An Equilibrium Model of Involuntary Employment

Rogerson, Richard

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Richard Rogerson*

University of Rochester
Rochester, NY 14627

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Abstract

This paper argues that one possibility of measuring involuntary unemployment empirically is to consider data on unemployment by reason of unemployment i.e. job losers, job leavers, entrants, re-entrants. It then gives a model in which involuntary unemployment is defined as a property of an allocation rather than a property of the equilibrium concept, and then shows for a particular class of environments that equilibrium allocations may involve involuntary unemployment. The result is obtained despite the fact that there are complete markets, no private information and no heterogeneity of agents.

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SECTION 1

INTRODUCTION

Considerable attention has been devoted in the last decade to models of aggregate fluctuations in the labour market which simultaneously satisfy some criterion of optimality and display some notion of involuntary unemployment. Among these is Chari [2]. In this paper we first argue that data exists which can help to distinguish voluntary and involuntary unemployment statistically. It then extends the environment of Rogerson [8] to include strictly convex preferences and shows that in this model equilbrium allocations may display involuntary unemployment even when there are complete markets, no heterogeneity of agents, no private information and all markets clear.

SECTION 2

INVOLUNTARY UNEMPLOYMENT

The usefulness of the concept of involuntary unemployment is a topic over which much heated debate has occurred. On one extreme people such as Lucas have argued:

> "Involuntary unemployment is not a fact or a phenomenon which it is the task of theorists to explain. It is, on the contrary, a theoretical construct which Keynes introduced in the hope that it would be helpful in discovering a correct explanation for a genuine phenomenon: large scale fluctuations in measured total employment. Is it the task of modern theoretical economics to explain the theoretical constructs of our predecessors, whether or not they have proved fruitful? I hope not for a surer route to sterility could scarcely be imagined."[1]

At the other extreme, Solow makes a strong appeal to one's casual empiricism and common sense when he states:

> "What looks like involuntary unemployment is involuntary unemployment."[2]

These two comments give one the impression that one of the main points of controversy concerns the issue of whether or not a state of unemployment exists which can in fact be identified in a well-defined manner statistically as being involuntary. Certainly one of the problems has been the nature of the original definition of involuntary unemployment put forth by Kenes:

> "Men are involuntarily unemployed if, in the event of a small rise in the price of wage goods, relative to the money wage, both the aggregate supply of labour willing to work for the current money-wage and the aggregate demand for it at that wage would be greater than the existing volume of employment."[3]

Clearly this does not correspond directly to any group of individuals who can be identified in labour force statistics.

In this section I argue that information which is currently collected in most Western European and North American countries does allow us to identify a group of unemployed individuals whose situation does display some feature of involuntary unemployment. Furthermore, on the basis of this identification it will be seen that large scale fluctuations in the amount of measured unemployment are accounted for mostly by fluctuations in the amount of involuntarily unemployed individuals.

Data collected about the unemployed in most countries now distinguishes among different causes of unemployment: job losers, job leavers, new entrants and (in some cases) reentrants. Data for the United States, Canada, France and West Germany are contained in the appendix. These figures display three prominant features:

1. The vast majority of the unemployed have been in the labour force previously.
2. On average a relatively small fraction of the unemployed are job leavers.
3. Fluctuations in unemployment are accounted for largely by fluctuations in the number of unemployed job losers.

The nature of the distinction between job leavers and job losers suggests that in one case the individual could have remained in his previous job but chose not to whereas in the other case the individual would have liked to remain in his job (as perhaps many of his coworkers did remain) but was unable to. Thus, one group has entered unemployment voluntarily while the other group has entered unemployment involuntarily. On this basis it seems natural to treat the job losers as being involuntarily unemployed. In a discussion of unemployment there are two questions to consider. One is why an individual becomes unemployed and the other is why an individual remains unemployed. Keynes emphasized the second question. He was not the first, however. Much earlier Beverirdge had written:

> "The distinction here made between the cause of displacement and the cause of continuing unemployment is no mere logical subtlety. It is indeed, a practical distinction. The most important practical question with regard to an unemployed man is not how he came to lose his last job but how it comes that he cannot get a fresh job now."[4]

The approach I have outlined emphasizes the reason for becoming unemployed. From the point of view of cyclical unemployment this seems to be appropriate. In a situation where most firms are dismissing workers the question of why a dismissed worker is not subsequently hired at a firm that is currently dismissing part of its own workforce appears to only be begging the question of why any of the workers were being dismissed in the first place.

