))f'(N_1) > m.$$

Since $N_2 \leq N_1$ it follows that

$$u'(f(N_2))f'(N_2) > u'(f(N_1))f'(N_1)$$

which contradicts that (3.2) holds with equality.

Finally, we show that condition (iv) holds if $\lambda^* < 1$ and $\lambda_w^* = 1$. In this case it is straightforword to show that (3.1) and (3.2) combine to give

$$u'(f(N_1))f'(N_1) < u'(f(N_2))f'(N_2)(1+s).$$

Since $N_1 < N_2$ it follows that $u'(f(N_1)) > u'(f(N_2))$ and hence the above expression yields

$$f'(N_1) < f'(N_2)(1+s)$$

This completes the proof.//

Note that there is no result concerning whether or not λ_W^* is strictly less than one. As the examples in the next section will show, λ_W^* may be either one or strictly less than one depending upon the preferences and other parameters of the model.

It is interesting to examine the properties of these allocations in light of the comments made in the introduction regarding the undesireable properties of other indivisible labor models, in particular the dynamic model of Hansen. We described two such features: One was that unemployment does not have any effect on future allocations at the individual level. The second is that in his framework the unemployed are always better off than the employed. The addition of experience accumulation changes the first of these properties but in fact makes the unemployed even better off relative to the employed. These two changes are closely connected; the above proposition showed that λ^* and λ_W^* are both strictly positive whereas λ_N^* is always zero. This means that the state of being unemployed is very persistent. Individuals who are unemployed in period one are always unemployed in period two, whereas workers who are employed in period one have probability $(1-\lambda_W^*) < 1$ of being unemployed in period two. However, this persistence serves only to make unemployment in period one more attractive, since consumption is independent of labor supply.

It is interesting to note the manner in which work is distributed across the population in this model. As mentioned above, no one who doesn't work in the first period ever works in the second period. On the one hand, this may seem efficient because of experience accumulation, but on the other hand, it seems suboptimal to concentrate labor supply on the same individuals for two consecutive periods. This last statement is false under the circumstances studied here. As is clear from problem (P-2) the assumption of indivisible labor in conjunction with lotteries makes individuals behave as if their preferences are linear in leisure. Hence, individuals are risk-neutral with respect to employment probabilities. Thus, distributing work more evenly over the workforce (ex-ante) is not of concern to the individuals. In section five we will argue that this allocation is not dynamically enforceable. This will suggest an alternative equilibrium concept in which the set of available lotteries is restricted. In section six this is done and it is shown how unemployment may display the properties outlined in the introduction.

The solution to the programming problem (P-2) may be achieved through a decentralized mechanism in several different manners. Following Rogerson [1985] one may simply price the lotteries produced by (P-2) and set up standard competitive markets. In this case one can introduce markets for risk sharing due to the presence of the lottery. This risk is perfectly diversifiable and explains why all agents receive the same consumption.

A second interpretation involves pricing the lotteries and having a government impose taxes and transfers on the agents after the outcome of the lottery has been determined to perform the insurance role. In this case, one can interpret the marginal product of labor as the wage and the difference between consumption and pre-tax wages will be the size of the tax. Under this interpretation the problem (P-2) characterizes the optimal tax-transfer policy. In this case, observe that proposition one predicts decreasing employment over time but increasing consumption. Since consumption is constant across workers, even the unemployed receive more consumption in period two. Thus, in period two, there are fewer employed workers financing an even higher consumption level of the unemployed workers. This implies that taxes must have increased over time. This condition has a simple intuition behind it. As time passes the economy accumulates experience, hence expanding production capabilities. The economy responds by financing a larger social welfare system. Condition (iv) says that wages have also increased over time.

Section 4: Examples With Perfect Pooling

This section works out examples of the perfect pooling equilibrium for different specifications of the economy in order to clarify and illustrate some of the characteristics of this equilibrium. Two cases are considered; one where u(c) is logarithmic and the other where u(c) is quadratic. In both examples we consider the case where technology is linear.

Example I: Logarithmic Utility function

Assume the following specification for the utility function and the production function.

$$u(c) = \alpha \ln c$$

$$f(N) = \theta N$$

The first order conditions (3.1) - (3.2) now take the following form:

(4.1.1) $\frac{2\alpha}{\lambda} \geqq m(1+\lambda_w)$ equality if $\lambda < 1$.

(4.1.2) $\frac{\alpha}{\lambda\lambda_w} \geqq m$, equality if $\lambda_w < 1$.

Suppose $\lambda_w < 1$. Then (4.1.2) holds with equality and

$$\frac{\alpha}{\lambda} = m\lambda_w.$$

Substituting into (4.1.1) gives

$$2m\lambda_w \geqq m(1+\lambda_w)$$

or

$$m\lambda_w \geqq m$$

which is a contradiction if $\lambda_w < 1$. Hence $\lambda_w = 1$ always. Setting $\lambda_w = 1$ in (4.1.1) gives

$$\lambda = \min(\frac{\alpha}{m}, 1).$$

In this specification employment is constant over time and the level of employment is independent of the level of experience. This is not surprising; with logarithmic utility the income and substitution effects cancel out. Figure one describes the results for this specification.

Example II: Quadratic Preferences and Linear Production Function

It should be noted that quadratic preferences do not satisfy the $u'(0) = \infty$ condition. Hence, results which depend upon this condition need not hold for this specification. Aside from this defect the quadratic example is a tractable framework and is worth investigating. One can interpret the quadratic specification as simply a local approximation which is accurate in some interval.

Let us assume the following specifications:

$$u(c) = c - \frac{\beta}{2} c^2$$

$$f(N) = \theta N$$

In this case, the first order conditions become:

(4.2.1) $(1-\beta\theta\lambda)\theta + (1-\beta\theta\lambda\lambda_w(1+s))\lambda_w\theta(1+s) \geqq m(1+\lambda_w)$, equality if $\lambda < 1$.

(4.2.2) $(1-\beta\theta\lambda\lambda_w(1+s))\theta(1+s) \geqq m$, equality if $\lambda_w < 1$.

Assuming that $\lambda < 1$ and $\lambda_w < 1$ the solutions for λ and λ_w are given by

$$\lambda = \frac{\theta-m}{\beta\theta^2}$$

$$\lambda_w = \frac{\theta(1+s)-m}{(\theta-m)(1+s)^2}$$

Assuming that $\lambda_w = 1$ and $\lambda < 1$ the solution is

$$\lambda = \frac{\theta(2+s)-2m}{\beta\theta^2(1+(1+s)^2)}$$

Whether or not λ_w is less than or equal to one depends on the effect of experience on the marginal product of working. It is easy to show that:

$$\lambda_w < 1 \text{ if } k > \frac{s+2}{s+1}$$

where k is such that

$$\theta = km.$$

Intuitively, for a given s, if the marginal product of inexperienced labor (θ) is low relative to the marginal disutility of working (m), i.e. $k \leq \frac{s+2}{s+1}$, then accumulated experience is extremely valuable, so that none is wasted: i.e. $\lambda_w = 1$. In other words, the substitution effect associated with an increase in the marginal product is larger than the income effect. On the contrary, if θ is high relative to m, i.e. $k \geq \frac{s+2}{s+1}$, then the marginal return (in utility terms) of experience is not so high, and the society tends to waste part of it: i.e. $\lambda_w < 1$. In order to clarify the above we consider specific values of k.

Example 2.a $k = 1,\ \theta = m = a$

In this case $\lambda_w = 1$, so that (4.2.5) implies

$$\lambda = \frac{s}{\beta a(1+(1+s)^2)}$$

The condition $\lambda \leq 1$ implies

$$s \leq \beta a(1+(1+s)^2)$$

which imposes restrictions on β (in particular $\beta > \frac{1}{2a}$ would be sufficient).

Marginal utility being positive requires:

$$s^2 + s + 2 > 0$$

which is always satisfied.

It is interesting to see how employment responds to variations in the return to experience;

$$\frac{\partial\lambda}{\partial s} = \frac{1}{a\beta}\,\frac{2-s^2}{[1+(1+s)^2]^2}.$$

Hence, for $s \lesseqgtr \sqrt{2}$, $\frac{\partial\lambda}{\partial s} \gtrless 0$.

We represent this relationship in Figure 2.

At $s = \sqrt{2}$ the income effect of increases in experience start dominating the substitution effect.

Example 2.b $\theta = 2a = 2m$.

In this case both λ and λ_w are less than one, given by:

$$\lambda \cdot \lambda_w = \frac{(2s+1)}{4\beta a(1+s)^2}$$

$$\lambda_w = \frac{1+2s}{(1+s)^2}$$

$$\lambda = \frac{2s + 1 + s^2}{4\beta a(1+s)^2} = \frac{(s+1)^2}{4\beta a(1+s)^2} = \frac{1}{4a\beta}$$

In this case the condition $\lambda \leq 1$ becomes:

$$\frac{1}{4a\beta} \leq 1 \Rightarrow \beta \geq \frac{1}{4a}$$

and the condition for positive marginal utility in the first period:

$$1 - \beta 2a \frac{1}{42\beta} = \frac{1}{2} > 0$$

and in the second period

$$\frac{2a(1+s)(2s+1)}{4\beta a(1+s)^2} < \frac{1}{\beta}$$

which implies

$$2s + 1 < 2 + 2s$$

which is always satisfied.

It is easy to show that $\frac{\partial \lambda_w}{\partial s} < 0$ for all s.

It is interesting to note that in this case, which is represented in Figure 3, the employment in the first period, is constant, independently of the return to experience, while increases in s reduce employment in the second period i.e., $\lambda \cdot \lambda_w$. Note however, that, even if employment decreases with s, total output increases as s increases. Total output in the second period is also greater than output in the first period, unless s = 0 in which case they are equal. In fact:

$$y_1 = \theta\lambda = \frac{1}{2\beta}$$

$$y_2 = \theta(1+s)\lambda\lambda_w = \frac{2s+1}{2\beta(1+s)}$$

Example 2c. $\theta = (3/2)a = (3/2)m$.

Setting $\lambda_w = 1$ gives s = 1. So for s < 1 we have $\lambda_w = 1$, and λ is given by:

$$\lambda = \frac{(3/2)(s+2)-2}{\beta(3/2)^2 a[1+(1+s)^2]}$$

while if s > 1 we have:

$$\lambda = \frac{(3/2)(s+1)-1}{\beta(3/2)a^2(1+s)^2}$$

and

$$\lambda_w = \frac{(1+3s)}{(1+s)^2}$$

When $\lambda_w = 1$ (i.e. s < 1) the effect of a variation in s on employment is given by

$$\text{sign}(\frac{\partial\lambda}{\partial s}) = \text{sign}(1-2s-(3/2)s^2)$$

the roots are $-(\frac{2\pm\sqrt{10}}{3})$ so that the derivative changes sign before s=1. When s > 1, employment in the first period is independent of s:

$$\lambda = \frac{2}{9a\beta}$$

As in the previous example, when s > 1 we have $\frac{\partial\lambda_w}{\partial s} < 0$. The above is summarized in Figure 4.

Section 5: Breaking the Perfect Pooling Equilibrium

> The just desire that all, rich and poor, should participate equally in the lottery, inspired an indignant agitation, the memory of which the years have not erased. Some obstinate people did not understand (or pretended not to understand) that it was a question of a new order, of a necessary historical stage.
>
> Jorge Luis Borges,
> The Lottery in Babylon

The purpose of this section is to demonstrate that there is a sense in which the arrangements implicit in the perfect pooling equilibrium are not dynamically incentive compatible. The exact details of this argument will be discussed later, but the intuition is quite clear and is as follows. At the end of period one workers are no longer identical; individuals who worked in period one have accumulated experience and are thus more productive than individuals who did not work. Individuals who did not work in period one will never work in period two whereas individuals who did work in period one face some strictly positive probability of working. Yet both types of agents receive the same consumption. From the point of view of the experienced worker this clearly seems undesireable. They are more productive, they have a higher probability of supplying labor yet they end up with the same consumption as the unemployed. It seems clear that this group of experienced workers has an incentive to "break away" from the inexperienced group and run a new lottery among themselves. Because they are more productive there are clear benefits in doing so. This argument is certainly very plausible, but it raises several questions. First, if the experienced workers run their own lottery in period two, does this destroy the lottery initially used in period one? The answer to this question is no. In the next section we analyze equilibrium allocations under this assumption. There it will be shown that even if all agents know that experienced and inexperienced workers will participate in separate lotteries in period two, it is still desireable to determine the employed individuals in period one by a lottery which provides compensation in period one for the unemployed. This raises the second important question: If the period one employed workers are going to run their own lottery in period two, why would they bother to give any compensation to the period one unemployed? If they are going to break away from this group next period, what is accomplished by honoring the compensation scheme implied by the lottery this period? Our answer depends upon distinguishing two types of lotteries: Lotteries which imply commitments about current allocations and lotteries which involve commitments about current and future allocations. In order to appreciate this distinction, it is useful to consider the indivisible labor economies of Rogerson and Hansen mentioned previously. We will argue that repetition of lotteries can act as an enforcement mechanism. Consider first the one period model of Rogerson. The lottery used to determine the allocation of resources creates winners and losers ex post. In particular employed people are worse off and unemployed people are better off. The reason unemployed people are better off is because they receive a transfer from the employed people. Clearly it seems that there is no reason for the employed people to carry out such transfers. If they don't then clearly the economy will end up at the competitive allocation achieved without lotteries. (The reader may feel that this discussion simply begs the question of commitment. We will return to this later.) Consider now the infinite horizon version of the indivisible labor economy studied by Hansen. For simplicity assume the environment is constant over time. Hansen produces the time profile of allocations by repeating the same lottery every period. The above argument is now clearly more complicated. If the lottery breaks down in period one because agents don't carry out the transfers ex post, then agents also lose the future benefits from having the lotteries instituted. Let u_W be the one period utility derived from the lottery ex post if the individual works, u_n the corresponding utility if the individual does not work, u_l the one period ex ante utility obtained according to the lottery, u_c the one period utility obtained from the competitive equilibrium without lotteries and β be the discount rate. In this model $\bar{u}_n > u_l > u_c > \bar{u}_w$. Define $\bar{u}_{wR}$ as the one period utility obtained by a worker who is employed and repudiates the agreement to transfer consumption to the unemployed. Clearly, $\bar{u}_{wR} > \bar{u}_w$. Consider the situation of a worker who in period one has been chosen to work. If they carry out the transfer and honor the lottery they obtain lifetime expected utility of

$$\bar{u}_w + \sum_{t=1}^{\infty} \beta^t u_l = \bar{u}_w + \frac{\beta u_l}{1-\beta}$$

If employed people refuse to carry out the transfers they destroy the system of lotteries, the economy reverts to the no-lottery competitive allocation and individuals receive utility:

$$u_c + \sum_{t=1}^{\infty} \beta^t u_c = \bar{u}_{wR} + \frac{\beta u_c}{1-\beta}$$

From above $\bar{u}_{wR} > \bar{u}_w$, so there is a one period gain from refusing to pay the transfer. However, $u_l > u_c$, so there is a future loss involved in this behavior. Clearly the optimal action will depend on the size of β. In particular, when $\beta = 0$ then the model reverts to the static case and it is optimal for the employed workers to break the system. But for sufficiently large β the opposite will be true. The driving force behind this result is that even though a worker is a "loser" in the current period lottery it is in their interest not to break the system because the system promises them higher average utility in future periods. In this case the fact that individuals will be involved in the lottery in the future acts so as to make it optimal for current "losers" to accept the outcome passively.