The distinction between job losers and job leavers is not without difficulty. Institutional factors can have an influence on the manner in which separations are recorded. Typically individuals classified as job leavers are not eligible for unemployment insurance benefits and in some firms will lose their eligibility for severance payments. Sometimes workers quit because they know they will be dismissed in the near future. Despite these problems the regularities displayed in Tables 1-4 seem to be sufficiently robust. (Similar problems occur when identifying individuals as unemployed or not in the labour force.)

A different argument which has been raised by people such as Becker is that no distinction should be made between job losers and job leavers because all separations which occur result from a joint decision of the employer and employee, and the separation occurs only if the match is suboptimal. In short, the distinction between job losers and job leavers has no economic significance. This argument suffers from several weaknesses. First it denies the importance of an observable and measurable phenomenon by showing it to be incompatible with a certain theoretical framework. Second there is a reasonably large amount of evidence indicating that there are substantial differences between job losers and job leavers. One is that on average income gains experienced by job leavers are greater than those experiienced by job losers.[5] A second is that the duration of unemployment for job leavers is considerably less on average than it is for job losers. (See Tables 5,6). Third, the ratio of quits to layoffs (which is not identical to the measure we have been discussing) has for many years shown a procyclical movement. As early as 1942 one author wrote: "The ratio of quits to total separations constitutes one of the most sensitive indexes of the labour market."[6]

The main point of this section is that in addition to requiring theories of the aggregate labour market to account for fluctuations in the level of employment they should also account for the nature of the fluctuations in unemployment. The distinction of job losers and job leavers seems to be one possibility. An alternative used by Jovanovic [3] was based on the fact that permanent separations are procyclical and temporary separations are countercyclical.

It should also be mentioned that the concept of involuntary unemployment has two different aspects. One, which has been stressed here, is the asymmetric treatment of individuals. The other, emphasized by Keynes, is that the aggregate level of employment is too low. Chari [2] shows how this second feature may be displayed when private information is introduced.

SECTION 3

THE ENVIRONMENT

The environment used here is a one state version of that used in Rogerson [8]. The economy lasts for two periods. There is a continuum of identical agents with names in [0,1]. There are three commodities output, labour and capital. Output is produced in two distinct sectors. Let

$$f_i^k(K,L): R_+ \times R_+ \to R_+$$

be the production function for sector k in period i, where i,k=1,2. K is the input of capital and L is the input of labour measured in man-hours of labour. For the same reasons as in [8] it is convenient to assume that

$$f_1^2(K,L) = 0 \text{ for all } K,L \geq 0.$$

For the remainder of the $f_i^k(K,L)$ it will be assumed that they satisfy:

(i) homogeneity of degree one in (K,L)

(ii) weakly concave in (K,L) jointly, strictly concave in each of K and L individually for a fixed value of the other argument.

(iii) $f_i^k(0,0) = 0$.

Capital is sector specific and cannot be accumulated or transformed and does not depreciate. Each agent (or worker) is endowed with one unit of each type of capital in both periods. Each individual is also endowed with one unit of time in both periods. Any fraction of this unit of time may be supplied as labour with the following restrictions:

(i) labour cannot be supplied in both sectors simultaneously

(ii) labour can only be supplied in sector two in period two and requires that the worker be idle in period one and suffer a psychic cost m associated with locating and adjusting to a new job.

The nature of the above restriction is such that it will be useful to distinguish formally between labour and capital supplied in sector one and labour and capital supplied in sector two. There will then be ten commodities: labour in two sectors in each of two periods, capital in each sector in each of two periods and output in each of two periods. We will use the following system to index commodities in each state of nature:

commodity 1 = output in period one

commodity 2 = output in period two

commodity 3 = labour in sector one in period one

commodity 4 = labour in sector two in period one

commodity 5 = labour in sector one in period two

commodity 6 = labour in sector two in period two

commodity 7 = capital in sector one in period one

commodity 8 = capital in sector two in period one

commodity 9 = capital in sector one in period two

commodity 10 = capital in sector two in period two

We can now proceed to define the economy more precisely, beginning with consumption sets. First define:

$$\bar{X} = \{x \in R^{10}: x_1 \geq 0,\ x_2 \geq 0,\ -1 \leq x_i \leq 0,\ i=3,4,\ldots,10,$$
$$x_4 = 0,\ x_5 \cdot x_6 = 0,\ x_3 \cdot x_6 = 0$$

The restriction $x_4 = 0$ implies labour cannot be supplied to sector two in period one; $x_5 \cdot x_6 = 0$ implies that labour cannot be supplied in both sectors in period 2; and $x_3 \cdot x_6 = 0$ implies that a worker must be idle in period one in order to work in sector two in period two.

The set $\bar{X}$ is non-convex. As shown in [7], in such cases equilibrium allocations can display undesirable properties. As a result it proves to be desirable to expand the consumption set to include lotteries. We therefore define the following sets:

$$X^1 = x \in \bar{X}: x_6 = 0$$
$$X^2 = x \in \bar{X}: x_6 \neq 0$$

Now define the consumption set X by:

$$X = X^1 \times X^2 \times [0,1]$$

The interpretation is that a worker chooses an allocation contingent upon remaining in sector one, an allocation contingent upon moving to sector two, and a probability of remaining in sector one. Note that X is convex. An element $x \in X$ will be written

$$x = (x^1, x^2, \phi) \text{ where } x^i \in X^i,\ i=1,2,\ \phi \in [0,1]$$

and

$x^i = (x_1^i,\ldots,x_{10}^i)$.

We now describe preferences. Define a function

$\overline{U}: \overline{X} \to R$ by

$$\overline{U}(x_1,\ldots,x_{10}) = \begin{cases} u(x_1)+u(x_2)+v(x_3)+v(x_5) & \text{if } x_6=0 \\ u(x_1)+u(x_2)+v(0)+v(x_6)-m & \text{if } x_6\neq 0 \end{cases}$$

where $m \geq 0$ and $u: R_+ \to R$ and $v: [-1,0] \to R$ satisfy:

(i) u,v strictly increasing,

(ii) u,v strictly concave.

Now, each agent has preferences over X specified by the utility function:

$$U: X \to R$$

specified by

$$U(x^1,x^2,\phi) = \phi\overline{U}(x^1) + (1-\phi)\overline{U}(x^2)$$

Using the same indexing system we can define a technology set by:

$$Y = \{y \in R^{10}: y_1 \geq 0,\ y_2 \geq 0,\ y_j \leq 0,\ j=3,4,\ldots,10,$$
$$y_1 \leq f_1^1(-y_3,-y_7),\ y_2 \leq f_2^1(-y_5,-y_9) + f_2^2(-y_6,-y_{10})\}$$

The economy described above is completely described by the following list:

$$\mathcal{E} = (X,u(\cdot),v(\cdot),m,Y)$$

As this model is virtually identical to that in [8] the reader is referred there for a discussion of the model's various features.

SECTION 4

EQUILIBRIUM AND OPTIMALITY

In this section we prove existence and optimality of equilibrium and that equilibrium is unique. The extension to include preferences which are strictly convex in consumption necessitates an alternative approach to that used in [8]. The problem arises due to the fact that the methods used there to show that the first order conditions had a unique solution do not work in the present situation. We begin with some definitions:

Definition: An allocation for $\mathcal{E}$ is a list (x,y) where $x\colon [0,1] \to R^{10}$ is measurable and $y \in R^{10}$.

The interpretation is as follows:

For each $t \in [0,1]$ $x(t)$ gives a complete description of agent t's allocation. We will write $x(t) = (x^1(t), x^2(t), \phi(t))$. y is the production vector.

Definition: A price system for $\mathcal{E}$ is a vector $p \in S_+^{10}$ where S_+^{10} is the subset of the unit simplex in R^{10} for which all components are non-negative.