We now want to address the point concerning the time horizon of commitment mentioned earlier. The sequential lotteries used by Hansen and discussed above maximize the ex ante expected utility of the representative worker. However, there are many other lotteries that could produce the same level of ex ante expected utility. In particular consider the following lottery, recalling that we are assuming technology and capital are constant over time. At time zero the workforce is randomly separated into two groups. One group works every period, the other group never works, but both groups have the same consumption every period. If the sizes of the two groups are chosen appropriately this lottery has the same expected utility as the sequential lotteries. Yet these two lotteries have very different dynamic enforceability characteristics. Repeating the above problem faced by an employed individual, the relevant utilities are:

$$\bar{u}_w + \frac{\beta \bar{u}_w}{1-\beta}$$

and $$\bar{u}_{wR} + \frac{\beta u_c}{1-\beta}$$

Since $u_c > u_w$ the individual would always choose to break the system. The important feature here is that the time zero lottery was committing the individual to a lifetime of transfers paid out to other individuals. There are no possibilities of gains from future lotteries.

This discussion has a simple conclusion: Lotteries which produce "temporary" winners and losers are sustainable because agents wish to benefit from future lotteries. But lotteries which produce "long term" winners and losers are not sustainable because the losers have no incentive to maintain the system.

We now apply this argument to the perfect pooling equilibrium concept of this paper. Note that the lottery of period one has two aspects. On the one hand it determines "temporary" winners and losers in that everyone receives the same period one consumption but not all individuals work. On the other hand it produces "long term" winners and losers because everyone is promised the same second period consumption but the losers (the employed) of period one will also be the losers (employed) in period two because their experience accumulation makes them more productive. The argument above claims that only the first of these aspects will be sustainable. What lies behind this is the following. In period two the experienced individuals do not want to include the inexperienced individuals in their transfer programs. But they do want to have a transfer program among themselves. If these individuals did not compensate the unemployed during period one this would indicate that all future lotteries among "equals" would also be destroyed. But these individuals do not want this outcome; they only want to exclude the less productive workers from their lotteries.

Note that according to the arguments outlined previously if the time horizon is finite then a recursive argument would show that even the sequential lotteries cannot be sustained. Since the model analyzed here is only two periods this should imply that even the one period lotteries should not be sustainable. While this argument is certainly valid we will choose to ignore it in the remainder of this paper, for the following reason. We could easily specify an infinite horizon version of the economy studied here. It should be clear that the kinds of results and intuition involved in analyzing the two period problem would continue to hold in essence in the infinite horizon model. At this stage the reward for having an infinite horizon model seems small in comparison with the difficulties associated with its solution. Therefore, we choose to work within the confines of the two period problem and ignore the incentive issues associated with the finite horizon.

Thus far the discussion has focused on theoretical grounds for limiting the available set of lotteries. The importance of this argument will become more apparent in the following sections where it will be demonstrated that this restriction on lotteries has important implications for the properties of unemployment. In particular, it will be seen that with this restriction the model appears to deal with the problems raised in the introduction in a more satisfactory manner. It is perhaps tempting for one to conclude that we are producing these properties by eliminating markets and hence the approach used here is merely a sophisticated version of models which assume wage inflexibility or liquidity constraints. We wish to emphasize that such a conclusion would be very misleading. The major reason for this is that even after ruling out certain types of lotteries the equilibrium we consider allows for more trades than the standard Arrow-Debreu equilibrium. All that we are restricting is the form of lotteries which are <u>added</u> to the standard Arrow-Debreu complete market structure. Second, we have specified an argument which produces the market structure endogenously, and thus the restriction placed on allowable lotteries has not been ad hoc.

One final comment concerns the notion of commitment. It is clear that if there is "perfect" commitment there is no problem in supporting the perfect pooling equilibrium. We feel that it is desireable to have some endogenous notion of which commitments are enforceable. This is what the above analysis has dealt with.

Section 6: Equilibrium With Imperfect Pooling

Earlier it was shown that introducing lotteries allows for Pareto improvements in the equilibrium allocations for the economy being studied. In the last section it was argued that lotteries with perfect pooling over time have a dynamic inconsistency property and should not be expected to exist. In this section we will study lotteries which don't allow for pooling over time. Prior to studying these lotteries in detail it is necessary to demonstrate that this restricted class of lotteries will still allow for welfare improvements relative to the equilibrium allocation generated in a world with no lotteries. This is not a very difficult exercise and the style of proof follows that used earlier in the paper to demonstrate the same claim in the case of lotteries with perfect pooling.

As remarked earlier, in a competitive equilibrium without lotteries there will potentially be four groups: A fraction λ_1 who work in both periods; a fraction λ_2 who work in period one but not in period two; a fraction λ_3 who work in period two but not in period one; and, a fraction λ_4 who don't work in either period. Let the consumption allocation of group i be (c_1^i, c_2^i). Define $c_1^* = \sum_{i=1}^{4} \lambda_i c_1^i$. Define $c_{2W}^* = \dfrac{\lambda_1 c_2^1 + \lambda_2 c_2^2}{\lambda_1 + \lambda_2}$, $c_{2N}^* = \dfrac{\lambda_3 c_2^3 + \lambda_4 c_2^4}{\lambda_3 + \lambda_4}$. Now construct the following lottery which is consistent with no pooling over time. A fraction λ_1 of the population works in both periods and consumes (c_1^*, c_{2W}^*). A fraction λ_2 works only in period one and consumes the same bundle. A fraction λ_3 work only in period two and consume (c_1^*, c_{2N}^*) and a fraction λ_4 don't work in either period and consume this same bundle. Notice that all workers have the same consumption in period one, but in period two the population is split into two groups. Those who worked in period one operate a lottery to smooth their consumption and those who didn't work in period one operate a separate lottery to smooth their consumption. It remains to show that this lottery improves welfare. Utilities in the competitive allocation without lotteries are given by:

$$u^1 = u(c_1^1) + u(c_2^1) - 2m$$
$$u^2 = u(c_1^2) + u(c_2^2) - m$$
$$u^3 = u(c_1^3) + u(c_2^3) - m$$
$$u^4 = u(c_1^4) + u(c_2^4)$$

In equilibrium it must be that $u^1 = u^2 = u^3 = u^4$ since all agents are identical. Hence,

$$u_c^* = u^j = \sum_{i=1}^{4} \lambda_i u^i, \quad j = 1,\ldots,4.$$

Utility from the above lottery is given by:

$$u_\ell^* = u\left(\sum_{i=1}^{4} \lambda_i c_1^i\right) + (\lambda_1+\lambda_2)u(c_{2W}^*) + (\lambda_3+\lambda_4)u(c_{2N}^*) - (2\lambda_1+\lambda_2+\lambda_3)m$$

By strict concavity of $u(\cdot)$ and the definitions above:

$$u_\ell^* > \sum_{i=1}^{4} \lambda_i u(c_1^i) + \frac{(\lambda_1+\lambda_2)(\lambda_1 u(c_2^1)+\lambda_2 u(c_2^2))}{(\lambda_1+\lambda_2)}$$
$$+ \frac{(\lambda_3+\lambda_4)(\lambda_3 u(c_2^3)+\lambda_4 u(c_2^4))}{(\lambda_3+\lambda_4)} - (2\lambda_1+\lambda_2+\lambda_3)m$$

$$= \sum_{i=1}^{4} \lambda_i (u(c_1^i)+u(c_2^i)) - (2\lambda_1+\lambda_2+\lambda_3)m$$

$$= \sum_{i=1}^{4} \lambda_i u^i = u_c^* \Rightarrow u_\ell^* > u_c^*$$

where the inequality is strict if $\lambda_1 \neq 1$. The allocation defined by this lottery is clearly feasible because aggregate labor supply and aggregate consumption are identical to the competitive equilibrium. This completes the proof.

In the last section it was demonstrated that the equilibrium with perfect pooling suffers from a problem of dynamic enforceability. This leads one to consider an equilibrium which is restricted in the sense that in each period there is a new drawing and workers with different histories participate in different lotteries.

Developing the details is very similar to the previous cases and won't be given here. It is convenient to consider the case where technology is linear and hence, this is the case studied here. The programming problem which will describe the optimal allocation with this restricted class of lotteries is the following:

$$\max_{\substack{c, c_w, c_N \\ \lambda, \lambda_w, \lambda_N}} \quad u(c) - \lambda m + \lambda[u(c_w) - \lambda_w m] + (1-\lambda)[u(c_N) - \lambda_N m]$$

$$\text{s.t. } 0 \leq \lambda \leq 1, \quad 0 \leq \lambda_i \leq 1, \quad i = N, w$$

$$0 \leq c \leq \lambda\theta$$

$$0 \leq c_w \leq \lambda_w(1+s)\theta$$

$$0 \leq c_N \leq \lambda_N\theta$$

The notation used to describe these outcomes is the following: c is the consumption of workers in period one and λ is the fraction of them who work. In period two, c_w is the consumption of all workers who worked in period one and λ_N is the fraction of them that work in period two. Similarly, c_N is the period two consumption of all workers who did not work in period one and λ_N is the fraction of them that work. Each lottery must be self-sufficient, that is, the consumption given to each participant must be produced through that group's collective labor. The last three constraints of the programming problem specify these conditions, where θ is the parameter specifying the linear technology. The objective function is simply the ex-ante expected utility of a worker facing the above sequence of lotteries. In period one a worker necessarily consumes c and works with probability λ. In period two there are two possibilities, depending upon the outcome of the first lottery. With probability λ, the individual is in the group of workers who worked in the first period. In this case, the worker necessarily gets c_w for second period consumption and works with probability λ_w. On the other hand, with probability $(1-\lambda)$ the worker is in the group that does not work in the first period, and then this individual receives c_N for period two consumption and works with probability λ_w.

The remainder of this section is devoted to deriving some general properties of the solution to this programming problem. Substituting the constraints on the consumptions into the objective function reduces the problem to choosing λ, λ_w, and λ_N. Because u'(0) is infinite, all of these variables will necessarily be positive. This is of some interest because in the case of perfect pooling it was seen that $\lambda_N = 0$ always held. Here it is true that λ_N is never equal to zero. If the other endpoint constraint is not binding then the solution will be characterized by the following three first order conditions:

(6.1) $u'(\lambda\theta)\theta - m + [u(\lambda_w(1+s)\theta)-\lambda_w m] - [u(\lambda_N\theta)-\lambda_N m] = 0$

(6.2) $u'(\lambda_w(1+s)\theta)(1+s)\theta - m = 0$

(6.3) $u'(\lambda_N\theta)\theta - m = 0$

If $s > 0$ it is necessarily true that $[u(\lambda_w(1+s)\theta)-\lambda_w m] > [u(\lambda_N\theta)-\lambda_N m]$ since the group with experience can always do better than the group without experience. Given this, equations (6.1) and (6.3) can be written as:

$$u'(\lambda\theta)\theta = m'$$

$$u'(\lambda_N\theta)\theta = m$$

where $m' < m$ by virtue of the last inequality above when $s > 0$. Hence, it follows that $\lambda > \lambda_N$ if $s > 0$. This implies that the probability of obtaining employment for inexperienced workers is decreasing over time.

It is of some interest to compare λ_N and λ_w because these two numbers determine the employment ratios of the two groups in the second period. Note that conditions (6.2) and (6.3) are of the same form and in fact are identical if $s = 0$. This suggests that by determining how λ_w changes when s increases it may be possible to infer the relative size of λ_w and λ_N. If λ_w is always increasing in s then λ_w will be larger than λ_N. If λ_w is always decreasing in s, then λ_w will be smaller than λ_N. Differentiating equation (6.2) with respect to s and using primes on λ_w to represent the derivative of λ_w with respect to s gives:

$$u'(\lambda_w(1+s)\theta)\theta + u''(\lambda_w(1+s)\theta)(1+s)\theta^2[\lambda_w'(1+s)+\lambda_w] = 0.$$

Solving for λ_w' gives:

$$\lambda_w' = \frac{-u'(\lambda_w(1+s)\theta) - \lambda_w(1+s)\theta u''(\lambda_w(1+s)\theta)}{u''(\lambda_w(1+s)\theta)(1+s)^2\theta}$$

$$= - \frac{-u'(\lambda_w(1+s)\theta)}{u''(\lambda_w(1+a)\theta)(1+s)^2\theta} - \frac{\lambda_w}{(1+s)}.$$

It follows that $\lambda_w' > 0$ iff

$$\frac{-u'(\lambda_w(1+s)\theta)}{\lambda_w(1+s)\theta u''(\lambda_w(1+s)\theta)} > 1.$$

But the left hand side is simply the inverse of the coefficient of relative risk aversion at $\lambda_w(1+s)\theta$. Hence, $\lambda_w' > 0$ iff the coefficient of relative risk aversion is smaller than one. From the previous discussion this implies that if the coefficient of relative risk aversion is uniformly smaller than (larger than) one then λ_w is larger than (smaller than) λ_N. It is interesting to see that this is simply the condition to insure that the labor supply function is not backward bending. That is to say, λ_w will be greater than λ_N if the income effect is not too large. This is definitely intuitive since the workers who have acquired the experience in period one differ from those who have not in the fact that they face higher wages in period two.

The same derivation used above demonstrates that $c_w'(s)$ is always positive. Hence, $c_w > c_N$ independently of the degree of risk aversion. These are the only results which seem to be true in general. The next section contains some examples to illustrate the types of results that can be obtained in more detail.

Before proceeding to the examples it is important to discuss the welfare implications of the present model for the employed and unemployed workers.

Recall that in period one the employed and unemployed consume the same amount, but only the employed supply labor. Hence, in period one, the unemployed are certainly better off. However, entering period two the period one employed are better off than the period one unemployed because they have superior production opportunities by virtue of their experience accumulation. A priori one might expect that lifetime utility conditional on being unemployed in period one may be larger or smaller than lifetime utility conditional upon being employed in period one. It turns out that this result will always be in one direction. The unemployed in period one are always better off (in a lifetime sense) than the employed in period one. This is due to a combination of two features of the environment in which the analysis has been carried out. These are the linearity of technology and the separability of preferences. Consider first the linearity of technology. Suppose that an equilibrium allocation is giving higher lifetime utility to the employed. If there is any consumption transfer it must be from the employed to the unemployed. Now imagine what happens if an unemployed person chooses to work instead of being unemployed. Because technology is linear no one's productivity is affected. Also, because there are less unemployed people each employed person has to give less of a transfer. Hence, not only is the person who decided to work better off, but so are all the employed people. The unemployed are unaffected. Continuing this argument leads to the conclusion that if the employed have a higher level of utility then everyone is better off if there is no unemployment. It is important to note that the reverse argument does not hold. If the unemployed are better off than the employed it does not follow that everyone is better off if everyone is unemployed. Increasing the number of unemployed increases the transfer that each employed worker must make.