We will write $p = (p_1,\ldots,p_{10})$. Equilibrium is then defined as:

Definition: A competitive equilibrium (CE) for $\mathcal{E}$ is a list (x,y,p) where (x,y) is an allocation for $\mathcal{E}$, p is a price system for $\mathcal{E}$ and

(i) for each $t \in [0,1]$, $x(t)$ is a solution to:

$$\begin{aligned} &\max_{x} U(x) \\ &\quad \text{s.t.} \quad x \in X \\ &\qquad\qquad p(\phi x^1+(1-\phi)x^2) \leq 0 \end{aligned}$$

(ii) y is a solution to:

$$\begin{aligned} &\max_{y} \; py \\ &\quad \text{s.t.} \; y \in Y \end{aligned}$$

(iii) $\int_0^1 [\phi(t)x^1(t) + (1-\phi(t))x^2(t)]dt \leqq y$

The interpretation of the above conditions is straightforward.

In [8] attention was devoted entirely to analysis of symmetric equilibria. The reason for this was twofold: Analysis of symmetric equilibria was much simpler, and symmetric and non-symmetric equilibria had the same aggregate properties. In the environment under consideration here a much stronger property than the latter exists: Any equilibrium will necessarily be a symmetric equilibrium. Formally we have the following:

Proposition 1: *If* (x,y,p) *is a* CE *for* $\mathcal{E}$ *then* x *is constant.*

Proof: See appendix.

Now consider the following problem.

(P-3) $$\max_{x,y} U(x)$$
$$\text{s.t. } x \in X$$
$$\phi x^1 + (1-\phi)x^2 \leq y.$$
$$y \in Y$$

The following is a standard result and hence we do not include the proof here.

<u>Proposition 2</u>: *If* (x,y p) *is a* CE *for* E *then* (x,y) *is a solution to* (P-3).

The following is also straightforward:

<u>Proposition 3</u>: *Problem* (P-3) *has a unique solution.*

<u>Proof</u>: Existence comes from the Weierstraas theorem and uniqueness is proven in exactly the same fashion as lemma two in the appendix.

The above two propositions together imply that if equilibrium exists, then the equilibrium allocation is unique. Hence we have still to demonstrate that equilibrium exists. The standard way of doing this is to show that the multipliers associated with problem (P-3) can be used as prices. However, this does not work here because (P-3) is not a concave programming problem and hence we cannot use results connecting maxima and saddlepoints. Alternatively, as was done in [8], we could show that the first order conditions for (P-3) include those for the individual agent's problem, using the multipliers as prices. However, this approach requires that the solution to the first order conditions be unique. Because I have been unable to prove this latter result, we take a slightly more abstract approach to the existence problem. The following is shown in the appendix.

Proposition 4: *A* CE *for* $\mathbf{E}$ *exists.*

SECTION 5

EQUILIBRIUM AND INVOLUNTARY UNEMPLOYMENT

In this section we define a concept of involuntary unemployment which is intended to coincide with the notion of a job loser and demonstrate that involuntary unemployment may exist in equilibrium. Since all equilibria are symmetric we will only concern ourselves with symmetric allocations here. Formally we have:

Definition: A symmetric allocation (x,y) for $\mathcal{E}$ is said to display involuntary unemployment if $\phi \in (0,1)$ and $\overline{U}(x^1) > \overline{U}(x^2)$.

The interpretation of the above is straightforward. The condition $\phi \in (0,1)$ implies that some workers will remain in sector one and some will become unemployed en route to sector two. The second part of the definition stipulates that ex post workers who remain are better off than workers who become unemployed. As mentioned above, this is meant to capture the notion of a job loser. This definition will be discussed in the next section.

The remainder of this section will be devoted to illustrating, by way of an example that equilibrium allocations for $\mathcal{E}$ may display involuntary unemloyment.

It is a difficult task to compute equilibrium allocations for $\mathcal{E}$

directly, since finding "a" solution to the first order conditions is not sufficient. However, we know that if $u(\cdot)$ was linear then this problem will not occur. Hence, the strategy employed will be to consider a sequence of economies $\{\mathcal{E}_\alpha, \alpha>0\}$ such that the $\mathcal{E}_\alpha$ are identical except that the $u(\cdot)$ functions are indexed by α, $u(\cdot,\alpha)$. Then, as α tends to zero the $u(\cdot,\alpha)$ converge uniformly to a linear function with slope one passing through the origin. We can easily calculate equilibrium allocations for this limit economy using the results of [8], and since the solution to (P-3) will be continuous in α, if we demonstrate existence of involuntary unemployment in the limit economy we have also demonstrated that involuntary unemployment occurs in equilibrium for each of the $\mathcal{E}_\alpha$ with $\alpha < \bar{\alpha}$ for some $\bar{\alpha}$.