The second property is the separability of preferences. If preferences are non-separable it is no longer true that the unemployed and employed necessarily have the same period one consumption. This potentially will lower the utility difference between the two groups resulting from the period one allocation.

Characterization of equilibrium allocations becomes more difficult when these two features are relaxed and won't be attempted in this paper. However, it is not difficult to generate examples where this kind of result can be obtained.[1] However, we wish to emphasize the dynamic nature of the welfare comparisons outlined. The unemployed are "happy" while they are receiving benefits from the employed but later on they are "unhappy" because their productive capabilities have not increased because they have not accumulated experience. We feel that these sorts of dynamic considerations are important in understanding unemployment and are clearly absent in any static model which compares utilities of the employed and unemployed.

Section 7: Examples with Imperfect Pooling

Similarly to section 4, this section works out some detailed examples to illustrate some of the features of the equilibrium concept developed in the last section.

Example I: Constant relative risk aversion utility function.

Assume the following utility function specification:

$$u(c) = \frac{1}{1-\alpha} c^{1-\alpha}$$

which is characterized by constant relative risk aversion given by:

$$- c \frac{u''(c)}{u'(c)} = \alpha$$

The first order condition (6.1) - (6.3) take the form:

(7.1.1) $(\lambda\theta)^{-\alpha}\theta - m + [\frac{1}{1-\alpha}(\lambda_w(1+s)\theta)^{1-\alpha}-\lambda_w m] - [\frac{1}{1-\alpha}(\lambda\theta)^{1-\alpha}-\lambda_N m] = 0$

(7.1.2) $[\lambda_w(1+s)\theta]^{-\alpha}(1+s)\theta = m$

(7.1.3) $[\lambda_N\theta]^{-\alpha}\theta = m$

The equilibrium values for λ, λ_w and λ_N are given by:

$$\lambda = [\frac{m}{\theta^{1-\alpha}}[1 + [\frac{\alpha}{1-\alpha}][\frac{\theta^{1-\alpha}}{m}]^{1/\alpha}[1-(1+s)^{\frac{1-\alpha}{\alpha}}]]^{-1/\alpha}$$

$$\lambda_w = [\frac{[\theta(1+s)]^{1-\alpha}}{m}]^{1/\alpha}$$

$$\lambda_N = [\frac{\theta^{1-\alpha}}{m}]^{1/\alpha}$$

Recall that λ, λ_w and λ_N must be all less than one. This will impose restrictions on the values of θ and m. A necessary, but not sufficient, condition is that $m > [\theta(1+s)]^{1-\alpha}$, i.e., the disutility of working must be sufficiently large with respect to its marginal product, otherwise there would always be full employment, and the lotteries would be irrelevant.

In this particular example $\lambda_w > \lambda_N$. This is because the degree of relative risk aversion (α) is always smaller than one, as was discussed in the previous section.

An interesting question is whether total employment increases or decreases over time. In order to have a feel for the problem let us assume the following specific parameter values: $m = 4$, $\theta = 1$, $\alpha = 1/2$. In this case $s < 12$ is necessary to guarantee that λ_1, λ_w and λ_N are all less than one. We then have the following results:

$$\lambda_N = \frac{1}{16}\lambda_w = \frac{1+s}{16} \qquad \lambda = \frac{16}{(16-s)^2}.$$

Total employment in the first period (λ_1) is equal to λ, while total employment in the second period (λ_2) is given by:

$$\lambda_2 = \lambda\,\lambda_w + (1-\lambda)\lambda_N = \frac{(16s)^2 + 16s}{16(16-s)^2}$$

Employment in the second period is always smaller than employment in the first period. The relationship between λ, λ_w, λ_N, λ_2 and s are pictured in Figure 5.

Finally, it should to be noticed that λ_w is increasing in s, (suggesting that the substitution effect is stronger than the income effect), and, as expected, that $\lambda_w > \lambda_N$, indicating that the probability of being employed is higher for experienced people, than for people previously unemployed. Figure 6 describes the equilibrium for m=3, θ=1, and α=1/2.

Example II: Logarithmic Utility Function:

This specification is a special case of the previous example when $\alpha \to 1$. This corresponds to considering u(c) = ln(c). It is eady to show that in this case the equilibrium is given by:

$$\lambda = \frac{1}{m-\ln(1+s)}$$

$$\lambda_w = \frac{1}{m}$$

$$\lambda_N = \frac{1}{m}$$

$$\lambda_2 = \frac{1}{m}$$

Note that λ_w and λ_N are equal and independent of the value s. This is consistent with our general result, since the relative degree of risk aversion for the logarithimic utility function is equal to one. In the following graph we represent the relationship between the employment ratios and experience. Recall that in this case, $m > 1$ and $s < e^{m-1} - 1$ are necessary conditions for λ, λ_w, λ_N, to be less than one. It is easy to compute the equilibrium levels of consumption:

$$c_1 = \lambda\theta = \frac{\theta}{m-\log(1+s)}$$
$$c_w = \frac{(1+s)\theta}{m}$$
$$c_N = \frac{\theta}{m}$$

If $m = 3$ and $\theta = 1$ then $c_1 = \frac{1}{3-\log(1+s)}$, $c_w = \frac{1+s}{3}$ $c_N = \frac{1}{3}$ and $\lambda = \frac{1}{3-\ln(1+s)}$, $\lambda_w = \lambda_N = \lambda_2 = \frac{1}{3}$, as is described in Figure 7 and 8.

Example III: Quadratic Utility Function

The same comments made earlier about the nature of quadratic preferences continue to apply here.

We assume the following specification:

$$u(c) = c - \frac{\beta}{2}c^2$$
$$f(N) = \theta N$$

For these functions the first order conditions (6.1) - (6.3) become:

$$(7.2.1) \quad (1-\beta\theta\lambda)\theta - m + (\lambda_w(1+s)\theta - \frac{\beta}{2}(\lambda_w(1+s)\theta)^2 - \lambda_w m) - (\lambda_N\theta - \frac{\beta}{2}(\lambda_N\theta)^2 - \lambda_N m) = 0$$

$$(7.2.2) \quad (1-\beta\lambda_w(1+s)\theta)(1+s)\theta - m = 0$$

$$(7.2.3) \quad (1-\beta\lambda_N\theta)\theta - m = 0$$

Solving these equations gives:

$$(7.2.4) \quad \lambda_w = \frac{(1+s)\theta-m}{\beta\theta^2(1+s)^2}$$

(7.2.5) $\lambda_N = \dfrac{\theta - m}{\beta\theta^2}$

(7.2.6) $\lambda = \dfrac{2\beta\theta^2(1+s)^2(\theta-m) + ms(1+s) - m(s+2)}{2\beta^2\theta^4(1+s)^2}$

Example III.a. Assume $\theta = m = 1$.

Equations (7.2.4) - (7.2.6) become

$$\lambda_w = \frac{s}{\beta(1+s)^2}$$

$$\lambda_N = 0$$

$$\lambda = \frac{s^2}{2\beta^2(1+s)^2}$$

This equilibrium is represented in Figure 9. People who were unemployed in the first period will not work in the second period either. It is interesting to compare λ_w and λ since this is the probability of employment over time for workers who work in period one. Comparing the expressions for λ_w and λ immediately gives:

$$\lambda_w > \lambda \text{ iff } s < 2\beta$$

This result claims that if the return to experience is not too large then the fraction of the first period employed people who work in the second period is greater than the fraction of the population working in the first period. In other words, if s is not too large, the substitution effect caused by the higher productivity is greater than the associated income effect. Note that because $\lambda_N = 0$, total employment will always be decreasing over time.

Example III.b. $m = \frac{\theta}{k} = 1$, where $k > 0$.

Substitution into (7.2.4) - (7.2.6) gives:

$$\lambda_w = \frac{(1+s)k-1}{\beta k^2(1+s)^2}$$

$$\lambda_N = \frac{k-1}{\beta k^2}$$

$$\lambda = \frac{2\beta k^2(1+s)^2(k-1) + s(2k(1+s) - (s+2))}{2\beta^2 k^4(1+s)^2}$$

This equilibrium is represented in Figure 10. Figure 11 depicts the equilibrium for $\theta=2$, $m=1$. Again, it is of interest to compare the relative magnitudes of the λ's. First, we consider λ_w and λ_N. Comparing the expressions for λ_w and λ_N gives

$$\lambda_w > \lambda_N \text{ if } k < \frac{s+2}{s+1} \text{ (or equivalently } s < \frac{2-k}{k-1})$$

$$\lambda_w < \lambda_N \text{ if } k > \frac{s+2}{s+1} \text{ (or equivalently } s > \frac{2-k}{k-1}).$$

Note that $k = \frac{\theta}{m}$ so k represents the ratio of the marginal product to the marginal disutility of work. Hence, the interpretation of the above result is that for a given value of s, when the marginal product is high relative to the marginal disutility of work, then $\lambda_N > \lambda_w$, i.e. a greater fraction of the first period unemployed work in period two than of the first period employed. Similarly, for k fixed, the lower the value of s, the more likely it is that $\lambda_N > \lambda_w$.

We now consider the comparison between λ_w and λ. Manipulation of the expressions for these two variables gives the following result:

$$\lambda_w > \lambda \text{ if } s^2(1-2k-2\beta k^2(k-1)) + 2s(\beta k^2-k+2) - 2\beta k^3 > 0$$

and $\lambda_w < \lambda$ otherwise.

It should be clear that depending upon the parameter values it is possible to get both $\lambda_w > \lambda$ and $\lambda_w < \lambda$.

Section 8: Evolution of the Society in a Multiperiod Environment

The most significant distinguishing feature between the perfect and imperfect pooling is the behaviour of individuals in period two conditional upon their period one allocations. In the perfect pooling case, all individuals have identical consumption in period two but the unemployed from period one never supply labor. In the imperfect pooling case the employed from period one always have a higher period two consumption than do the period one unemployed. Also, the period one unemployed face some non-zero probability of working in period two. Stated more generally, the imperfect pooling case seems to offer more interesting possibilities in analyzing differential behaviour over time among employed and unemployed workers.

Moreover, it is interesting to note the implications that the imperfect pooling arrangement has on the evolution of the social structure of the society. Each period, new lotteries are added and these lotteries, in turn, produce a further separation of the society into new economic groups.

The exact way in which the class division operates depends crucially on the assumption about the way in which experience accumulates overtime. As an example, consider the intuitive case in which experience accumulation depends only on the total number of periods in which the individual was employed, independently of whether the employment periods were consecutive or not.

Consider Figure 11A. In this particular example, at the beginning of period n, n-economic groups will be present, and n-lotteries will be organized. In general, groups will differ in their consumption bundle, their productivity, their wage and their probability of subsequent employment. The complexity of the social structure increases as time elapses.

We believe that the ability to produce such a rich economic structure is an appreciable characteristic of this model.

Moreover, this particular framework is able to generate dynamic mobility across different groups. In other words, every individual in the society faces a positive probability of moving upward or downward in the social structure. This extended framework not only determines the allocation of income among economic groups in the society, but also endogenously generates dynamic mobility of individuals across these economic groups. This setting, therefore, seems to be well suited to dealing with problems related to income distribution which are rarely addressed and difficult to handle in strictly representative agent models.

Section 9: Imperfect Pooling With Borrowing and Lending

In the last two sections we characterized equilibrium allocations for the imperfect pooling case. One of the properties of the equilibrium allocations was that all agents received the same consumption in the first period but different consumptions in the second period. This suggests that opportunities exist for borrowing and lending. Recall that in the perfect pooling equilibrium no such opportunities exist because all agents had the same consumption in any given period (although consumption is not constant across time). The objective of this section is to discuss how credit markets can be introduced into the analysis and to demonstrate by way of an example that allowing for a credit market does not appreciably change the qualitative differences between the perfect and imperfect pooling equilibria.

In the imperfect pooling equilibrium some care must be taken as to how the credit market is introduced. Consideration of the imperfect pooling equilibrium is equivalent to assuming that certain markets do not exist, in particular, markets for insurance which link outcomes of the period one lottery to period two allocations. Hence, we do not want to assume markets for credit which imply transactions which were previously ruled out. What this amounts to is that borrowing and lending decisions cannot be made ex ante conditional on the outcome of the lottery, because this would be tantamount to allowing for the markets which were earlier ruled out. This implies that agents should only be allowed to borrow and lend after the realization of the first period lottery. This, of course, does not imply that agents do not take this into account when the first period lottery is determined.

Intuitively, it is fairly clear how introducing the credit market will affect allocations. Without these credit markets we know that all agents have the same period one consumption but that experienced workers consume more in period two. Because both experienced and inexperienced workers would like to smooth consumption over time (given the nature of preferences) it follows that experienced people would like to borrow from the inexperienced workers in period one and pay them back in period two. Given the consumption profiles, it is also clear that inexperienced workers are also willing to take part in this transaction. The aspect of the analysis which is difficult is that these borrowing and lending arrangements will affect the labor supply in period one, hence affecting the relative sizes of the experienced and inexperienced groups which, in turn, affects the possibilities for borrowing and lending which, of course, affects labor supply in period one.

Technically, it is not difficult to deal with this problem. The solution involves solving for the equilibrium recursively. That is, taking period one employment decisions as given, it is possible to determine the equilibrium interest rate and the lifetime allocations that result. Note that λ, the fraction of agents who work, is the only variable needed to characterize the first period decision. This procedure produces a function which maps λ into lifetime allocations for all individuals. Given this function, the last step is to find the value of λ which maximizes expected lifetime utility. Although this problem is not difficult in principle, it turns out to be considerably more intractable than any of the other cases dealt with. As a result, this section demonstrates how to solve for equilibrium with borrowing and lending for one specification of the environment. Later we compare the results with those obtained for the other equilibrium concepts and the same specification.