Consider the following features of the limit economy:

$$f_2^2(K,H) = K + \theta H \qquad \theta > 0$$
$$v(h) = h + \frac{b}{2}h^2, \qquad b < 0$$

It turns out that for our purposes we will not need to specify the technology in sector one explicitly. Implicitly, we assume that the technology in sector one is constant through time and is such that the equilibrium value of ϕ is in the open interval (0,1). It is clear that the arguments in [8] would continue to hold for the above specification (assuming technology is such as to generate an interior solution for ϕ). The following first order conditions from [8] are sufficient for our purposes:

(1) $\quad v'(-h_2) = \theta$

$$(2) \quad 2[v'(-h_1)h_1 + v(-h_1)] = v'(-h_2)h_2 + v(-h_2) + v(0) - m$$

where h_1 is hours/worker in sector 1 in both periods and h_2 is hours/worker in sector 2 in period 2. Substitution gives:

$$(3) \quad h_2 = \frac{1-\theta}{b}$$

$$(4) \quad h_1 = (h_2^2/2 + m/b)^{1/2}$$

Since consumption is constant across individuals we need only evaluate the expression:

$$(5) \quad 2v(-h_1) - v(-h_2) - v(0) + k.$$

If this expression is positive then we have involuntary unemployment according to the definition given above. First note that from (3) and (4) the above expression is increasing in m. This follows from the fact that h_2 is independent of m, h_1 is decreasing in m (recall $b < 0$) and $v(\cdot)$ is monotone. Hence for given values of θ and b it follows that if $m = \bar{m}$ results in involuntary unemployment in equilibrium then the same holds for any $m > \bar{m}$ (assuming ϕ remains greater than zero).

It is straightforward to verify that if we take:

$b = -1$

$\theta = 3/2$

$m = 1$

that the solution to (3) and (4) is such that (5) is positive. We summarize this as:

Proposition 5: *There exists an equilibrium for* Σ *which displays involuntary unemployment.*

The involuntary unemployment that occurs in this model exists even though markets are complete, markets clear, there is no private information and all agents behave competitively. It illustrated one possible factor which may cause unemployment to be involuntary as defined in the previous section. The functional form used, in which m enters separately from c and h assumes that looking for alternative employment is an activity intrinsically different (and hence not expressed in terms of) from either consuming or working. Hence, individuals who experience m are not compensated directly in terms of consumption as would happen if we considered utility of the form u(c-am).

CONCLUSION

The paper has three aspects. First, it extended the analysis of Rogerson [8] to the case where preferences are strictly convex. Second, it argued that data on job leavers and job losers is one way of identifying voluntary and involuntary unemployment statistically. Third, in the context of a specific model it has identified involuntary unemployment as a property of an allocation, not an equilibrium, and shown that in this model equilibrium allocations may display involuntary unemployment.

APPENDIX

Proof of Proposition 1

The proposition will be proved as a series of lemmas, as many of the intermediate results established here will be used later. Consider the worker's problem rewritten using a more suggestive notation:

$$\text{(P-1)} \quad \max_{c_{ij},h_{ij},\phi} \ \phi[u(c_{11}) + u(c_{12}) + v(-h_{11}) + v(-h_{12})] + (1-\phi)[u(c_{21}) + u(c_{22}) + v(0) + v(-h_{22}) - m]$$

$$\text{s.t.} \quad 0 \leq h_{ij} \leq 1 \qquad i,j = 1,2$$

$$0 \leq \phi \leq 1$$

$$c_{ij} \geq 0 \qquad i,j = 1,2$$

$$\phi[p_1c_{11} + p_2c_{12}] + (1-\phi)[p_1c_{21} + p_2c_{22}] \leq I + \phi[w_{11}h_{11} + w_{12}h_{12}] + (1-\phi)w_{22}h_{22}.$$

The notation is as follows: c_{ij}, and h_{ij} are consumption and hours of work in period j contingent upon being in sector i; p_j is the price of output in period j; w_{ij} is the wage in period j in sector i; and I is income from capital.

Lemma 1: If (c_{ij},h_{ij},ϕ) is a solution to (P-1) then $c_{ij} = \bar{c}_i$, all i,j.

Proof: Suppose this doesn't hold for some i. Define $\bar{c}_{ij}$ by:

$$\bar{c}_{ij} = \frac{1}{2}(c_{i1} + c_{i2}) \qquad i,j=1,2.$$

Consider $(\bar{c}_{ij}, h_{ij}, \phi)$. This satisfies the constraints and by strict concavity of $u(\cdot)$ this dominates the alternative. //

We can now consider the following problem:

$$\text{(P-2)} \qquad \max_{c_i, h_{ij}, \phi} \; u(c_1) + u(c_2) + \phi[v(-h_{11}) + v(-h_{12})] + (1-\phi)[v(-h_{22}) + v(0) - m]$$

$$\text{s.t.} \qquad 0 \le h_{ij} \le 1 \qquad i,j = 1,2$$

$$0 \le \phi \le 1$$

$$c_i \ge 0, \; i=1,2.$$

$$p_1c_1 + p_2c_2 \le I + \phi[w_{11}h_{11} + w_{12}h_{12}] + (1-\phi)w_{22}h_{22}$$

Lemma 2: If a solution exists to (P-2), then it is unique.

Proof: Assume that (c_i, h_{ij}, ϕ) and $(\bar{c}_i, \bar{h}_{ij}, \bar{\phi})$ are two distinct solutions. Define:

$$c_i^* = \frac{1}{2}(c_i + \bar{c}_i) \qquad i=1,2$$

$$h_{ij}^* = \frac{\phi}{\phi+\bar{\phi}} h_{ij} + \frac{\bar{\phi}}{\phi+\bar{\phi}} \bar{h}_{ij} \qquad j=1,2$$

$$h_{22}^* = \frac{1-\phi}{2-\phi-\bar{\phi}} h_{22} + \frac{(1-\bar{\phi})}{2-\phi-\bar{\phi}} \bar{h}_{22}$$

$$\phi^* = \frac{1}{2}(\phi+\bar{\phi})$$

We now show that $(c_i^*, h_{ij}^*, \phi^*)$ is feasible and dominates the other two solutions. It is enough to check the budget constraint to demonstrate feasibility. To see that this holds note that:

$$\phi^*[w_{11}h_{11}^* + w_{12}h_{12}^*] + (1-\phi^*)w_{22}h_{22}^* + I$$

$$= \frac{1}{2}(\phi+\bar{\phi})[w_{11}(\frac{\phi}{\phi+\bar{\phi}} h_{11} + \frac{\bar{\phi}}{\phi+\bar{\phi}} \bar{h}_{11}) + w_{12}(\frac{\phi}{\phi+\bar{\phi}} h_{12} + \frac{\phi}{\phi+\bar{\phi}} \bar{h}_{12})]$$

$$+ \frac{1}{2}(2-\phi-\bar{\phi})[w_{22}(\frac{1-\phi}{2-\phi-\bar{\phi}} h_{22} + \frac{1-\bar{\phi}}{2-\phi-\bar{\phi}} \bar{h}_{22})] + I$$

$$= \frac{1}{2}[w_{11}(\phi h_{11} + \bar{\phi}\bar{h}_{11}) + w_{12}(\phi h_{12} + \bar{\phi}\bar{h}_{12})$$

$$+ w_{22}((1-\phi)h_{22} + (1-\bar{\phi})\bar{h}_{22})] + I$$

$$= \frac{1}{2}[\phi(w_{11}h_{11} + w_{12}h_{12}) + (1-\phi)w_{22}h_{22}]$$

$$+ \frac{1}{2}[\bar{\phi}(w_{11}\bar{h}_{11} + w_{12}\bar{h}_{12}) + (1-\bar{\phi})w_{22}\bar{h}_{22}] + I$$

$$= \frac{1}{2}(p_1c_1 + p_2c_2) + \frac{1}{2}(p_1\bar{c}_1 + p_2\bar{c}_2) = p_1c_1^* + p_2c_2^*$$