Assume the following specification:

$$u(c) = \ell n(c)$$

$$f(N) = \theta N$$

We will compute the equilibrium recursively. Hence, assume λ has been determined and the result of the first period lottery is known, we solve for the resulting equilibrium. Each agent has income in period one equal to $\lambda\theta$ as a result of the first period lottery. Each group will then solve the following problems:

Employed individuals:

(E)
$$\max_{c_{1w},c_{2w},\lambda_w} \ln(c_{1w}) - m + \ln(c_{2w}) - \lambda_w m$$
$$\text{s.t. } c_{1w} + \frac{c_{2w}}{1+r} \leq \lambda\theta + \frac{\lambda_w(1+s)\theta}{1+r}$$
$$c_{1w} \geq 0 \quad c_{2w} \geq 0 \quad 0 \leq \lambda_w \leq 1$$

Unemployed individuals:

(U)
$$\max_{c_{1N},c_{2N},\lambda_N} \ln(c_{1N}) + \ln(c_{2N}) - \lambda_N m$$
$$\text{s.t. } c_{1N} + \frac{c_{2N}}{1+r} < \lambda\theta + \frac{\lambda_N\theta}{1+r}$$
$$c_{1N} \geq 0 \quad c_{2N} \geq 0 \quad 0 \leq \lambda_N \leq 1.$$

substituting for c_{1w} and c_{1N} using the respective budget constraints, produces the following first order conditions:

(E.1) $$\left(\lambda\theta + \frac{\lambda_w\theta(1+s)}{1+r} - \frac{c_{2w}}{1+r}\right)^{-1}\left(\frac{\theta(1+s)}{1+r}\right) = m \qquad (\lambda_w)$$

(E.2) $$\left(\lambda\theta + \frac{\lambda_w\theta(1+s)}{1+r} - \frac{c_{2w}}{1+r}\right)^{-1}\left(\frac{1}{1+r}\right) = \frac{1}{c_{2w}} \qquad (c_{2w})$$

(U.1) $$\left(\lambda\theta + \frac{\lambda_N\theta}{1+r} - \frac{c_{2N}}{1+r}\right)^{-1}\left(\frac{\theta}{1+r}\right) = m \qquad (\lambda_N)$$

(U.2) $$\left(\lambda\theta + \frac{\lambda_N\theta}{1+r} - \frac{c_{2N}}{1+r}\right)^{-1}\left(\frac{1}{1+r}\right) = \frac{1}{c_{2N}} \qquad (c_{2N})$$

Substituting (E.1) into (E.2) gives

(E.3) $$c_{2w} = \frac{\theta(1+s)}{m}$$

Similarly, combining (U.1) and (U.2) gives

(U.3) $$c_{2N} = \frac{\theta}{m}$$

Note that second period consumptions are determined independently of (1+r) and λ, i.e. independently of the interest rate and initial income.

(E.2) and (U.2) can now be solved for λ_w and λ_N, giving:

(E.4) $$\lambda_w = \frac{2}{m} - \frac{(1+r)\lambda}{(1+s)}$$

(U.4) $$\lambda_N = \frac{2}{m} - (1+r)\lambda$$

Substituting into the budget equations gives c_{1w}, c_{1N}:

(E.5) $$c_{1w} = \frac{(1+s)\theta}{(1+r)m}$$

(U.5) $$c_{1N} = \frac{\theta}{(1+r)m}$$

The interest rate is determined by the condition that net saving is equal to zero, or equivalently, total production is equal to total consumption. So the condition

$$\lambda c_{1w} + (1-\lambda)c_{1N} = \lambda\theta$$

will determine the equilibrium interest rate as a function of λ. Once this is determined, we then know c_{1w}, c_{2w}, λ_w, c_{1N}, c_{2N} and λ_N as a function of only λ.

This condition is:

$$(1-\lambda)\frac{\theta}{(1+r)m} + \frac{\lambda(1+s)\theta}{(1+r)m} = \lambda\theta$$

Manipulation of this yields

$$(1+r) = \frac{1+\lambda}{\lambda m}$$

As a function of λ the equilibrium is:

$$\lambda_w = \frac{2}{m} - \frac{(1+\lambda s)}{(1+s)m}$$

$$c_{1w} = \frac{(1+s)\lambda\theta}{(1+\lambda s)}$$

$$c_{2w} = \frac{\theta(1+s)}{m}$$

$$\lambda_N = \frac{2}{m} - \frac{(1+\lambda s)}{m}$$

$$c_{1N} = \frac{\lambda\theta}{(1+\lambda s)}$$

$$c_{2N} = \frac{\theta}{m}$$

Using these values as functions of λ we can now determine the value of λ that maximizes expected utility of the representative worker. This corresponds to the following problem:

$$\max_{\lambda} \lambda[\ln(c_{1w}) - m + \ln(c_{2w}) - \lambda_w m] + (1-\lambda)[\ln(c_{1N}) + \ln(c_{2N}) - \lambda_N m]$$

$$\text{s.t. } 0 \leq \lambda \leq 1$$

where it is understood that all of the arguments are functions of λ. The first order condition for this problem is

$$\ln(c_{1w}) + \ln(c_2 w) - m - \lambda_w m - \ln(c_{1N}) - \ln(c_{2N}) + \lambda_N$$

$$+ \lambda[\frac{1}{c_{1w}} c'_{1w} + \frac{1}{c_{2w}} c'_{2w} \lambda - m\lambda'_w]$$

$$+(1-\lambda)[\frac{1}{c_{1N}} c'_{1N} + \frac{1}{c_{2N}} c'_{2N} - m\lambda'_N] = 0$$

Substitution gives:

$$2\ln(1+s) - m - \frac{(1+\lambda s)s}{1+s} + \frac{s\lambda}{1+s} + \frac{1}{\lambda(1+\lambda s)} + s - \lambda s = 0$$

Figure 12 through 17 give a graphic representation of the equilibrium values of the principal variables, as a function of s, when m = 3. As expected, when s = 0, the equilibrium with and without borrowing coincide, with the difference that now we are able to compute the equilibrium interest rate. Since there is no heterogeneity in the population, no trade in the credit market actually occurs.

For values of s > 0, λ_1 and λ_w are greater when credit markets are present, while λ_N and λ_2 are smaller. This is in accord with our intuition, i.e. experienced people have the incentive to increase their labor supply (λ_w) in the second period since the return can be spread intertemporally. For the same reason the marginal utility of being experienced increases, so that λ increases. This is reflected in a larger consumption in the first period (c_{1w}) than without the credit market. It is interesting to notice that consumption in the first period for the unemployed is lower than without the credit market, as expected, but that consumption in the second period is unaltered. This seemingly puzzling result is resolved by observing that the unemployed increase their utility by greatly reducing the amount of labor supplied in the second period (λ_N).

Finally, the interest rate tends to be increasing in s, even if not strictly monotonically. This reflects an increase in the relative abundance of goods in the second period caused by the higher level of s

Section 10: Conclusions

In this paper we show how the theoretical device of the lottery can be fruitfully used to describe the intertemporal allocation of resources in a world characterized by technological imperfection (i.e. labor indivisibility and non-instantaneous learning process).

The major motivation for this was to examine to what degree this type of environment could produce interesting results concerning the behaviour of unemployed individuals. Previous models utilizing indivisible labor without experience accumulation resulted in unemployed individuals having higher instantaneous utility than employed individuals and both groups having identical future opportunities. Future employment and consumption paths are independent of current employment status. This paper has shown that experience accumulation leads to a restricted class of lotteries which are sustainable and that this fact produces interesting behavioural differences for employed and unemployed workers. The two main implications are that although unemployed workers have higher instantaneous utility than the employed, their future utility prospects are lowered as a result of being unemployed. Second, employment probabilities are state dependent; unemployed and employed workers have different probabilities of future employment. It should be emphasized that all of this behaviour is due entirely to the real rigidities of indivisible labor and experience accumulation. Without these two features the economy would become a standard example of a purely convex representative agent economy with no unemployment, no ex post heterogeneity and none of the above results.

Finally, we would like to address here the frequently posed problem of the empirical relevance of the lottery. Our point of view is that it is not crucial that in reality we do not observe entities that perfectly replicate a fair lottery. The essential features of the lottery are the random allocation of a limited amount of jobs across the population, and the redistribution of income from the recipients of a salary to the unemployed. We believe that several modern societies share these characteristics. Even if we do not observe formal drawings which divide the population into employed and unemployed, searching procedures could be interpreted as a special kind of lottery. Moreover, it is likely that in the real world, job allocation does not obey an ex-ante fairness criteria typical of a lottery. However, in our opinion, what is essential is that there exists some randomness associated with the job allocation mechanism so that individuals are willing to pool in order to insure themselves against the unlucky outcome. We think that this pooling mechanism is actually observable in institutions like unemployment insurance programs. The view that the actual existence of the lottery is, in a sense, irrelevant is shared by a non economist, who we already cited several times before in the paper. Borges concludes his short story "The Lottery in Babylon", as follows:

> One abominably insinuates that the Company [Lottery] has not existed for centuries and that the sacred disorder of our lives is purely hereditary, traditional. Another judges it eternal and teaches that it will last until the last night, when the last god annihilates the world. Another declares that the Company is omnipotent, but that it only has influence in tiny things: in a bird's call, in the shadings of rust and of dust, in the half dreams of dawn. Another, in the words of masked heresiarch, that is has never existed and will not exist. Another, no less vile, reasons that it is indifferent to affirm or deny the reality of the shadowy corporation, because Babylon [life] is nothing else than an infinite game of chance.