This shows that the budget constraint is satisfied. Note that we have also shown that

$$\phi^* h^*_{ij} = \frac{1}{2}(\phi h_{ij} + \bar{\phi}\bar{h}_{ij}) \qquad j=1,2$$

$$(1-\phi^*)h^*_{22} = \frac{1}{2}((1-\phi)h_{22} + (1-\bar{\phi})\bar{h}_{22})$$

To show that the starred allocation dominates the other two note that if

$$U_1 = u(c_1) + u(c_2) + \phi[v(-h_{11}) + v(-h_{12})] + (1-\phi)[v(-h_{22}) + v(0) - m]$$

$$U_2 = u(\bar{c}_1) + u(\bar{c}_2) + \bar{\phi}[v(-\bar{h}_{11}) + v(-\bar{h}_{12})] + (1-\bar{\phi})[v(-\bar{h}_{22}) + v(0) - m]$$

then $U_1 = U_2$ and

$$\frac{1}{2}U_1 + \frac{1}{2}U_2 \leq u(c^*_1) + u(c^*_2)$$

$$+ \frac{1}{2}(\phi+\bar{\phi})[\frac{\phi}{\phi+\bar{\phi}}\{v(-h_{11}) + v(-h_{12})\} + \frac{\bar{\phi}}{\phi+\bar{\phi}}\{v(-\bar{h}_{11}) + v(-\bar{h}_{12})\}]$$

$$+ \frac{1}{2}(2-\phi-\bar{\phi})[\frac{1-\phi}{2-\phi-\bar{\phi}}\{v(-h_{22}) + v(0) - m\} + \frac{1-\bar{\phi}}{2-\phi-\bar{\phi}}\{v(-\bar{h}_{22}) + v(0) - m\}]$$

$$\leq u(c^*_1) + u(c^*_2) + \phi^*[v(-h^*_{11}) + v(-h^*_{12})] + (1-\phi^*)[v(-h^*_{22}) + v(0) - m]$$

with at least one of the two inequalities strict since the two original allocations were distinct. This completes the proof. //

Proof of Proposition 4

We only sketch the proof here. The argument is that standard existence proofs (e.g. Debreu [ ]) can be used. The fact that we are only looking for a symmetric equilibrium (any equilibrium will be symmetric) the economy is formally identical to a single agent economy. Essentially then, we need only assure that the excess demand correspondence has the appropriate properties. Since the production side of the economy is standard, it creates no difficulties. On the worker side, the only potential problem is in guaranteeing that the excess demand is convex-valued, since the upper contour sets of the objective are not convex sets. However, we have demonstrated previously that demand is single valued, and hence convex-valued. This completes a sketch of the proof. //

TABLE 1: Composition of Unemployment for U.S.

	Unemployment Rate	Job Losers	Job Leavers	Re-entrants	New Entrants
69	3.4	35.9	15.4	34.1	14.6
70	4.8	44.3	13.4	30.0	12.3
71	5.8	46.3	11.8	29.4	12.5
72	5.5	43.1	13.1	29.8	13.9
73	4.8	38.7	15.7	30.7	14.9
74	5.5	43.5	14.9	28.4	13.2
75	8.3	55.4	10.4	23.8	10.4
76	7.5	49.8	12.2	26.0	12.1
77	6.9	45.2	13.0	28.1	13.7
78	5.9	41.5	14.1	30.0	14.3
79	5.7	42.8	14.3	29.5	13.4
80	7.0	51.9	11.6	25.2	11.4
81	7.5	51.7	11.1	25.4	11.9

Source: BLS Employment and Earnings, various issues

TABLE 2: Composition of Unemployment for Canada

	Unemployment Rate	Job Losers	Job Leavers	Re-entrants	New Entrants
75	6.9	40	28	6	26
76	7.1	45	24	6	25
77	8.0	49	23	6	23
78	8.3	48	23	6	23
79	7.4	47	22	4	25
80	7.5	48	21	5	25
81	7.5	48	21	5	26
82	11.0	59	15	4	21