Footnotes

1. In fact, in a slightly different setting, Greenwood and Huffman provide such an example.

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FIGURE 1

PERFECT POOLING, LOGARITHMIC UTILITY, THETA=1 M=1 BETA=1
RELATIONSHIP BETWEEN EMPLOYMENT RATIO AND EXPERIENCE

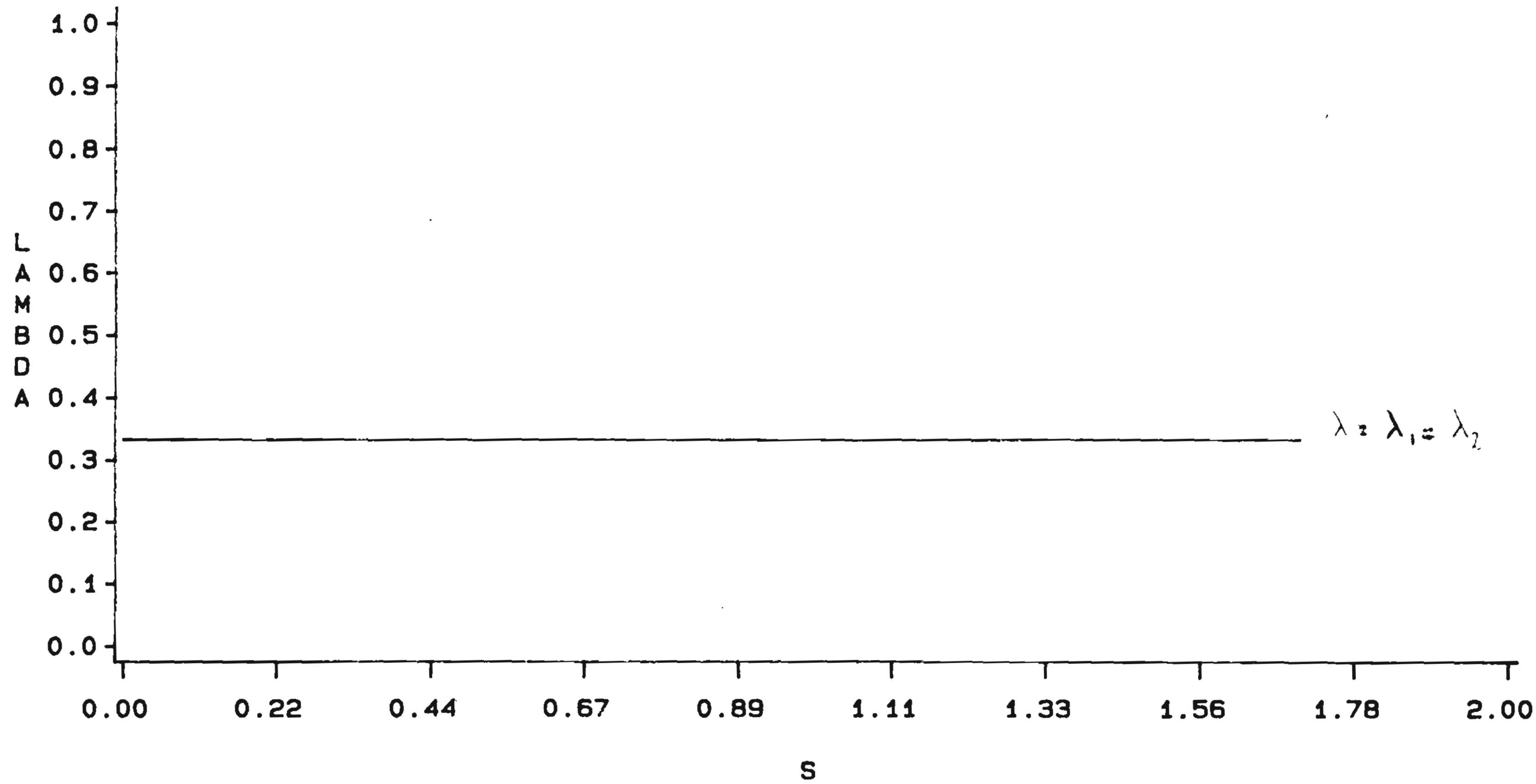


FIGURE 2

PERFECT POOLING, QUADRATIC UTILITY, THETA=1 M=1 BETA=1
RELATIONSHIP BETWEEN EMPLOYMENT RATIO AND EXPERIENCE

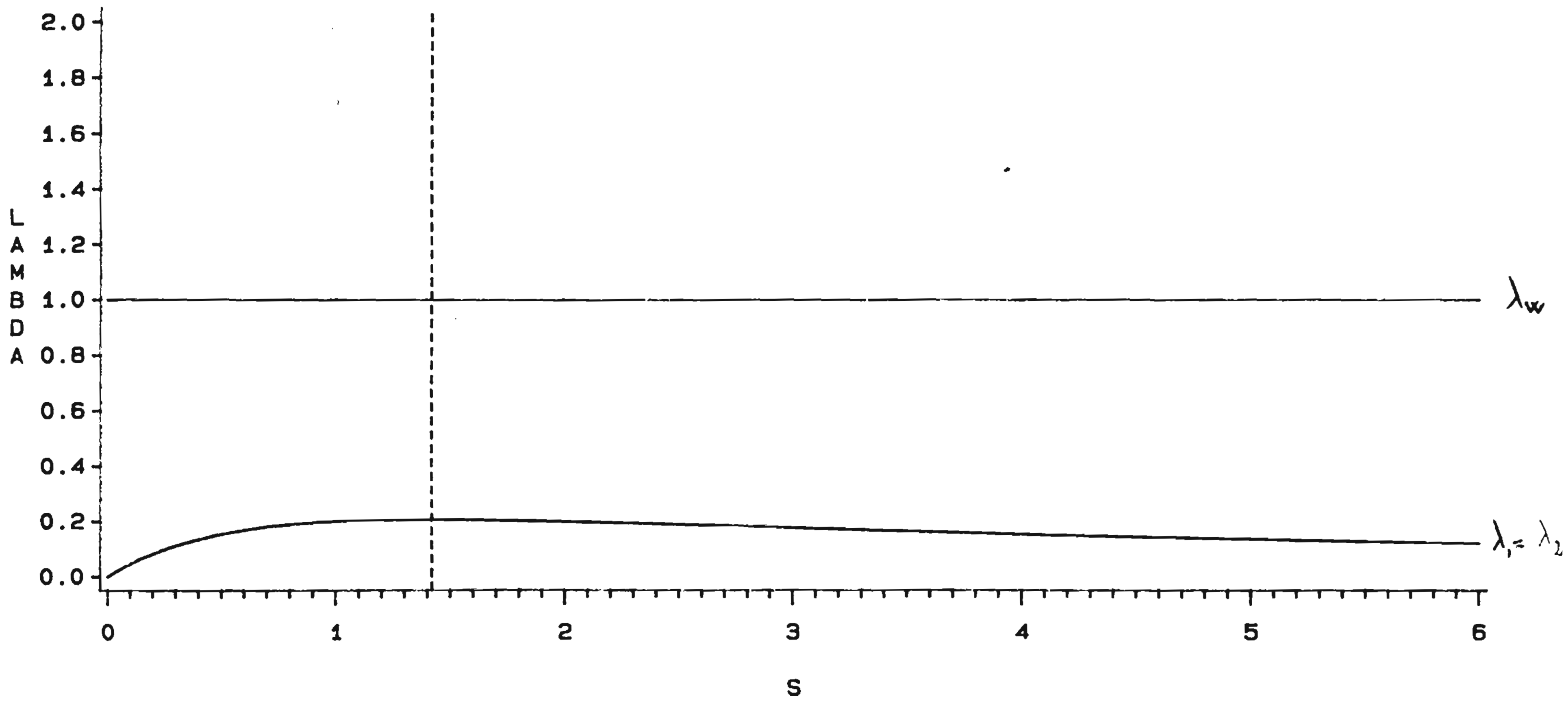


FIGURE 3

PERFECT POOLING, QUADRATIC UTILITY, THETA=2 M=1 BETA=1
RELATIONSHIP BETWEEN EMPLOYMENT RATIOS AND EXPERIENCE

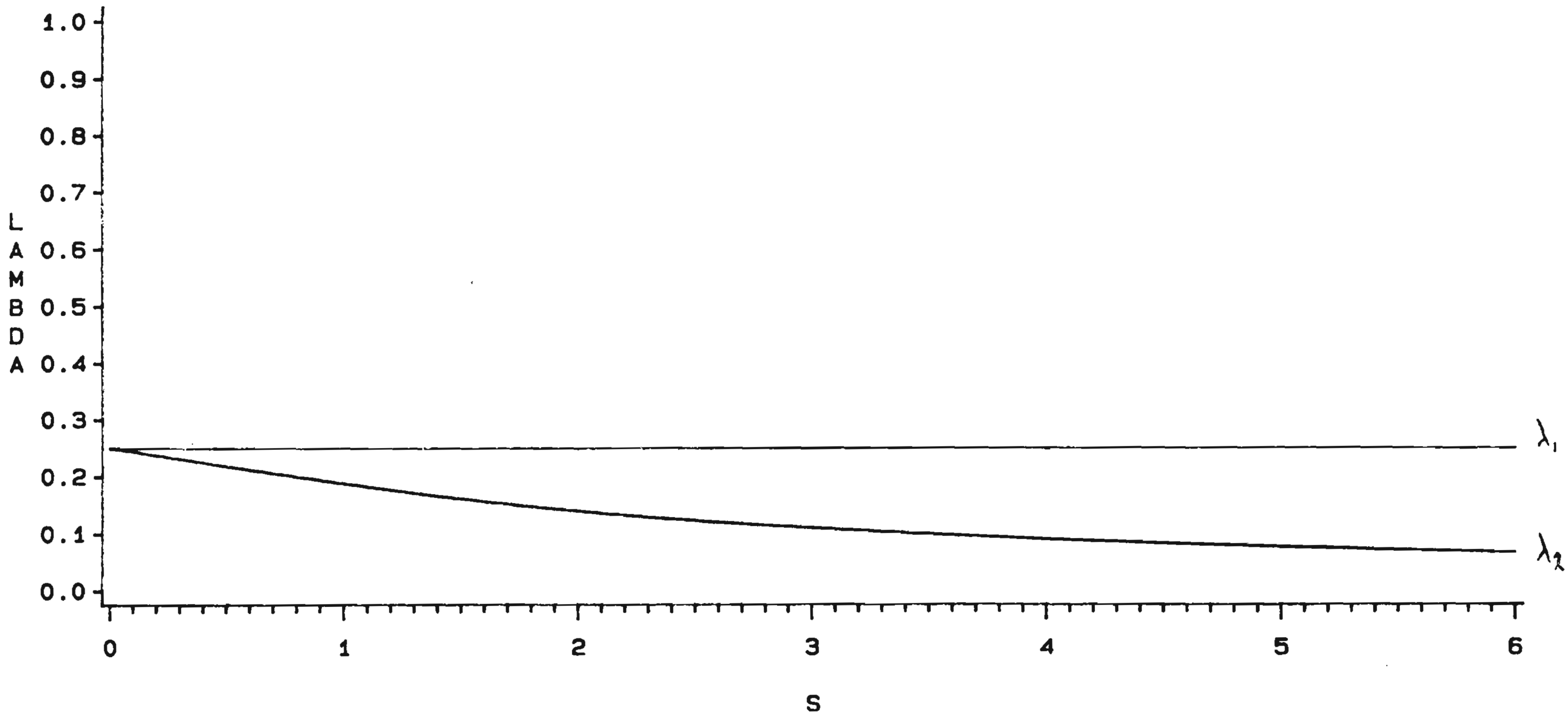


FIGURE 4

PERFECT POOLING, QUADRATIC UTILITY, THETA=3/2 M=1 BETA=1
RELATIONSHIP BETWEEN EMPLOYMENT RATIOS AND EXPERIENCE

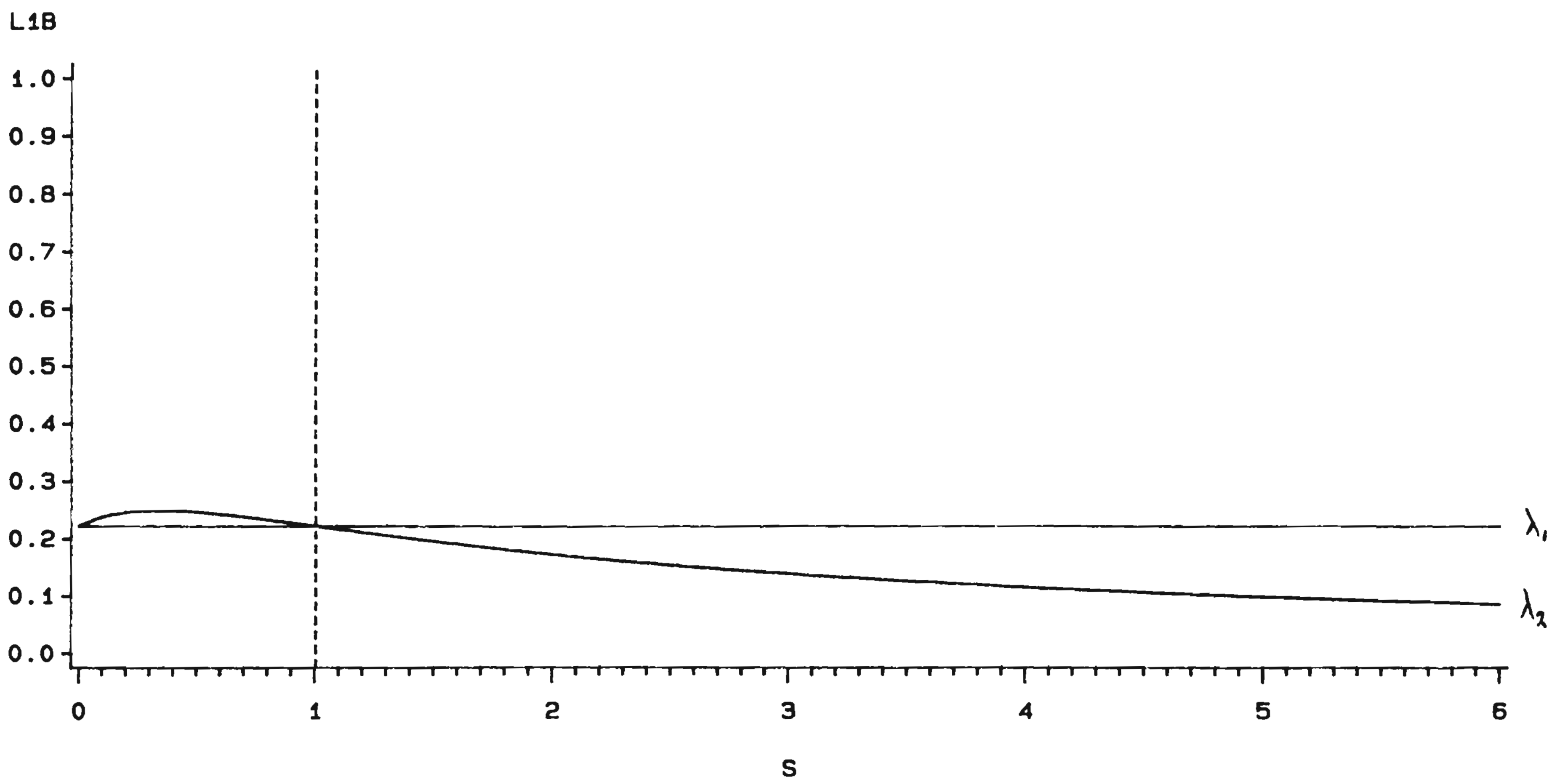


FIGURE 5

IMPERFECT POOLING, CONSTANT RELATIVE RISK AVERSION UTILITY, THETA=1 M=4

RELATIONSHIP BETWEEN EMPLOYMENT RATIOS AND EXPERIENCE

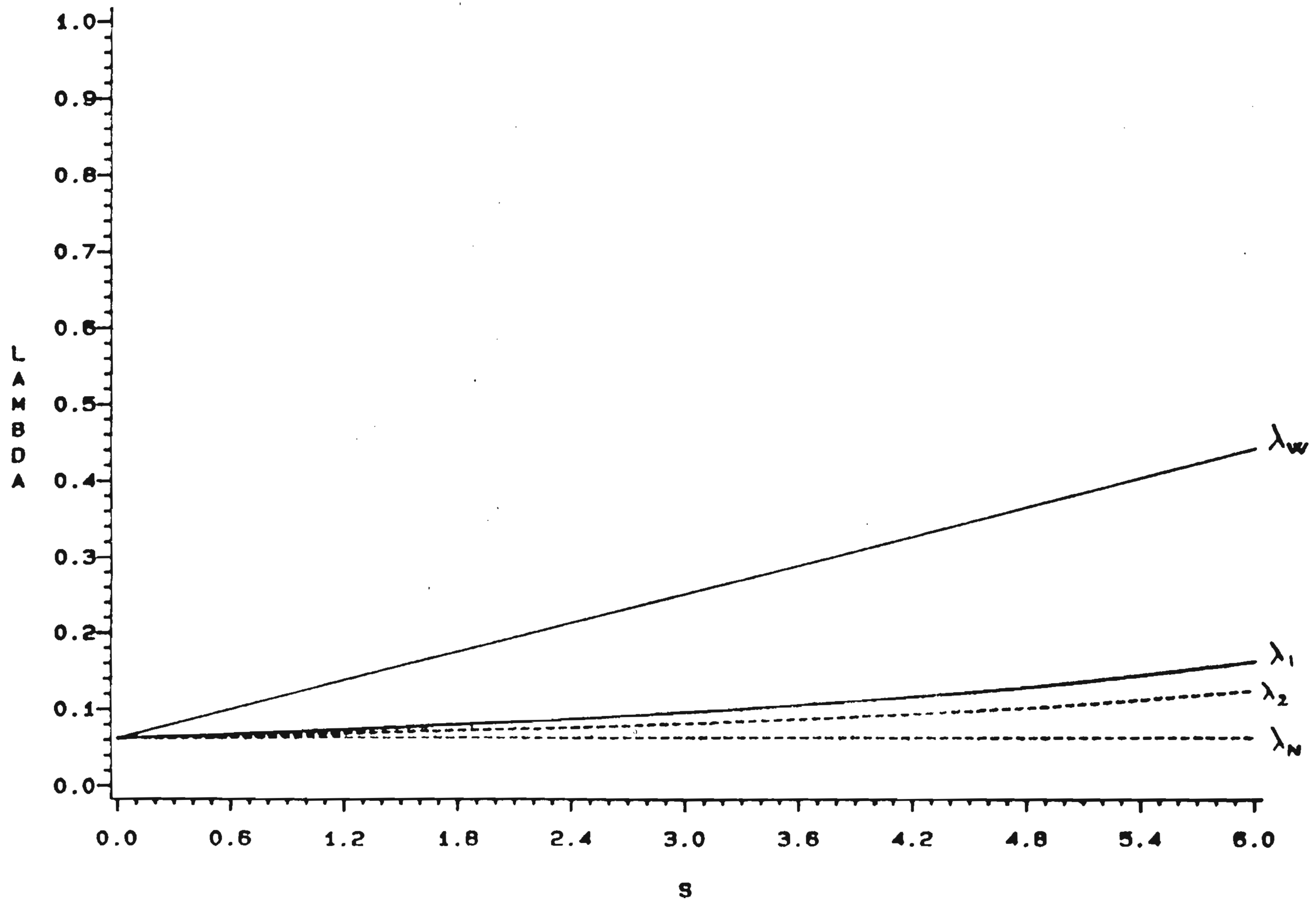


FIGURE 6

IMPERFECT POOLING, CONSTANT RELATIVE RISK AVERSION UTILITY, THETA=1 M=3
RELATIONSHIP BETWEEN EMPLOYMENT RATIOS AND EXPERIENCE