Source: Statistics Canada

TABLE 3: Composition of Unemployment for West Germany

	Unemployment Rate	Job Losers	Job Leavers	New Entrants
68	1.2	65.3	11.9	3.4
69	0.7	54.1	26.7	3.4
70	0.6	42.2	28.5	4.9
71	0.7	45.3	35.2	4.1
73	1.0	28.3	26.2	19.9
75	4.0	62.3	9.8	12.9
77	3.9	42.3	12.4	14.2
79	3.3	32.2	15.8	17.4

Source: Eurostat

TABLE 4: Composition of Unemployment for France

	Unemployment Rate	Job Losers	Job Leavers	New Entrants
68	2.1	45.3	20.8	18.5
69	2.3	43.7	20.8	16.0
70	2.4	39.8	23.1	20.0
71	2.6	41.8	22.5	17.4
73	2.6	38.4	20.6	20.0
75	4.1	43.1	19.0	14.4
77	4.7	41.1	17.0	17.9
79	5.9	43.5	15.3	15.8

Source: Eurostat

TABLE 5: Distribution of Duration of Unemployment, Job Losers, U.S.

	Less than 5 weeks	5-14 weeks	15 weeks and over
70	44.6	34.7	20.7
71	36.3	32.7	31.0
72	36.9	30.8	32.3
73	42.5	32.1	25.5
74	43.5	32.9	23.6
75	29.3	32.0	38.7
76	29.7	28.3	42.0
77	33.6	30.3	36.2
78	38.6	32.1	29.3
79	41.0	33.7	25.3
80	35.4	33.6	31.0
81	34.8	30.9	34.2

Source: BLS Employment and Earnings, various issues

TABLE 6: Distribution of Duration of Unemployment, Job Leavers, U.S.

	Less than 5 weeks	5-14 weeks	15 weeks and over
70	57.3	28.5	14.3
71	46.3	32.5	21.2
72	50.6	29.9	19.5
73	55.3	28.3	16.3
74	53.2	30.6	16.3
75	41.1	29.7	29.1
76	41.9	29.8	28.3
77	45.7	29.7	24.8
78	50.1	30.3	19.6
79	49.8	32.5	17.7
80	47.5	31.6	21.0
81	45.1	31.7	23.2

Source: BLS, Employment and Earnings, various issues.

FOOTNOTES

1. Lucas, R., "Unemployment Policy," AER, May 1978, page 354.

2. Solow, R., "On Theories of Unemployment, " AER, 1980, page 3.

3. Keynes, J.M., "General Theory of Employment, Interest and Money," Harcourt Brace, New York, 1935, page 15.

4. Beveridge, W., "Unemployment: A Problem of Industry, " Langman, Greens and Co., New York, 1930, page 115.

5. OECD, "Urban Mobility," Chapter 5.

6. Woytinsky, "Three Aspects of Labour Dynamics, Report Prepared for Commission on Social Security, Social Science Council, 1942, page 52.

References

[1] Beveridge, W., "Unemployment: A Problem of Industry," Langman, Greens and Co., New York, 1930.

[2] Chari, V., "Involuntary Unemployment and Implicit Contracts," QJE, Supplement 1983, 107-122.

[3] Jovanovic, B., "Turnover, Unemployment and the Cycle," unpublished paper, Bells Labs., 1983.

[4] Keynes, J., "General Theory of Employment, Interest and Money," Harcourt Brace, New York, 1935.

[5] Lucas, R., "On Unemployment Policy," AER, 1977.

[6] OECD, "Urban Mobility," 1965.

[7] Rogerson, R., "Indivisible Labour, Lotteries and Equilibrium," unpublished paper, University of Minnesota, 1984.

[8] Rogerson, R., "Hours Per Worker, Employment, Unemployment and Duration of Unemployment: An Equilibrium Model," unpublished paper, University of Minnesota, 1984.

[9] Solow, R., "On Theories of Unemployment," AER, 70(1980), 1-11.

[10] Woytinsky, W., "Three Aspects of Labour Dynamics," Report prepared for Commission on Social Security, Social Science Research Council, 1942.

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