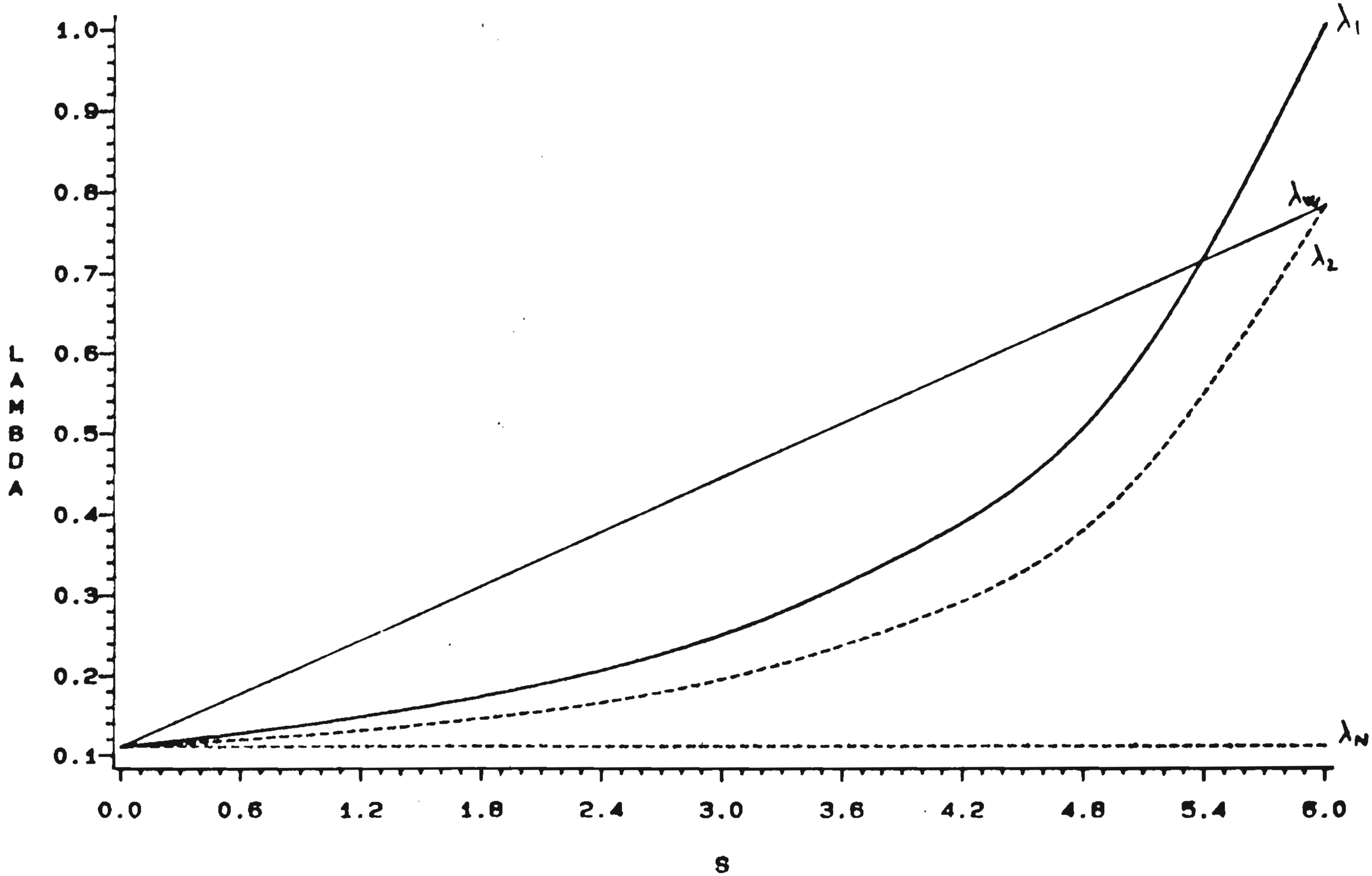


FIGURE 7

IMPERFECT POOLING, LOGARITHMIC UTILITY, THETA=1 M=3
RELATIONSHIP BETWEEN EMPLOYMENT RATIOS AND EXPERIENCE

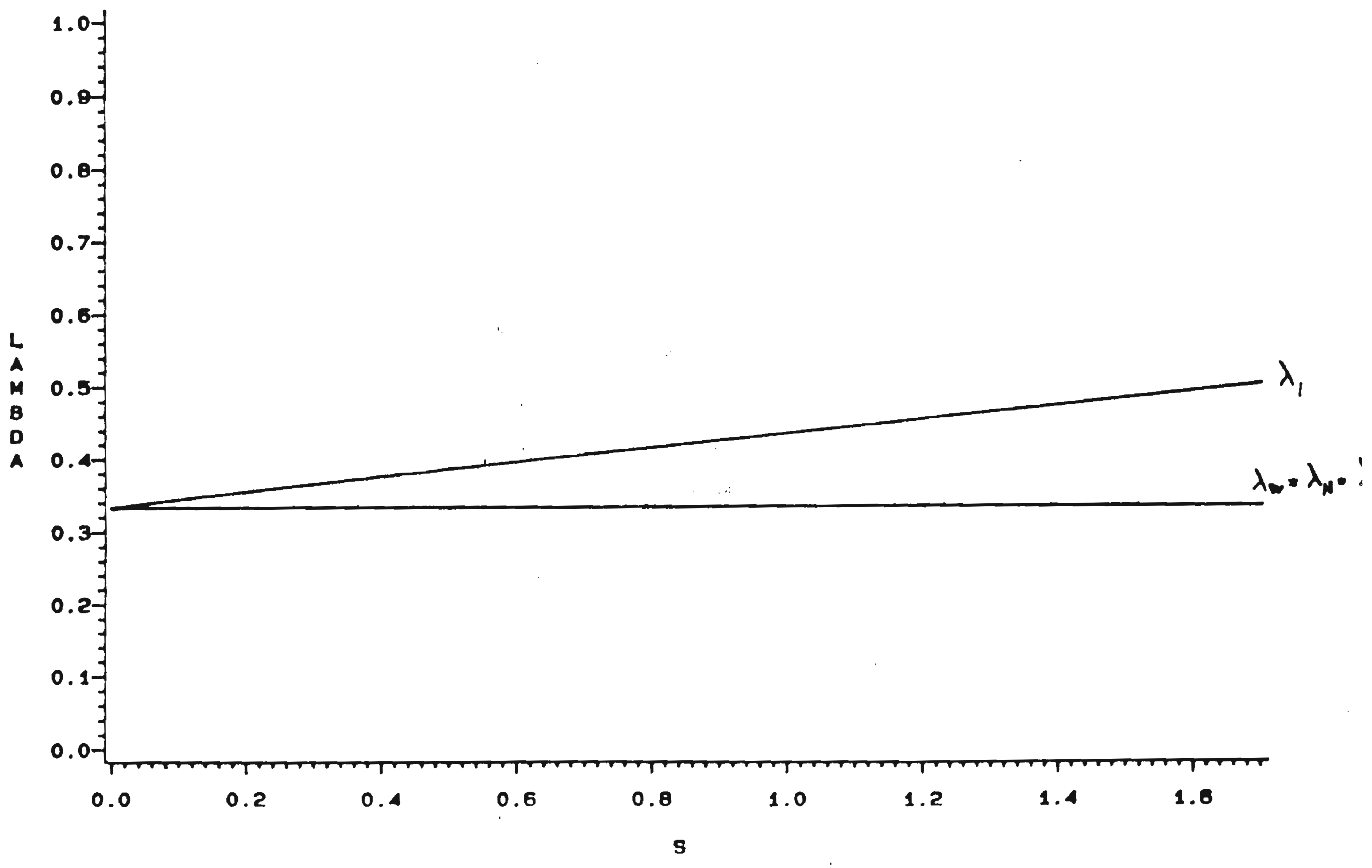


FIGURE 8

IMPERFECT POOLING, LOGARITHMIC UTILITY, THETA=1 M=3
RELATIONSHIP BETWEEN CONSUMPTION AND EXPERIENCE

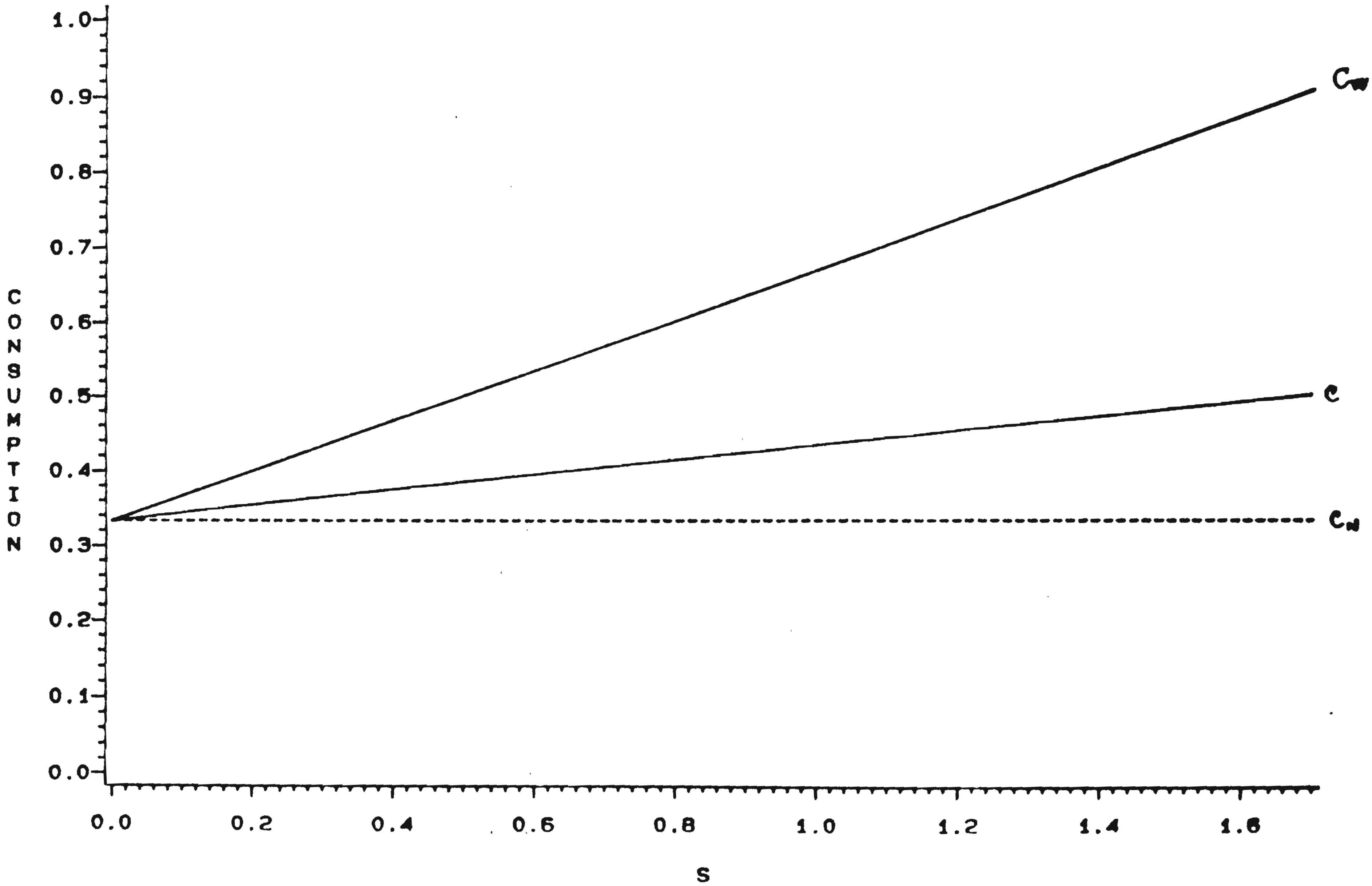


FIGURE 9

IMPERFECT POOLING, QUADRATIC UTILITY, THETA=1 M=1 BETA=1
RELATIONSHIP BETWEEN EMPLOYMENT RATIOS AND EXPERIENCE

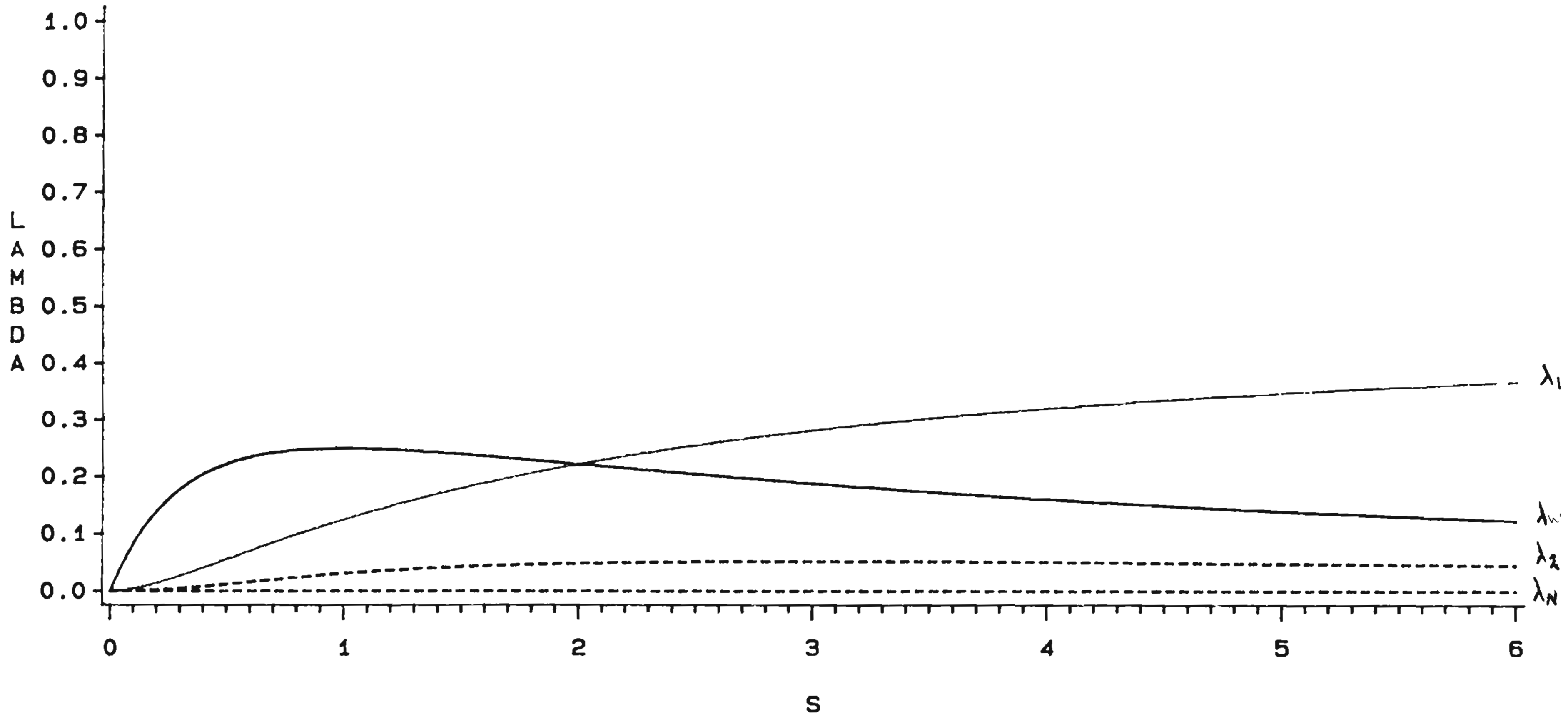


FIGURE 10

IMPERFECT POOLING, QUADRATIC UTILITY, THETA=2 M=1 BETA=1
RELATIONSHIP BETWEEN EMPLOYMENT RATIOS AND EXPERIENCE

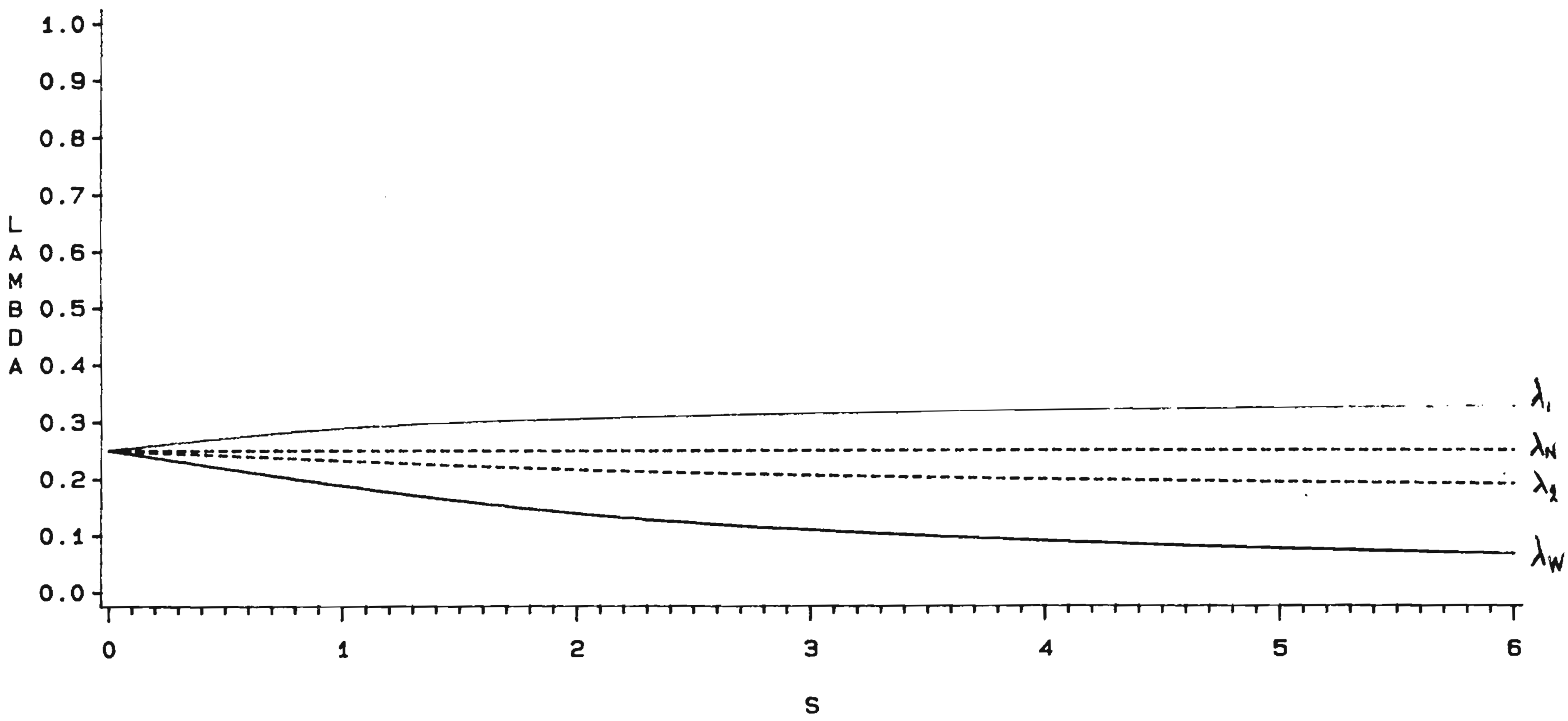


FIGURE 11

IMPERFECT POOLING, QUADRATIC UTILITY, THETA=3 M=1 BETA=1
RELATIONSHIP BETWEEN EMPLOYMENT RATIOS AND EXPERIENCE

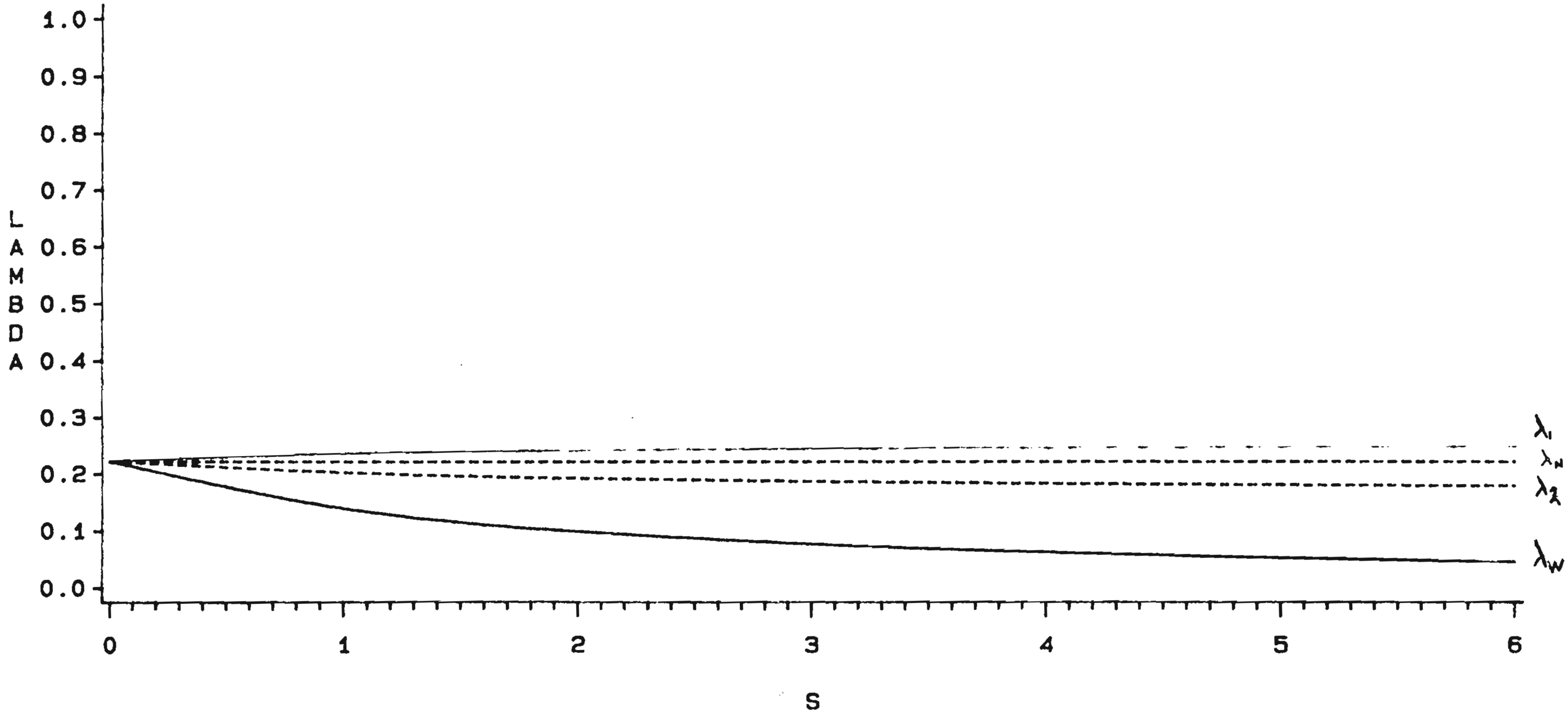


FIGURE 12

IMPERFECT POOLING WITH CREDIT MARKETS. LOGARITHMIC UTILITY. THETA=1 M=3

RELATIONSHIP BETWEEN EMPLOYMENT RATIOS AND EXPERIENCE

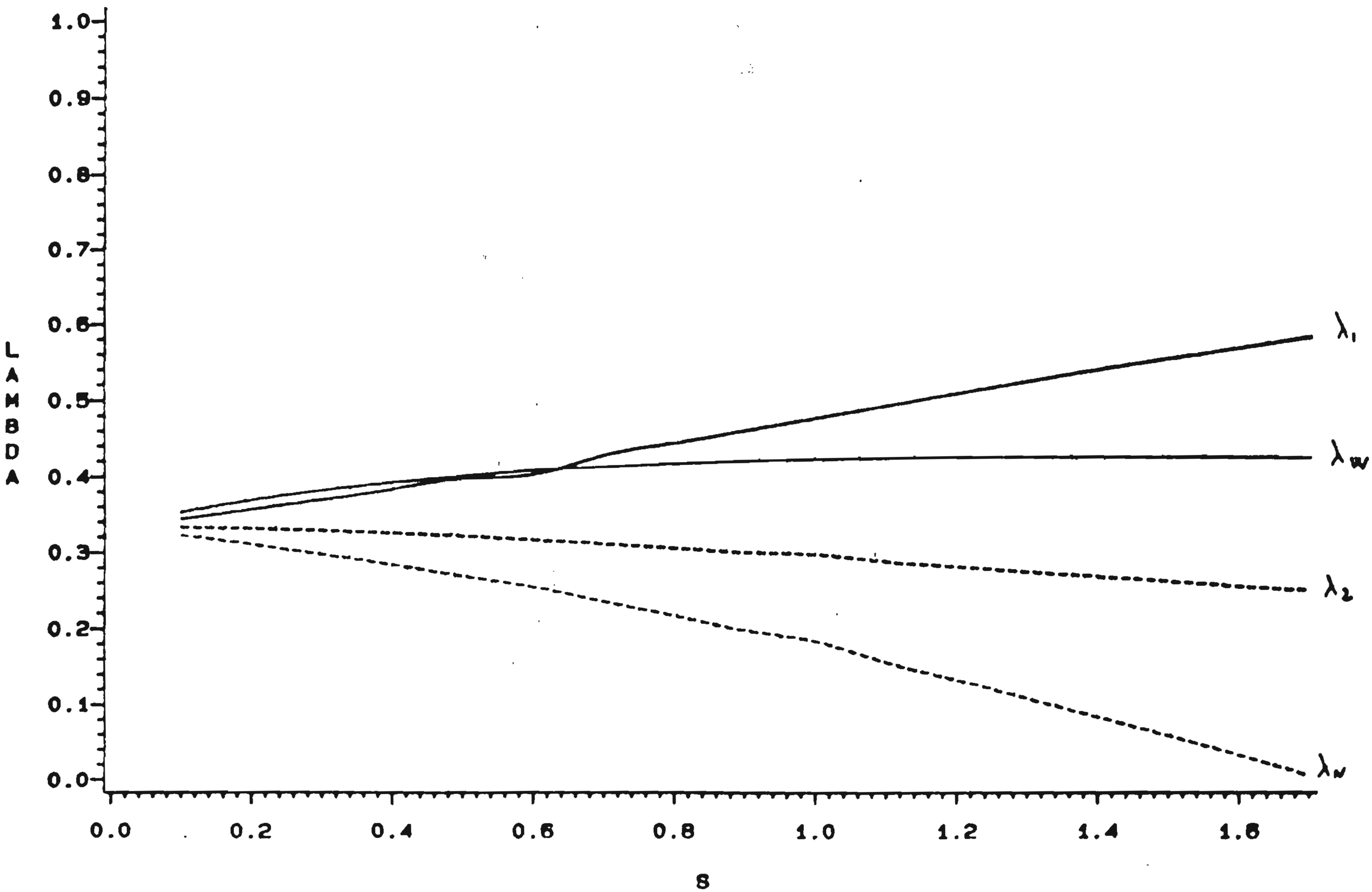


FIGURE 13

IMPERFECT POOLING WITH CREDIT MARKETS, LOGARITHMIC UTILITY, RELATIONSHIP BETWEEN CONSUMPTION AND EXPERIENCE THETA=1 M=3

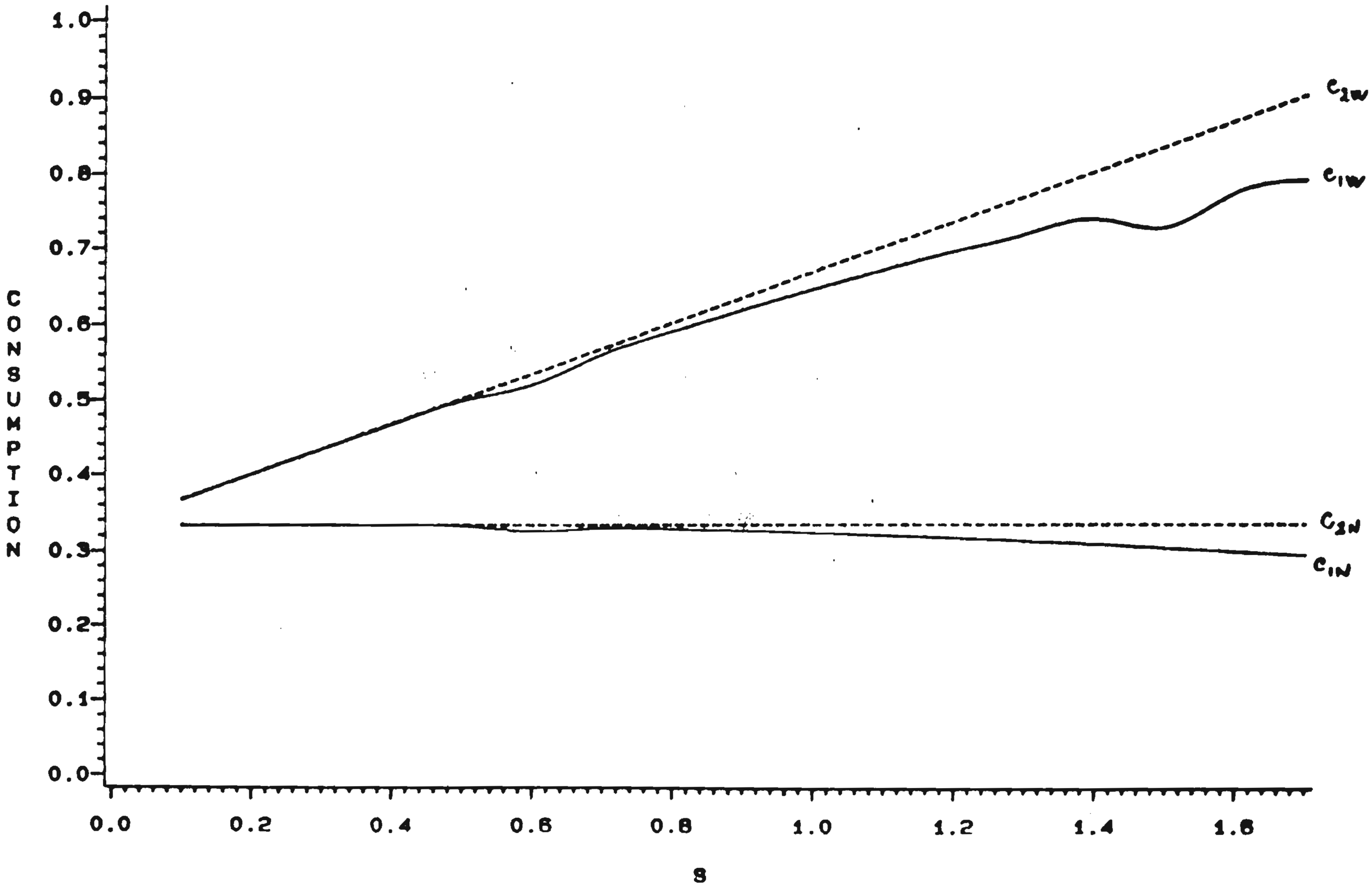


FIGURE 14

IMPERFECT POOLING WITH CREDIT MARKETS, LOGARITHMIC UTILITY, THETA=1 M=3
POPULATION EMPLOYED
RELATIONSHIP BETWEEN CONSUMPTION,
INTEREST RATE AND EXPERIENCE

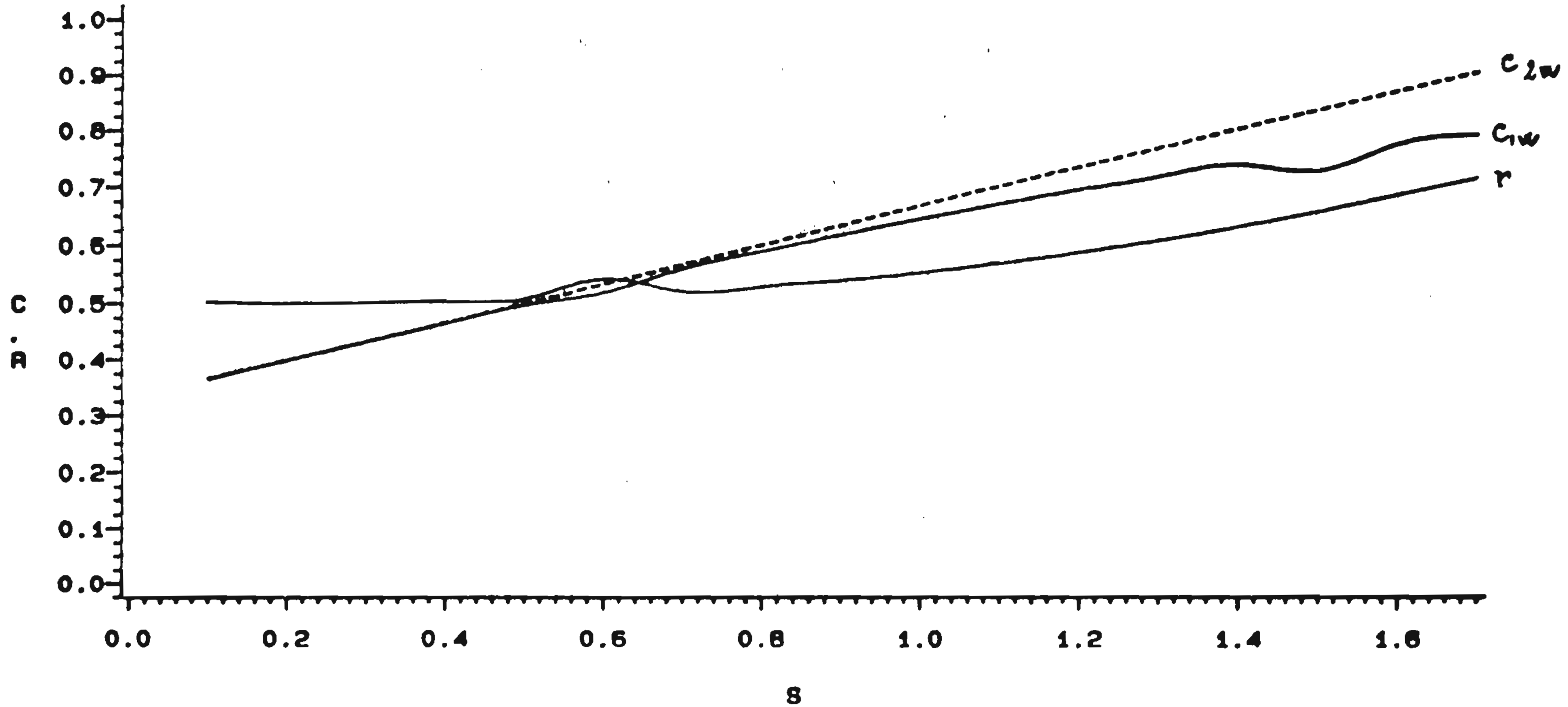


FIGURE 15

IMPERFECT POOLING WITH CREDIT MARKETS, LOGARITHMIC UTILITY, POPULATION EMPLOYED

RELATIONSHIP BETWEEN EMPLOYMENT RATIOS, INTEREST RATE AND EXPERIENCE

THETA=1 M=3

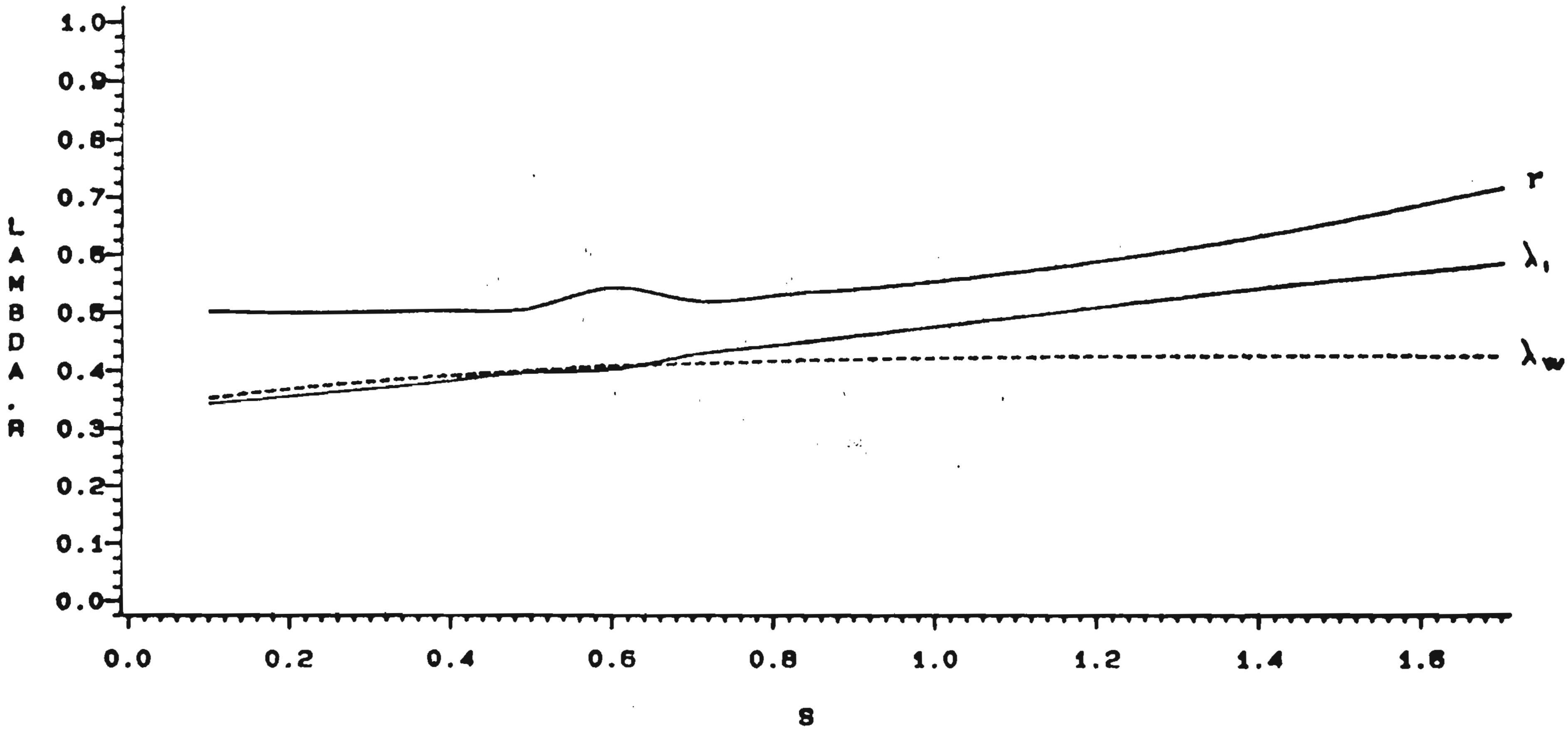


FIGURE 16

IMPERFECT POOLING WITH CREDIT MARKETS. LOGARITHMIC UTILITY. THETA=1 M=3

POPULATION UNEMPLOYED

RELATIONSHIP BETWEEN CONSUMPTION, INTEREST RATE AND EXPERIENCE

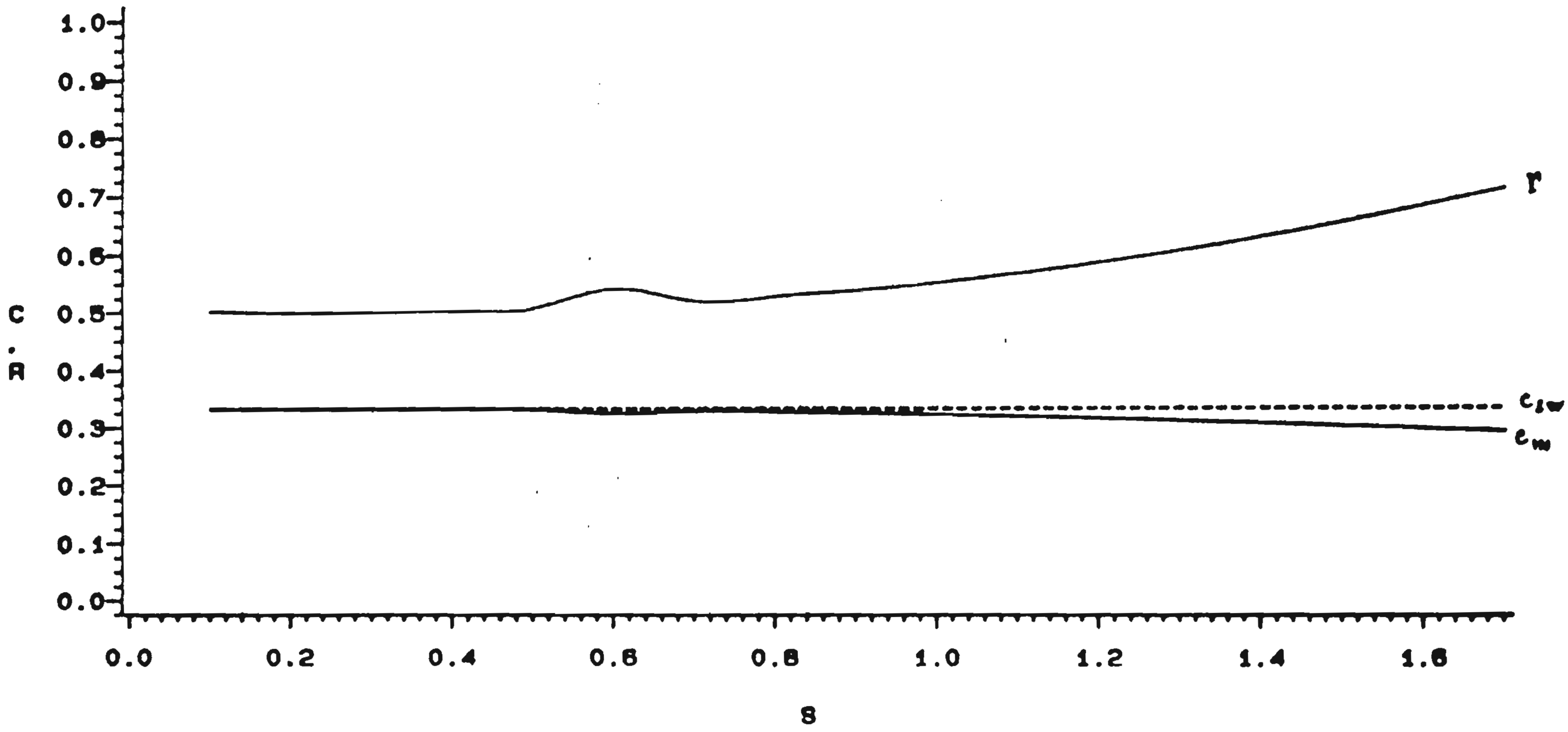
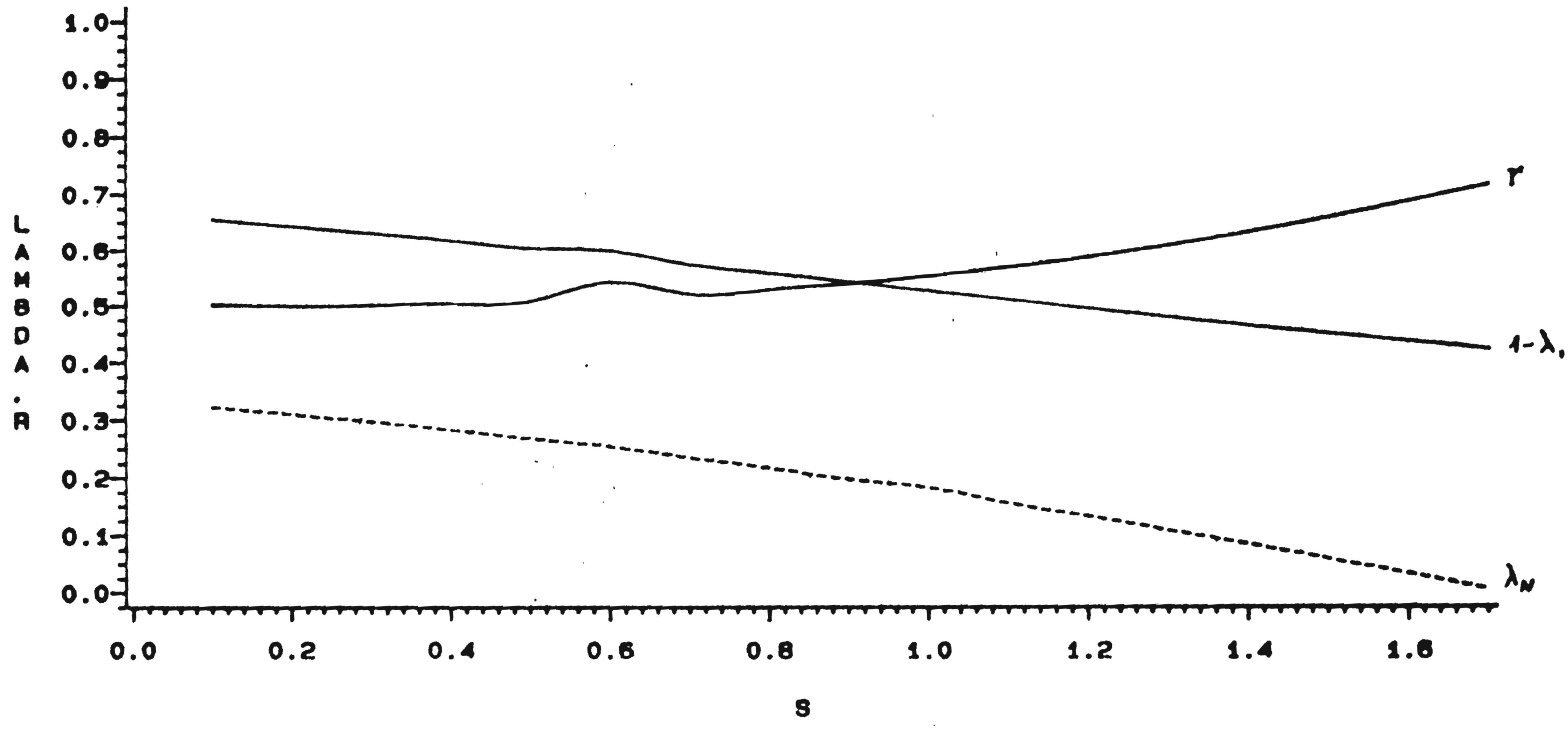


FIGURE 17

IMPERFECT POOLING WITH CREDIT MARKETS, LOGARITHMIC UTILITY,
POPULATION UNEMPLOYED
RELATIONSHIP BETWEEN EMPLOYMENT RATIOS,
INTEREST RATE AND EXPERIENCE

THETA=1 M=3



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